Convolutional Neural Networks II

CS 6384 Computer Vision

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Supervised Learning

$\mathbf{x}$ $\Rightarrow$  $\Rightarrow$ $\mathbf{y}$

Image

Object class

$$f(\mathbf{x})$$

Training Data $\{\mathbf{x}_i, \mathbf{y}_i\}_{i=1}^{N}$

Input

Output

Convolutional Neural Networks

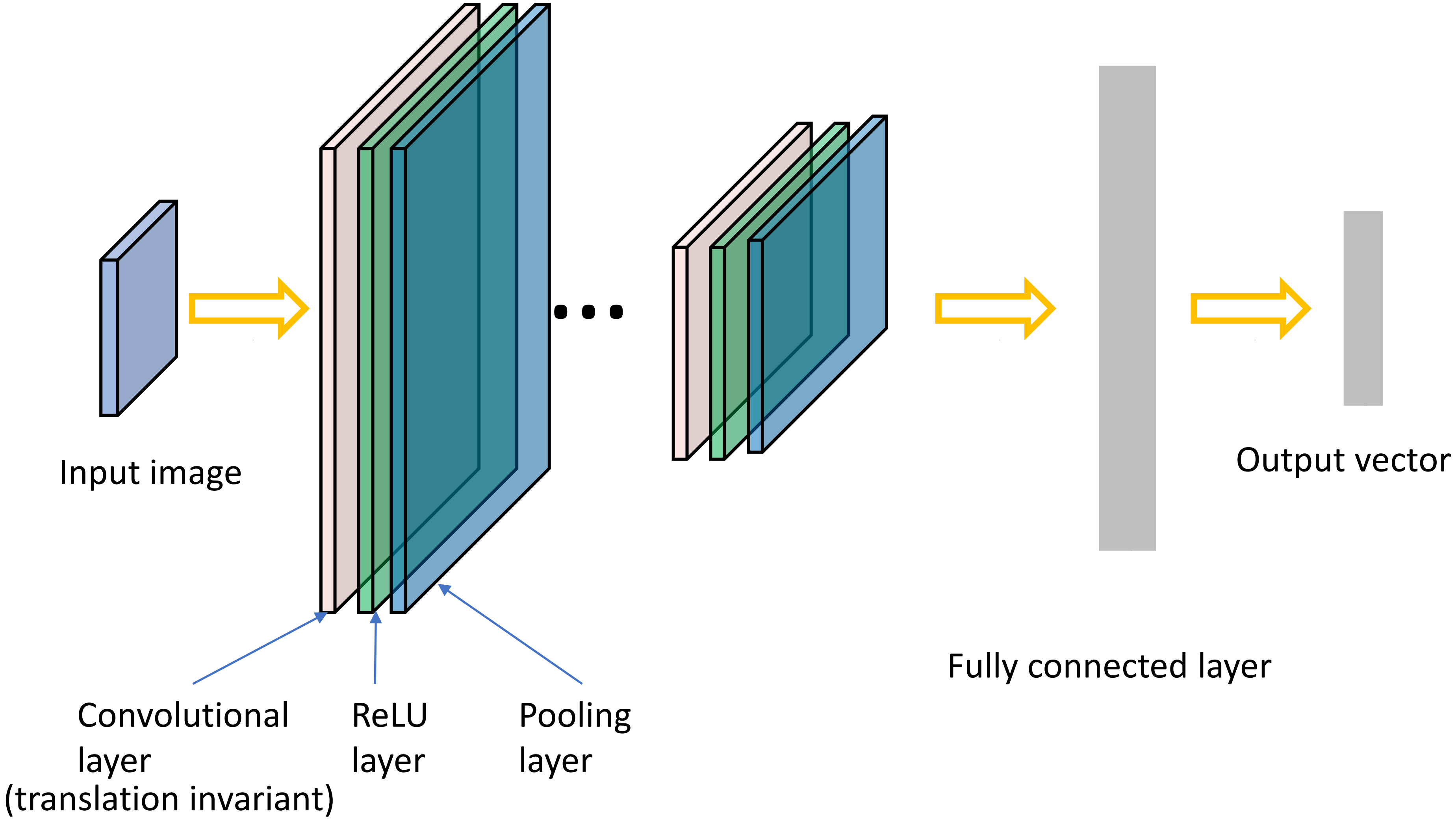


Image Classification

- ImageNet dataset
  - Training: 1.2 million images
  - Testing and validation: 150,000 images
  - 1000 categories

n02119789: kit fox, Vulpes macrotis
n02100735: English setter
n02096294: Australian terrier
n02066245: grey whale, gray whale, devilfish, Eschrichtius gibbosus, Eschrichtius robustus
n02509815: lesser panda, red panda, panda, bear cat, cat bear, Ailurus fulgens
n02124075: Egyptian cat
n02417914: ibex, Capra ibex
n02123394: Persian cat
n02125311: cougar, puma, catamount, mountain lion, painter, panther, Felis concolor
n02423022: gazelle

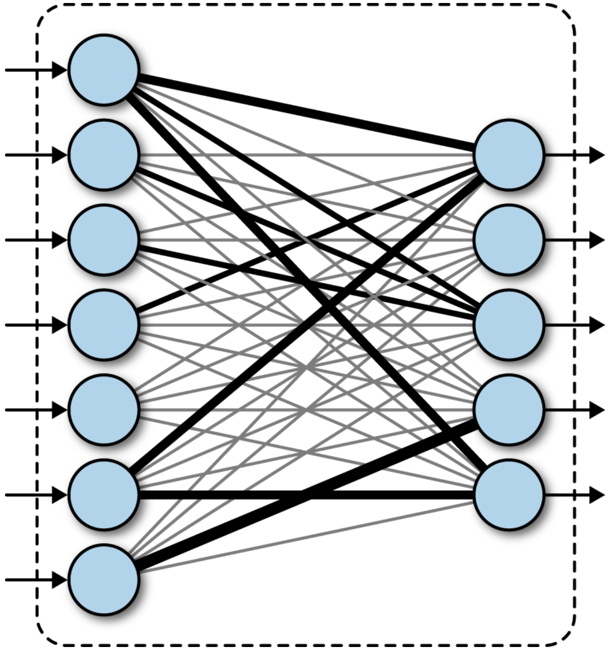
https://image-net.org/challenges/LSVRC/2012/index.php

Image Classification

Let's consider only using one FC layer

$224 \times 224 \times 3$

$W : m \times n$



$x : n \times 1$

150528

$y : m \times 1$

1000

$$\mathbf{y} = W\mathbf{x}$$

$\sigma(\mathbf{y})$ Probability distribution

Softmax function

$$\sigma(\mathbf{y})_i = \frac{e^{y_i}}{\sum_j^m e^{y_i}}$$

Image Classification

- Training data $\{\mathbf{x}_i, \mathbf{y}_i\}_{i=1}^{N}$

Image   label

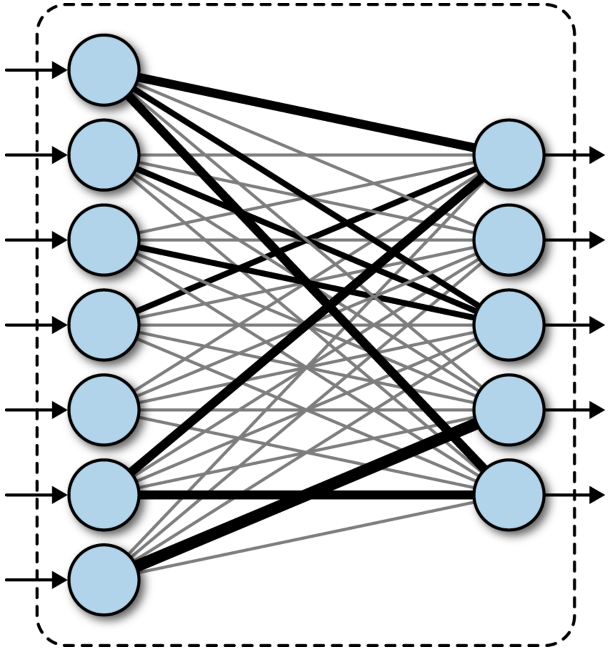
- One-hot vector $\mathbf{y}_i = 000\ldots1\ldots000$

Ground truth category

Image Classification

$224 \times 224 \times 3$

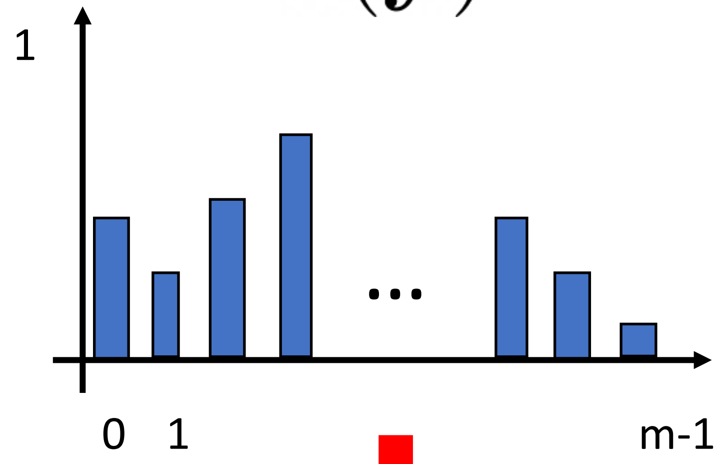
$W : m \times n$



$x : n \times 1$ $\quad$ $y : m \times 1$

$$\mathbf{y} = W\mathbf{x}$$

$\sigma(\mathbf{y})$



The goal of training

Ground truth

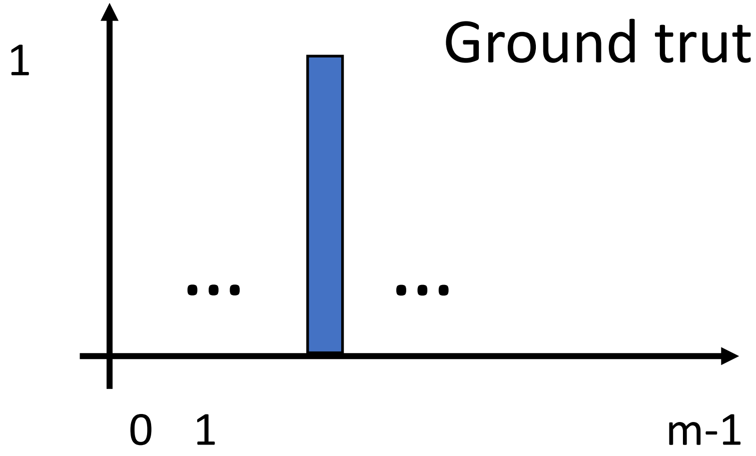


Image Classification

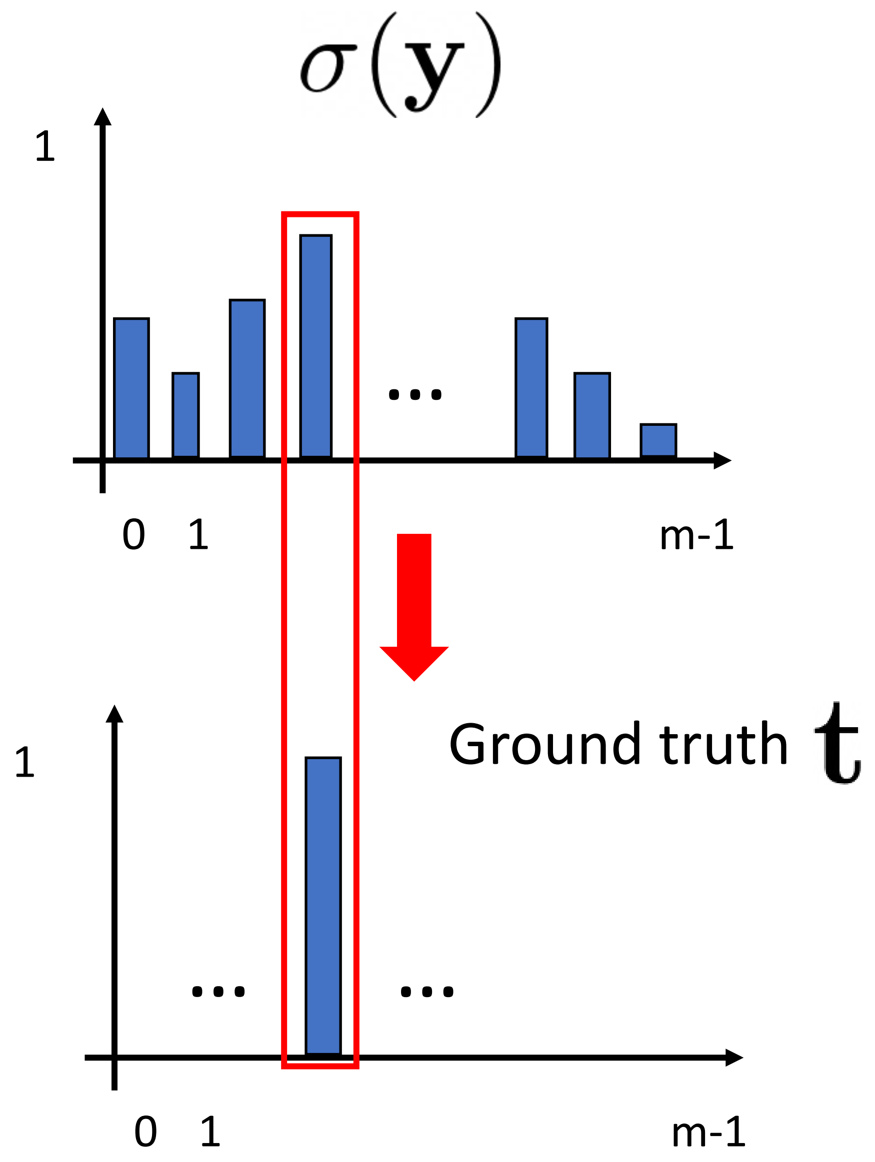
- Cross entropy loss function

Cross entropy between two distributions
(measure distance between distributions)

$$H(p,q) = -\operatorname{E}_p[\log q]$$

$$H(p,q) = -\sum_{x\in\mathcal{X}} p(x)\, \log q(x)$$

$$L_{CE} = -\sum_{i=0}^{m-1} t_i \log \sigma(\mathbf{y})_i$$



Training

- Cross entropy loss function

$$\mathbf{y} = W\mathbf{x}$$

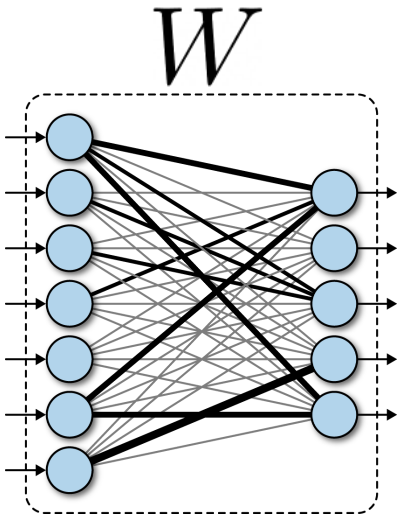
Minimize

$$L_{CE} = -\sum_{i=0}^{m-1} t_i \log \sigma(\mathbf{y})_i$$

$$\sigma(\mathbf{y})_i = \frac{e^{y_i}}{\sum_j^m e^{y_j}}$$

With respect to weights W

$\mathbf{x}$



$$\mathbf{y} \longrightarrow \sigma(\mathbf{y}) \longrightarrow L_{CE}$$

Training

- Gradient descent

$$W \leftarrow W - \gamma \frac{\partial L}{\partial W}$$

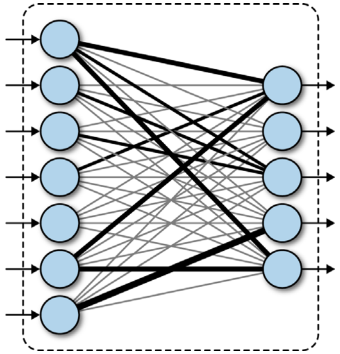
Learning rate

- Chain rule

$$\frac{\partial L}{\partial W} = \frac{\partial L}{\partial \sigma(\mathbf{y})} \frac{\partial \sigma(\mathbf{y})}{\partial \mathbf{y}} \frac{\partial \mathbf{y}}{\partial W}$$

W

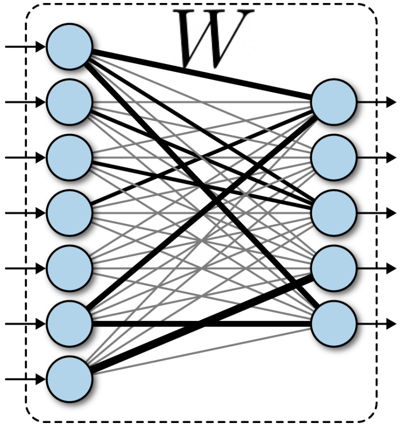
$\mathbf{x}$



$$\mathbf{y} \longrightarrow \sigma(\mathbf{y}) \longrightarrow L_{CE}$$

Training

- Gradient descent $$L_{CE} = -\sum_{i=0}^{m-1} t_i \log \sigma(\mathbf{y})_i = -\mathbf{t} \cdot \log \sigma(\mathbf{y})$$

$\mathbf{x}$  $\mathbf{y} \longrightarrow \sigma(\mathbf{y}) \longrightarrow L_{CE}$

How to compute gradient? $$\frac{\partial L}{\partial \mathbf{y}} \quad \left[\frac{\partial L}{y_1} \quad \frac{\partial L}{y_2} \quad \cdots \quad \frac{\partial L}{y_m} \right]$$

$$1 \times m$$

Training

$$L_{CE} = -\sum_{i=0}^{m-1} t_i \log \sigma(\mathbf{y})_i = -\mathbf{t} \cdot \log \sigma(\mathbf{y})$$

$$\sigma(\mathbf{y})_i = \frac{e^{y_i}}{\sum_j^m e^{y_i}}$$

- Chain rule

$$\frac{\partial L}{\partial \mathbf{y}} = \frac{\partial L}{\partial \sigma(\mathbf{y})} \cdot \frac{\partial \sigma(\mathbf{y})}{\partial \mathbf{y}}$$

$$1 \times m \qquad 1 \times m \qquad m \times m$$

$$\frac{\partial \mathbf{y}}{\partial \mathbf{x}} = \begin{bmatrix} \nabla f_1(\mathbf{x}) \\ \nabla f_2(\mathbf{x}) \\ \cdots \\ \nabla f_m(\mathbf{x}) \end{bmatrix} = \begin{bmatrix} \frac{\partial}{\partial \mathbf{x}} f_1(\mathbf{x}) \\ \frac{\partial}{\partial \mathbf{x}} f_2(\mathbf{x}) \\ \cdots \\ \frac{\partial}{\partial \mathbf{x}} f_m(\mathbf{x}) \end{bmatrix} = \begin{bmatrix} \frac{\partial}{\partial x_1} f_1(\mathbf{x}) & \frac{\partial}{\partial x_2} f_1(\mathbf{x}) & \ldots & \frac{\partial}{\partial x_n} f_1(\mathbf{x}) \\ \frac{\partial}{\partial x_1} f_2(\mathbf{x}) & \frac{\partial}{\partial x_2} f_2(\mathbf{x}) & \ldots & \frac{\partial}{\partial x_n} f_2(\mathbf{x}) \\ & & \cdots & \\ \frac{\partial}{\partial x_1} f_m(\mathbf{x}) & \frac{\partial}{\partial x_2} f_m(\mathbf{x}) & \ldots & \frac{\partial}{\partial x_n} f_m(\mathbf{x}) \end{bmatrix}$$

Jacobian matrix

$$\frac{\partial L}{\partial \sigma(\mathbf{y})} = -\mathbf{t} \cdot \frac{1}{\sigma(\mathbf{y})}$$

$$\frac{\partial \sigma(\mathbf{y})_i}{\partial y_j} = \sigma(\mathbf{y})_i(\delta_{ij} - \sigma(\mathbf{y})_j) \qquad \delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$$

https://eli.thegreenplace.net/2016/the-softmax-function-and-its-derivative/

Training

- Gradient descent

$$L_{CE} = -\sum_{i=0}^{m-1} t_i \log \sigma(\mathbf{y})_i = -\mathbf{t} \cdot \log \sigma(\mathbf{y})$$

$$\mathbf{y} = W\mathbf{x}$$

$$\frac{\partial L}{\partial W} = \frac{\partial L}{\partial \sigma(\mathbf{y})} \frac{\partial \sigma(\mathbf{y})}{\partial \mathbf{y}} \frac{\partial \mathbf{y}}{\partial W}$$

$$\frac{\partial L}{\partial \sigma(\mathbf{y})} = -\mathbf{t} \cdot \frac{1}{\sigma(\mathbf{y})}$$

$$\frac{\partial \sigma(\mathbf{y})_i}{\partial y_j} = \sigma(\mathbf{y})_i(\delta_{ij} - \sigma(\mathbf{y})_j)$$

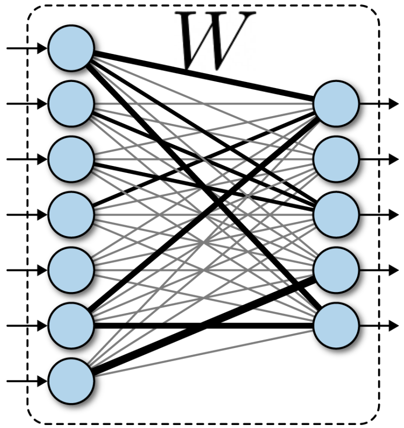
$$\delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$$

$$\frac{\partial y_i}{\partial W_{jk}} = \begin{cases} 0 & \text{if } i \neq j \\ x_k & \text{otherwise} \end{cases}$$

$$W \leftarrow W - \gamma \frac{\partial L}{\partial W}$$

Learning rate

Back-propagation

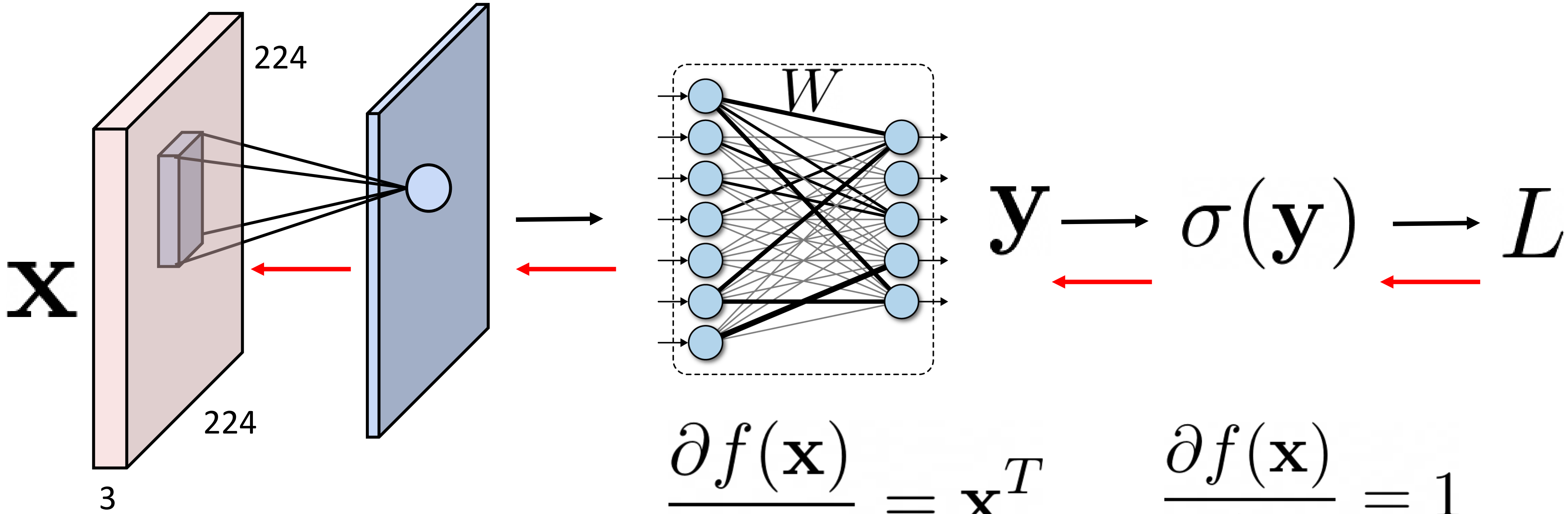


$$\mathbf{x} \qquad \mathbf{y} \longrightarrow \sigma(\mathbf{y}) \longrightarrow L$$

$$\frac{\partial \mathbf{y}}{\partial W} \qquad \frac{\partial \sigma(\mathbf{y})}{\partial \mathbf{y}} \qquad \frac{\partial L}{\partial \sigma(\mathbf{y})}$$

$$\frac{\partial L}{\partial W} = \frac{\partial L}{\partial \sigma(\mathbf{y})} \frac{\partial \sigma(\mathbf{y})}{\partial \mathbf{y}} \frac{\partial \mathbf{y}}{\partial W}$$

Back-propagation



$$\frac{\partial f(\mathbf{x})}{\partial \mathbf{w}} = \mathbf{x}^T \qquad \frac{\partial f(\mathbf{x})}{\partial b} = 1$$

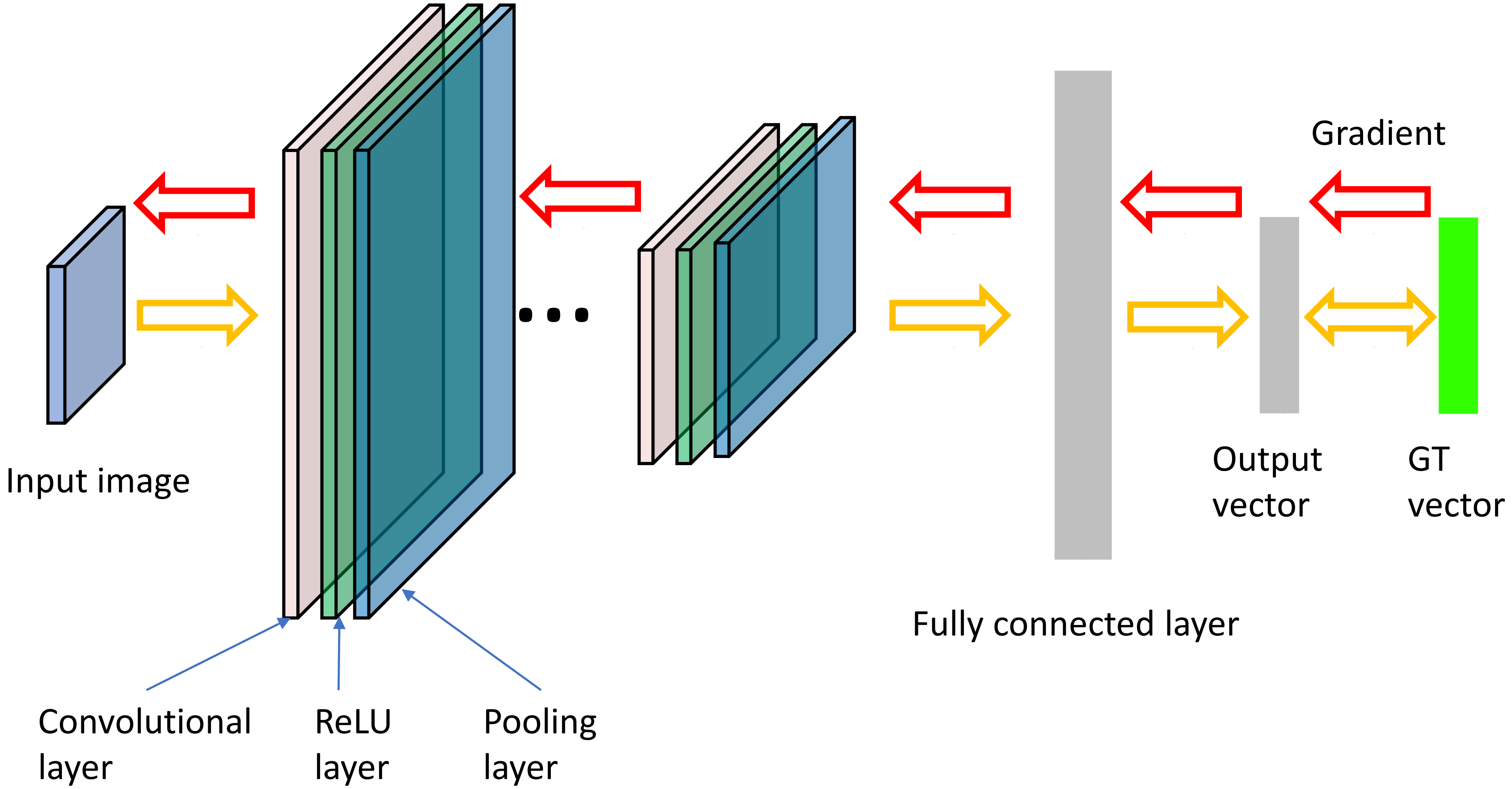
$$f(\mathbf{x}) = \mathbf{w}^T\mathbf{x} + b$$

Conv filter

Bias

$$\frac{\partial L}{\partial \mathbf{w}} = \sum_i \frac{\partial L}{\partial f(\mathbf{x}_i)} \frac{\partial f(\mathbf{x}_i)}{\partial \mathbf{w}}$$

Training: back-propagate errors



Back-propagation

- For each layer in the network, compute local gradients (partial derivative)
  - Fully connected layers
  - Convolution layers
  - Activation functions
  - Pooling functions
  - Etc.

- Use chain rule to combine local gradients for training

$$\frac{\partial L}{\partial W} = \frac{\partial L}{\partial \sigma(\mathbf{y})} \frac{\partial \sigma(\mathbf{y})}{\partial \mathbf{y}} \frac{\partial \mathbf{y}}{\partial W}$$

Classification Loss Functions

- Cross entropy loss

$$L_{CE} = -\sum_{i=0}^{m-1} t_i \log \sigma(\mathbf{y})_i$$

Binary ground truth label

Logit

- Hinge loss for binary classification

$$L = \max(0, 1 - t \cdot y)$$

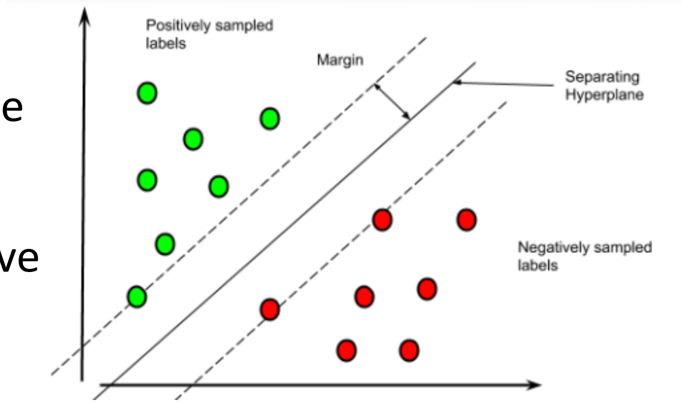
Classification score

ground truth label $t \in \{-1, +1\}$

$y \geq 0$ Predict positive

$y < 0$ Predict negative

Max margin classification



Classification Loss Functions

- Hinge loss for multi-class classification

$$\ell(y) = \max(0, 1 + \max_{y \neq t} \mathbf{w}_y \mathbf{x} - \mathbf{w}_t \mathbf{x})$$

margin

Score corresponds to the most wrong label

Score corresponds to the ground truth label

Regression Loss Functions

- Mean Absolute Loss or L1 loss

$$L_1(x) = |x|$$

$$f(y, \hat{y}) = \sum_{i=1}^{N} |y_i - \hat{y}_i|$$

- Mean Square Loss or L2 loss

$$L_2(x) = x^2$$

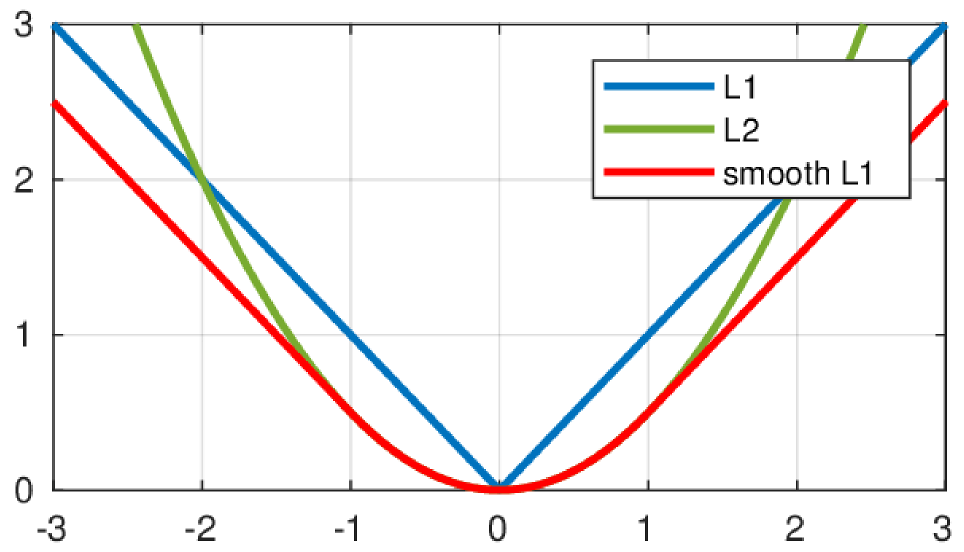
$$f(y, \hat{y}) = \sum_{i=1}^{N} (y_i - \hat{y}_i)^2$$

Regression Loss Functions

- Smooth L1 loss

$$\text{smooth } \mathrm{L}_1(x) = \begin{cases} 0.5x^2 & if|x| < 1 \\ |x| - 0.5 & otherwise \end{cases}$$

$$f(y, \hat{y}) = \begin{cases} 0.5(y - \hat{y})^2 & \text{if } |y - \hat{y}| < 1 \\ |y - \hat{y}| - 0.5 & otherwise \end{cases}$$

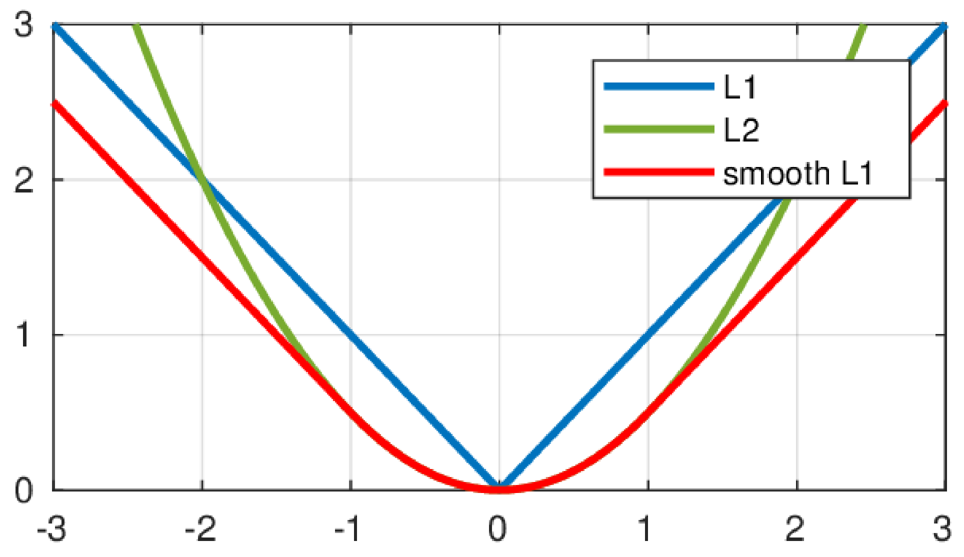


Regression Loss Functions

- Huber loss
  - Generalization of smooth L1 loss ($\delta = 1$)

$$L_\delta(a) = \begin{cases} \frac{1}{2}a^2 & \text{for } |a| \le \delta, \\ \delta(|a| - \frac{1}{2}\delta), & \text{otherwise.} \end{cases}$$

$$L_\delta(y, f(x)) = \begin{cases} \frac{1}{2}(y - f(x))^2 & \text{for} |y - f(x)| \le \delta, \\ \delta\,(|y - f(x)| - \frac{1}{2}\delta), & \text{otherwise.} \end{cases}$$



Optimization

- Gradient descent
  - Gradient direction: steepest direction to increase the objective
  - Can only find local minimum
  - Widely used for neural network training (works in practice)
  - Compute gradient with a mini-batch (Stochastic Gradient Descent, SGD)

$$W \leftarrow W - \gamma \frac{\partial L}{\partial W}$$

Learning rate

Optimization

- Gradient descent with momentum
    - Add a fraction of the update vector from previous time step (momentum)
    - Accelerated SGD, reduced oscillation

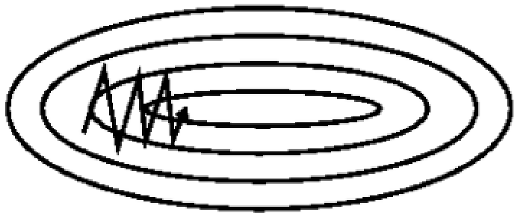


Image 2: SGD without momentum

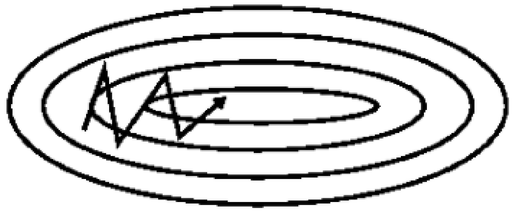


Image 3: SGD with momentum

momentum Learning rate

$$v_t = \gamma v_{t-1} + \eta \nabla_\theta J(\theta)$$
$$\theta = \theta - v_t$$

https://ruder.io/optimizing-gradient-descent/

Optimization

- Adam: Adaptive Moment Estimation
  1. Exponentially decaying average of gradients and squared gradients

$$m_t = \beta_1 m_{t-1} + (1 - \beta_1) g_t$$
$$v_t = \beta_2 v_{t-1} + (1 - \beta_2) g_t^2$$

$$\beta_1 = 0.9, \beta_2 = 0.999$$

Start m and v from 0s

  2. Bias-corrected 1st and 2nd moment estimates

$$\hat{m}_t = \frac{m_t}{1 - \beta_1^t} \qquad \hat{v}_t = \frac{v_t}{1 - \beta_2^t}$$

  3. Updating rule

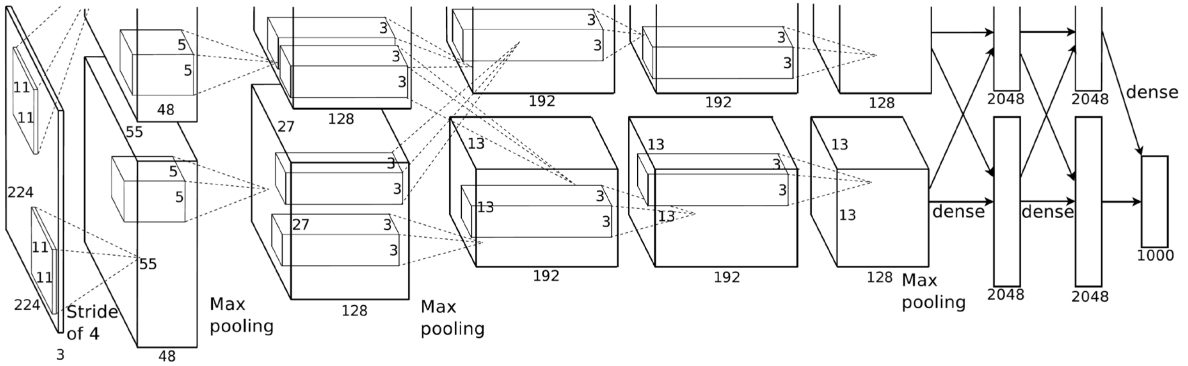
Learning rate

$$\theta_{t+1} = \theta_t - \frac{\eta}{\sqrt{\hat{v}_t} + \epsilon} \hat{m}_t \qquad \epsilon = 10^{-8}$$

Adaptive learning rate

Case Study: Training AlexNet

- Data augmentation
  - Extracting random 224x224 patches from 256x256 images
  - Change RGB intensities



$$[I_{xy}^R, I_{xy}^G, I_{xy}^B]^T$$

$$+ \quad [\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3][\alpha_1\lambda_1, \alpha_2\lambda_2, \alpha_3\lambda_3]^T$$

Eigen vectors of 3x3 covariance matrix of RGB values on training set

Random variable N(0, 0.1)

Eigen values

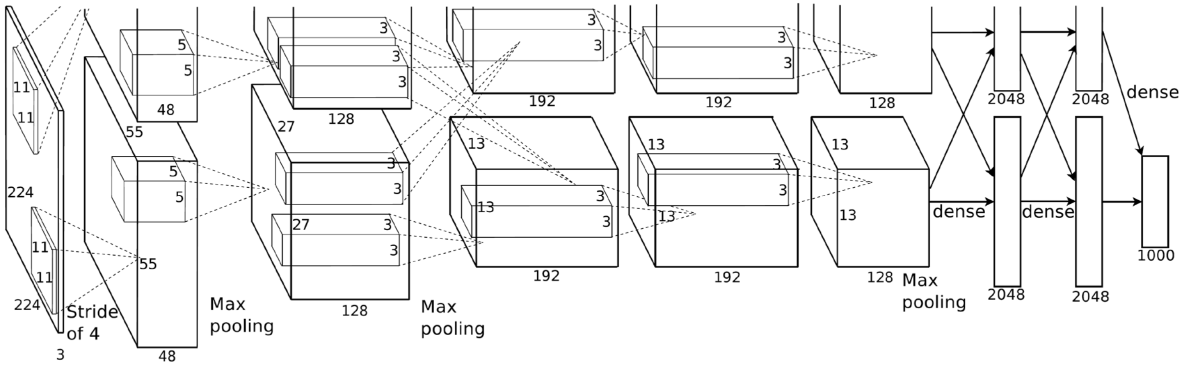
covariance matrix

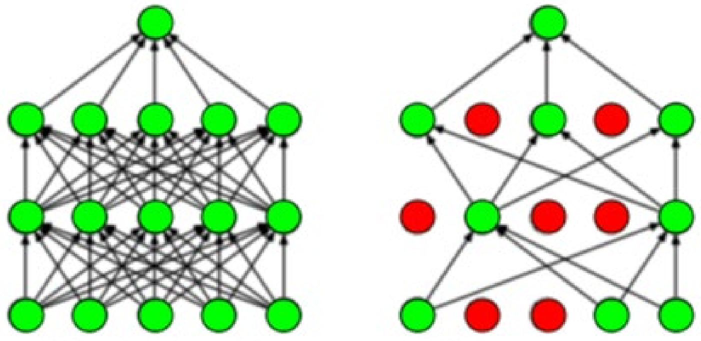
$$S = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})(X_i - \bar{X})'$$

https://papers.nips.cc/paper/2012/hash/c399862d3b9d6b76c8436e924a68c45b-Abstract.html

Case Study: Training AlexNet

- Dropout
  - Set to zero the output of each hidden neuron with probability 0.5
  - Apply to the first two FC layers
  - Prevent overfitting



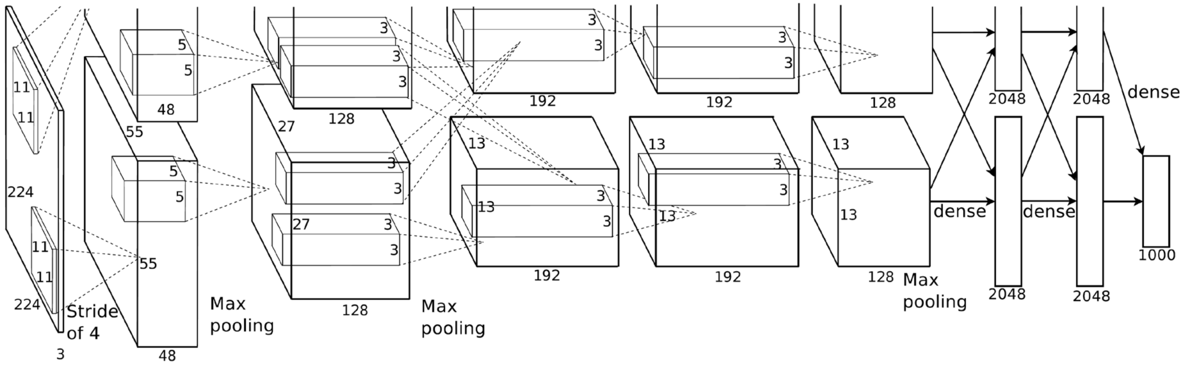


(a) Standard Neural Net (b) After applying dropout.

https://papers.nips.cc/paper/2012/hash/c399862d3b9d6b76c8436e924a68c45b-Abstract.html

Case Study: Training AlexNet

- Batch size: 128
- Updating rule



$$w_{i+1} := w_i + v_{i+1}$$

$$v_{i+1} := 0.9 \cdot v_i - 0.0005 \cdot \epsilon \cdot w_i - \epsilon \cdot \left\langle \frac{\partial L}{\partial w}\Big|_{w_i} \right\rangle_{D_i}$$

Momentum Weight Decay Learning rate Gradient

Five to six days on two NVIDIA GTX 580 3GB GPUs, 2012

https://papers.nips.cc/paper/2012/hash/c399862d3b9d6b76c8436e924a68c45b-Abstract.html

Further Reading

- Stanford CS231n, lecture 3 and lecture 4, http://cs231n.stanford.edu/schedule.html
- Deep learning with PyTorch https://pytorch.org/tutorials/beginner/deep_learning_60min_blitz.html
- Dropout: A Simple Way to Prevent Neural Networks from Overfitting https://jmlr.org/papers/v15/srivastava14a.html
- Matrix Calculus: https://explained.ai/matrix-calculus/