Visual Rendering

• Converting 3D scene descriptions into 2D images

• The graphics pipeline
Vertex Transform

\[
\mathbf{v}_{\text{window}} = \begin{pmatrix}
\frac{x_{\text{window}}}{z_{\text{window}}} \\
\frac{y_{\text{window}}}{z_{\text{window}}} \\
1
\end{pmatrix} \in \left(0, \frac{\text{width}}{z_{\text{window}}}, 0, \frac{\text{height}}{z_{\text{window}}}, 1\right)
\]

vertex in window coords
Rasterization

- Determine which pixels are inside the triangles
- Interpolate vertex attributes (e.g., color)
Pixels vs. Fragments

• Pixels are dots on the screen: (x, y) and RGB color
• Fragments: (x, y, z), z is the depth and other attributes (color, normal, texture coordinates, alpha value, etc.)
Rasterization

• Determine which fragments are inside the triangle

\[ e_1 = p_2 - p_1 \]
\[ e_2 = p_3 - p_2 \]
\[ e_3 = p_1 - p_3 \]

\( p \) is inside if and only if

\[ (p - p_1) \times e_1 < 0 \]
\[ (p - p_2) \times e_2 < 0 \]
\[ (p - p_3) \times e_3 < 0 \]

magnitude of the cross products
Barycentric Coordinates

- Interpolate attributes of the vertices

\[ p = \alpha_1 p_1 + \alpha_2 p_2 + \alpha_3 p_3 \]

\[ 0 \leq \alpha_1, \alpha_2, \alpha_3 \leq 1 \]

\[ \alpha_1 + \alpha_2 + \alpha_3 = 1 \]
Barycentric Coordinates

\[
p_1 = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} \quad p_2 = \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} \quad p_3 = \begin{bmatrix} x_3 \\ y_3 \\ z_3 \end{bmatrix} \quad p = \begin{bmatrix} x \\ y \\ z \end{bmatrix}
\]

\[
p = \alpha_1 p_1 + \alpha_2 p_2 + \alpha_3 p_3
\]

\[
0 \leq \alpha_1, \alpha_2, \alpha_3 \leq 1 \quad \alpha_1 + \alpha_2 + \alpha_3 = 1
\]

\[
\begin{align*}
\alpha_1 &= \frac{(y_2 - y_3)(x - x_3) + (x_3 - x_2)(y - y_3)}{(y_2 - y_3)(x_1 - x_3) + (x_3 - x_2)(y_1 - y_3)}, \\
\alpha_2 &= \frac{(y_3 - y_1)(x - x_3) + (x_1 - x_3)(y - y_3)}{(y_2 - y_3)(x_1 - x_3) + (x_3 - x_2)(y_1 - y_3)}, \\
\alpha_3 &= 1 - \alpha_1 - \alpha_2.
\end{align*}
\]

https://en.wikipedia.org/wiki/Barycentric_coordinate_system
Barycentric Coordinates

\[ p = \alpha_1 p_1 + \alpha_2 p_2 + \alpha_3 p_3 \]

Color

\[
\begin{align*}
R &= \alpha_1 R_1 + \alpha_2 R_2 + \alpha_3 R_3 \\
G &= \alpha_1 G_1 + \alpha_2 G_2 + \alpha_3 G_3 \\
B &= \alpha_1 B_1 + \alpha_2 B_2 + \alpha_3 B_3
\end{align*}
\]

Apply to other attributes, e.g., depth, texture coordinates, alpha value, etc.
Depth Buffer for Visibility Testing

• When drawing multiple triangles, determine which one to draw and which one to discard

• If depth of fragment is smaller than the current value is the depth buffer, overwrite color and depth value using the current fragment
Lighting and Shading

• How to determine color and what attributes to interpolate after rasterization

Rasterization: determine which fragments are inside the triangles
Basic Behavior of Light

• Light can be described in three ways
  • Photons: tiny particles of energy moving through space at high speed
  • Waves: ripples through space
  • Rays: a ray traces the motion of a single hypothetical photon
Interactions with Materials

Reflection

Absorption

Transmission

Specular

Diffuse
Wavelengths and Colors

Wavelength: \( \lambda = \frac{v}{f} \)

Speed: meters per second

Frequency: how many cycles per second

Electromagnetic spectrum

<table>
<thead>
<tr>
<th>Radiation type</th>
<th>Radio waves</th>
<th>Microwaves</th>
<th>Infrared</th>
<th>Ultraviolet</th>
<th>X-rays</th>
<th>Gamma rays</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wavelength (approximate)</td>
<td>30 mm</td>
<td>1 mm</td>
<td>10 nm</td>
<td>0.01 nm</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Visible light

700 nm, 600 nm, 500 nm, 400 nm
Reflection of Materials

• We see objects with different colors because the materials reflect specific colors differently
Lambertian Lighting

Think about this point as a vertex of a 3D mesh. We want to compute its color on the image.

Diffuse reflection

\[ R = d_R I_R \max(0, n \cdot \ell) \]
\[ G = d_G I_G \max(0, n \cdot \ell) \]
\[ B = d_B I_B \max(0, n \cdot \ell) \]
\[ n \cdot \ell = \cos \theta \]

Reflectance property of the material (triangle)

Spectral power distribution of the light source

\[ (d_R, d_G, d_B) \]
\[ (I_R, I_G, I_B) \]

\[ L = d I \max(0, n \cdot \ell) \]
\[ n \cdot \ell < 0 \]

Light behind triangle
Blinn-Phong Lighting

Related to specular reflection

\[ b = \frac{\ell + v}{\|\ell + v\|} \]

Material property that expresses the amount of surface shininess

- \( x = 100 \), mild amount of shininess
- \( x = 10000 \), almost like a mirror

Specular reflectance property of the material

\[ L = dI \max(0, n \cdot \ell) + sI \max(0, n \cdot b)^x \]

Think about this point as a vertex of a 3D mesh. We want to compute its color on the image.
Ambient Lighting

• Independent of light/surface position, viewer, normal

• Adding some background color

\[ L = dI \max(0, n \cdot \ell) + sI \max(0, n \cdot b)^x + L_a \]

Ambient light
Multiple Light Sources and Attenuation

• N light sources

\[ L = L_a + \sum_{i=1}^{N} dI_i \max(0, n \cdot l_i) + sI_i \max(0, n \cdot b_i)^x \]

• Attenuation: the greater the distance, the low the intensity

\[ L = L_a + \sum_{i=1}^{N} \frac{1}{k_c + k_l c + k_q c^2} \left( dI_i \max(0, n \cdot l_i) + sI_i \max(0, n \cdot b_i)^x \right) \]

\( c \) Light source distance to surface

constant linear quadratic attenuation

Used by OpenGL for ~25 years
Phong Reflection Model

Ambient + Diffuse + Specular = Phong Reflection
Bidirectional Reflectance Distribution Function (BRDF)

For Lambertian shading, BRDF is a constant
- The surface reflects equally in all directions

Shading in a more precise and general way

\[ f(\theta_i, \phi_i, \theta_r, \phi_r) = \frac{\text{radiance}}{\text{irradiance}} \]

- Radiance: light energy reflected from the surface
- Irradiance: light energy arriving at the surface

For Lambertian shading, BRDF is a constant
- The surface reflects equally in all directions
Lighting Calculations

• All lighting calculations can happen in world space
  • Transform vertices and normal into world space

• Calculate lighting, i.e., compute vertex color given material properties, light source color and position, vertex position, normal position, view position

Think about this point as a vertex of a 3D mesh. We want to compute its color on the image
Lighting vs. Shading

• Lighting: interaction between light and surface
  • Different mathematic models exist, e.g., Phong lighting model
  • What formula is being used to calculate intensity/color

• Shading: how to compute color for each fragment
  • What attributes to interpolate
  • Where to do lighting calculation
Flat Shading

• Compute color only once per triangle (i.e., with Phong lighting)
  • Compute color for the first vertex or the centroid

• Pro: fast to compute

• Con: create a flat, unrealistic appearance
Gouraud or Per-vertex Shading

- Compute color only once per vertex (i.e., with Phong lighting)
- Interpolate per-vertex color to all fragments within the triangle
- Pro: fast to compute
- Con: flat, unrealistic specular highlights
Gouraud or Per-vertex Shading
Phong Shading or Per-fragment Shading

• Compute color only once per fragment (i.e., with Phong lighting)
• Need to interpolate per-vertex normal to all fragments to do the lighting calculation
• Pro: better appearance of specular highlights
• Con: slower to compute
Shading

Flat Shading

Gouraud Shading

Phong Shading

http://www.decew.net/OSS/timeline.php
Shader

- Vertex shader
  - Lighting computation for each vertex

- Fragment shader
  - Lighting computation for each fragment
Shader

• Shaders are small programs that are executed in parallel on GPUs for each vertex (vertex shader) or each fragment (fragment shader).

• Vertex shader (before rasterization)
  • Modelview projection transform of vertex and normal
  • If per-vertex lighting, compute lighting for each vertex

• Fragment shader (after rasterization)
  • If per-vertex lighting, assign color to each fragment
  • If per-fragment lighting, compute lighting for each fragment
Texture Mapping

• Map textures (2D images) to 3D models

1. Without texture
   • Need to specify vertex colors

2. With texture
   • Vertex colors from texture
Texture Mapping

• UV coordinates (normalized)
Texture Mapping

• Same texture, different UV coordinates for mapping
Texture Mapping

- Texture filtering: the resolution of the texture image is different from the displayed fragment
  - Magnification
  - Minification

![Texture Mapping Diagram]

Texture image is larger
Texture Mapping
Review of the Graphics Pipeline

- **Vertex shader**
  - Vertex transforms
  - Per-vertex lighting

- **Fragment shader**
  - Texturing
  - Per-fragment lighting

- Combine the fragments of all primitives into 2D color-pixel for display
Further Reading

• 3D graphics with OpenGL, Basic Theory