

# Convolutional Neural Networks I

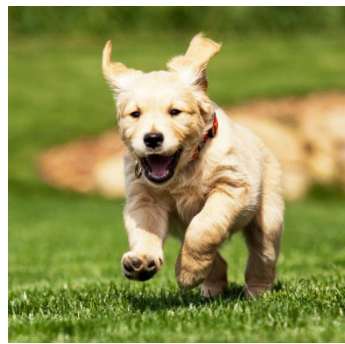
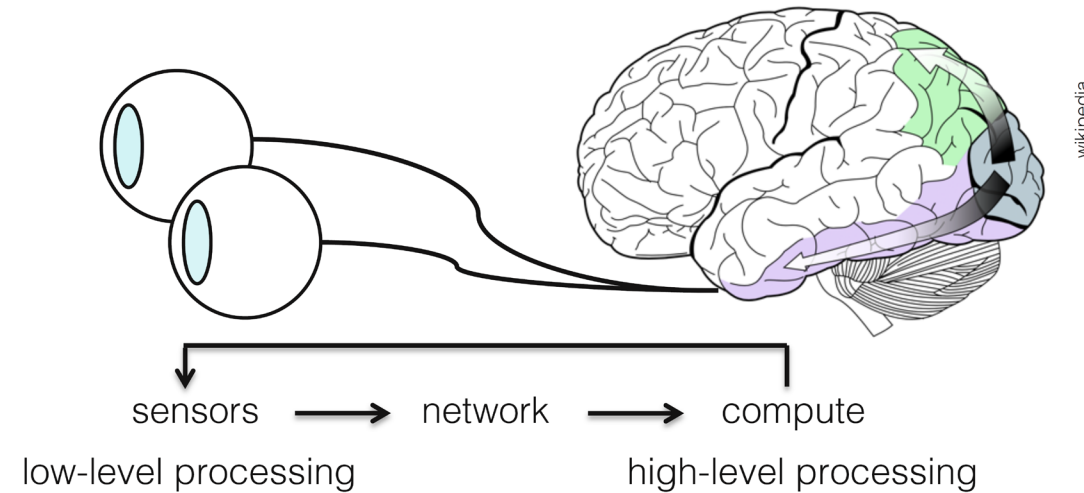
CS 6384 Computer Vision

Professor Yu Xiang

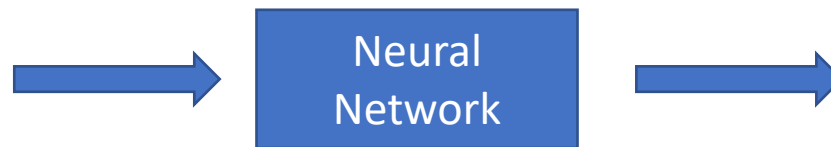
The University of Texas at Dallas

Some slides of this lecture are courtesy Stanford CS231n

# Visual Perception vs. Computational Perception



Image

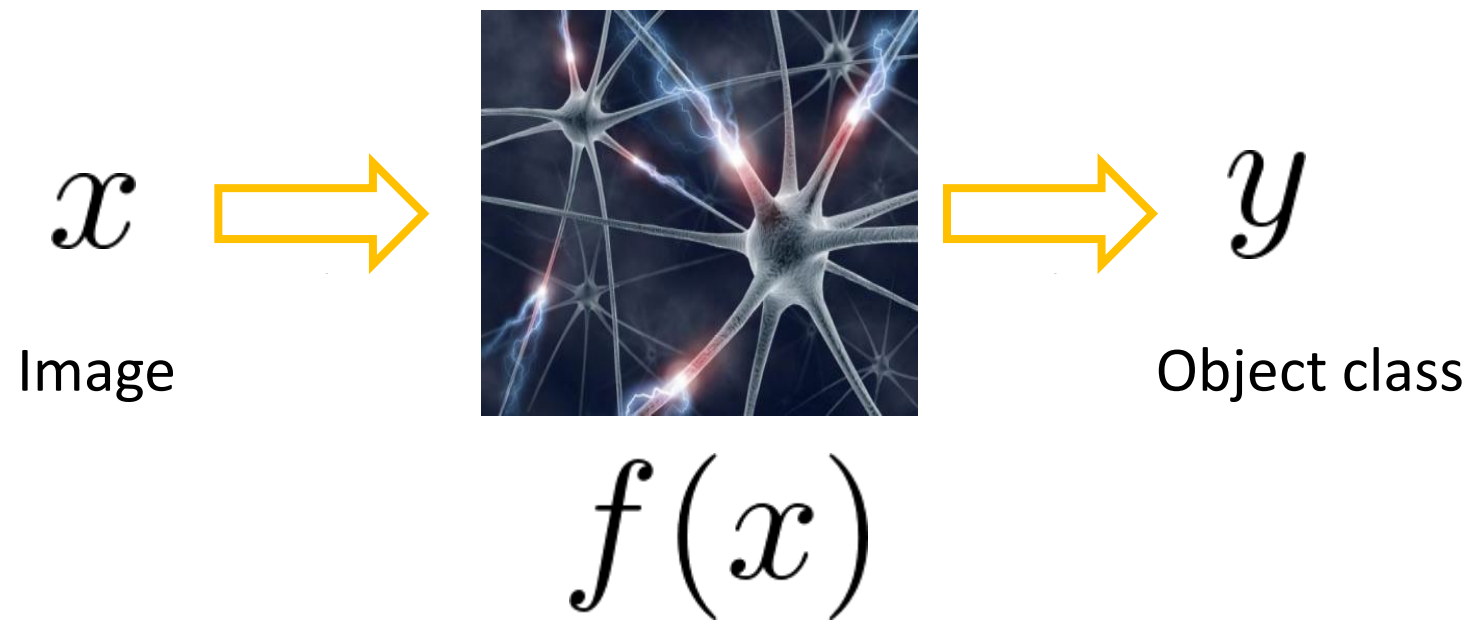


## High-level information

- Depth
- Motion
- Object classes
- Object poses
- Etc.

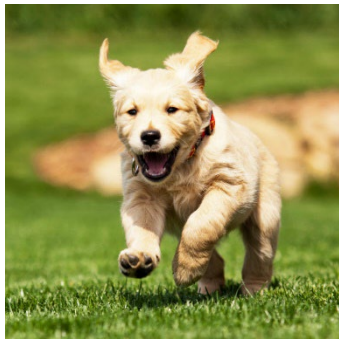
# Mathematic Models

- Try to model the human brain with computational models, e.g., neural networks

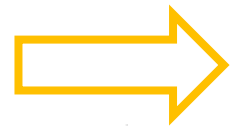


# Mathematic Models

- What is the form of the function  $f(x)$ ?
  - No idea!
  - Concatenate simple functions (neurons)



$x$



$f(x)$



$y \in \{+1, -1\}$

Dog

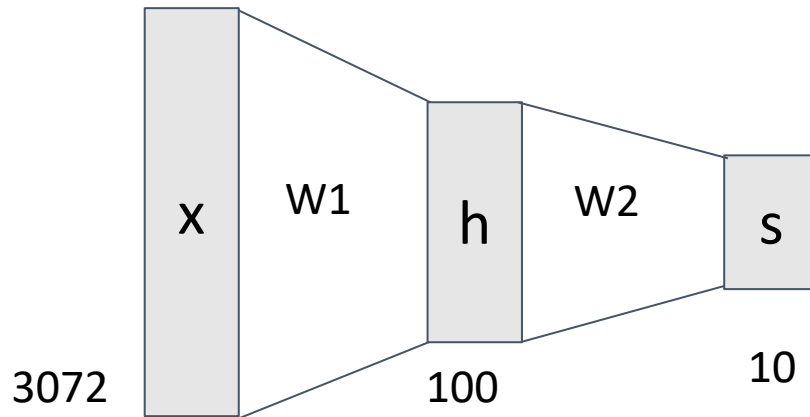
# Neural Network: Concatenation of functions

Linear score function:  $f = Wx$

2-layer Neural Network

$$f = f_2(f_1(x)) = W_2 \max(0, W_1 x)$$

Non-linearity

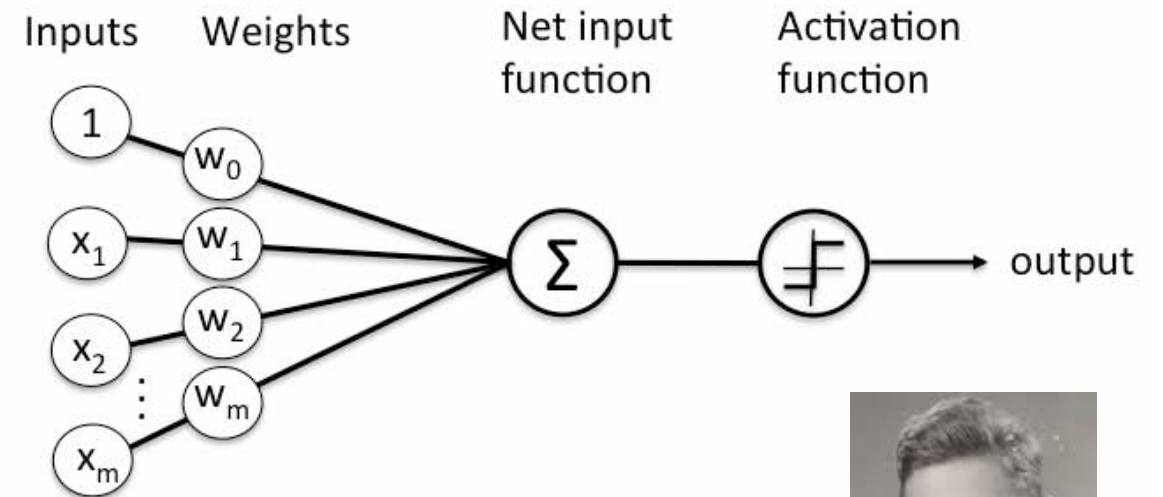
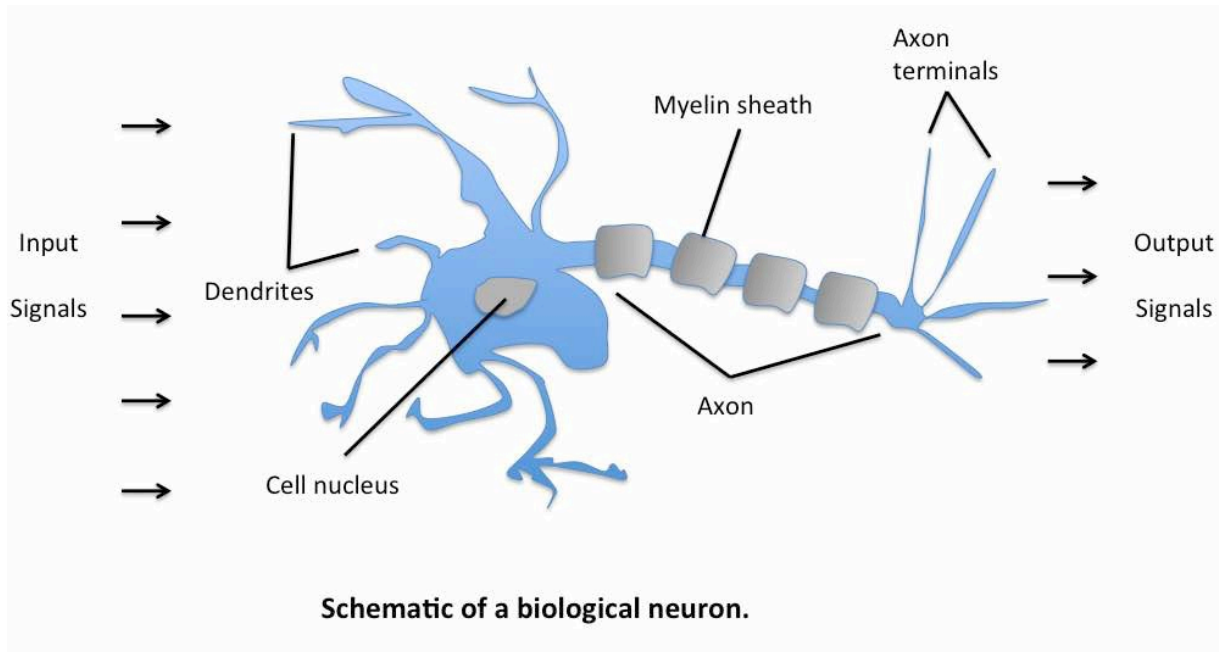


$$h = f_1(X)$$

$$s = f_2(h)$$

Need to learn the weights!

# Frank Rosenblatt's Perceptron



$$\sigma(\mathbf{w}^T \mathbf{x} + b) = \begin{cases} 1 & \text{if } \mathbf{w}^T \mathbf{x} + b \geq 0, \\ 0 & \text{otherwise.} \end{cases}$$



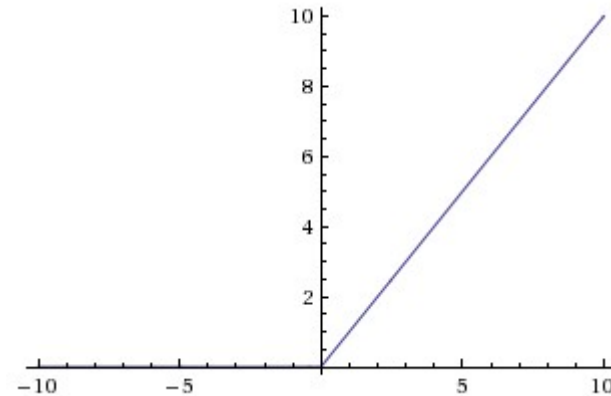
Frank Rosenblatt  
(1928-1971)

# Activation Functions

2-layer Neural Network

$$f = f_2(f_1(x)) = W_2 \max(0, W_1 x)$$

**Rectified Linear Unit (ReLU)**  
 $\max(0, x)$

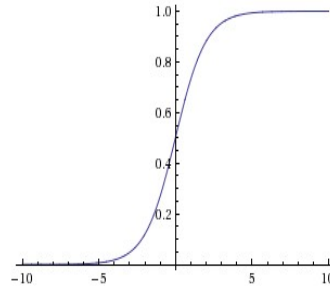


Introduce non-linearity to the network

# Activation Functions

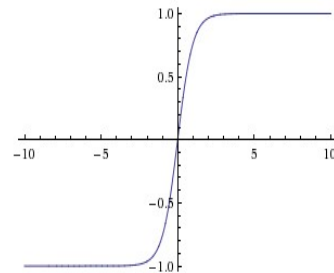
**Sigmoid**

$$\sigma(x) = 1/(1 + e^{-x})$$

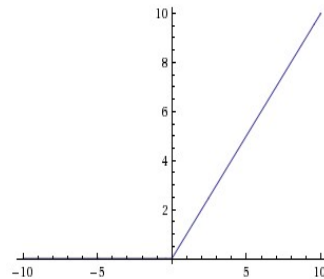


**tanh**  $\tanh(x)$

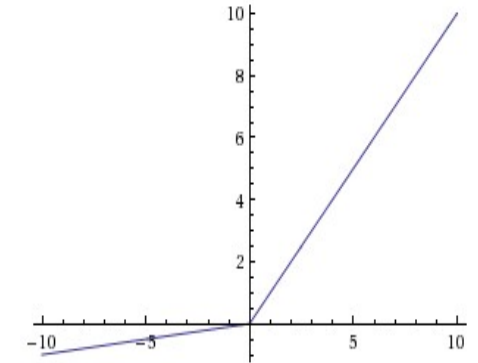
$$\frac{e^{2x} - 1}{e^{2x} + 1}$$



**ReLU**  $\max(0, x)$

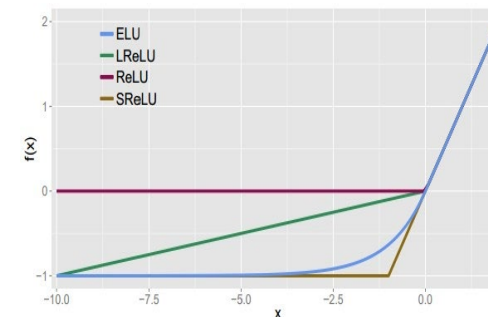


**Leaky ReLU**  
 $\max(0.1x, x)$



**Maxout**  $\max(w_1^T x + b_1, w_2^T x + b_2)$

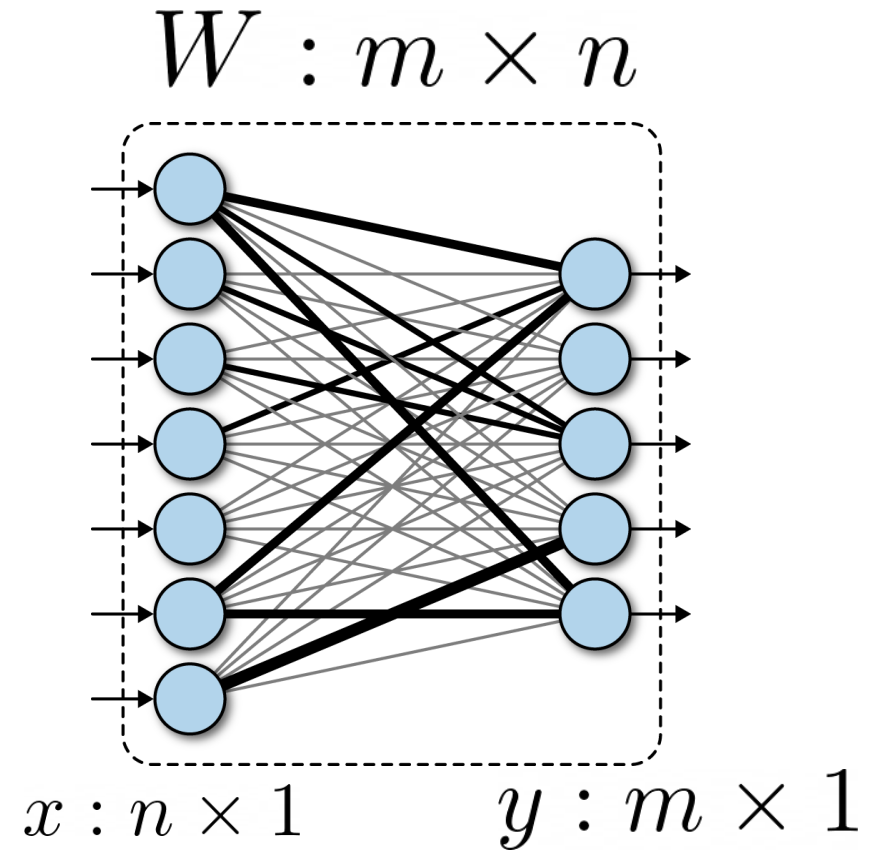
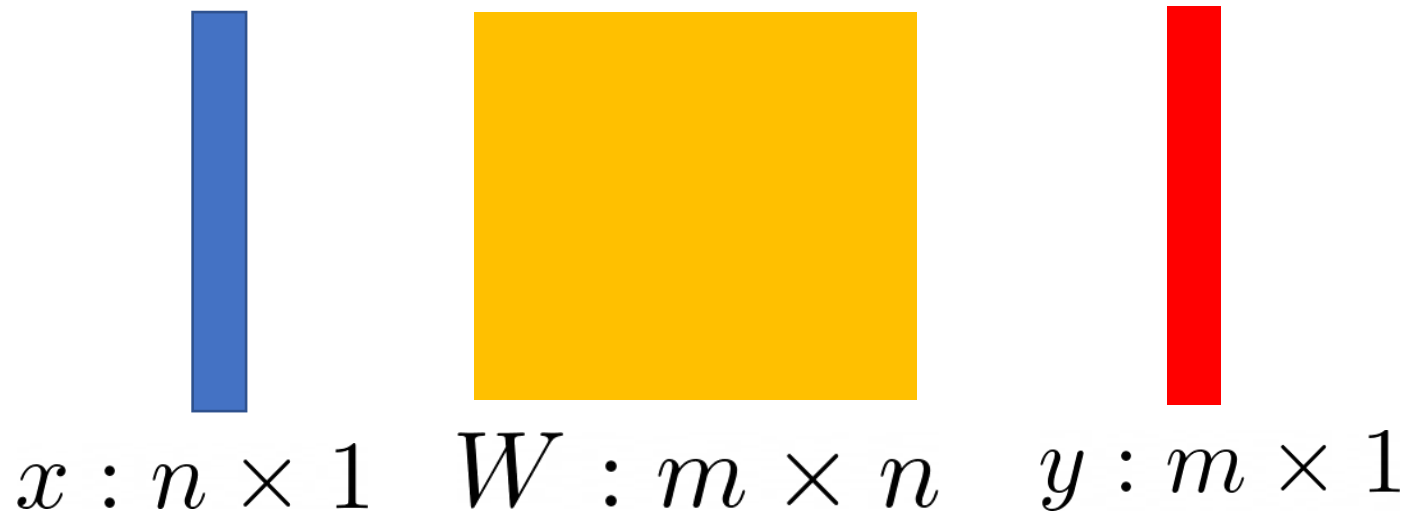
**ELU** Exponential Linear Unit  
 $f(x) = \begin{cases} x & \text{if } x > 0 \\ \alpha (\exp(x) - 1) & \text{if } x \leq 0 \end{cases}$





# Fully Connected Layer

$$y = Wx$$



# Fully Connected Layer

- What is the drawback of only using fully connected layers?

$$y = Wx$$

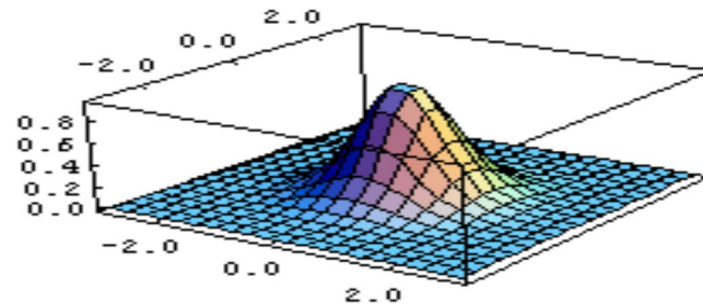
- Consider an image with 640 x 480
  - $x$  is with dimension 307,200
  - The weight matrix of the fully connect layer is too large

# Convolutional Layers

- Consist of convolutional filters
- Share weights among different image locations

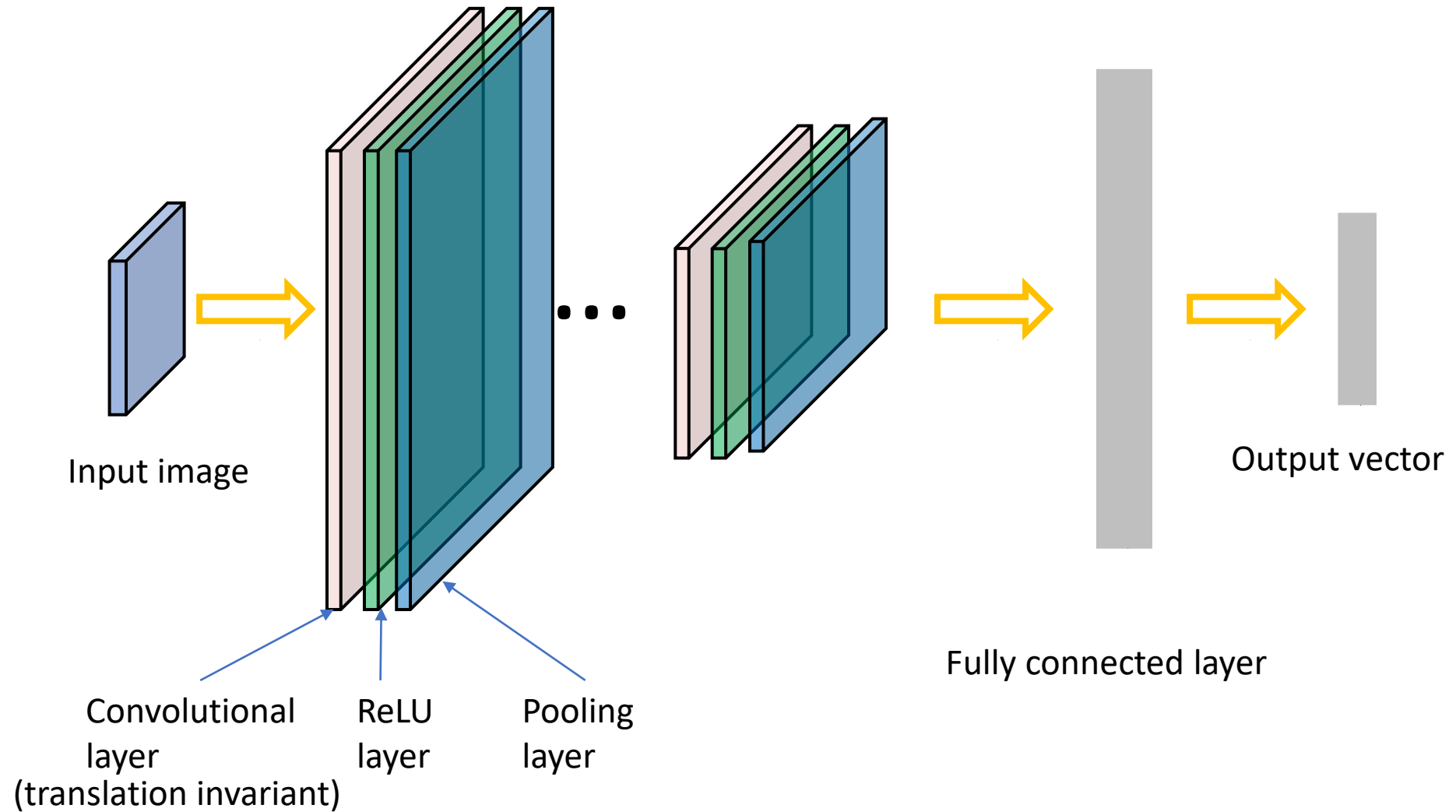
$$g(x, y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

Gaussian Filter

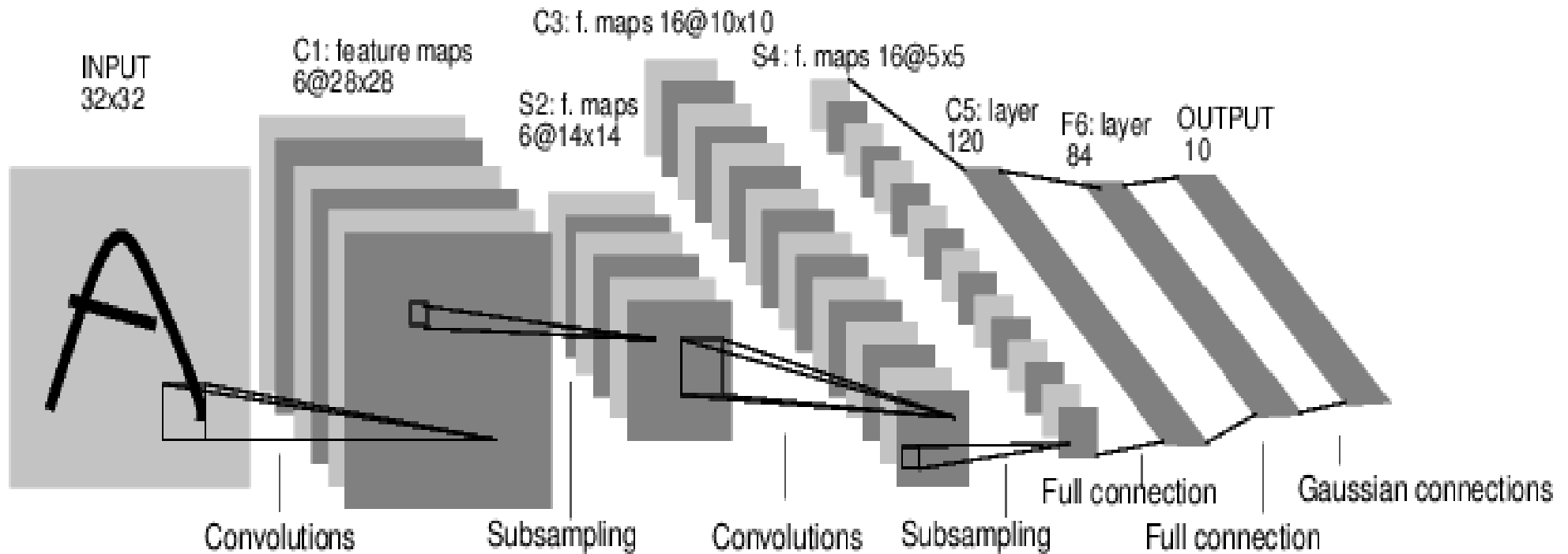


Learn the weights!

# Convolutional Neural Networks



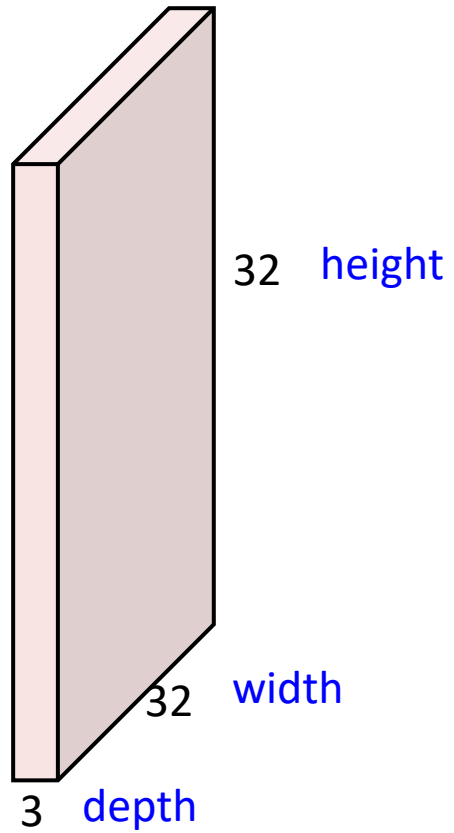
# Convolutional Neural Networks



[LeNet-5, LeCun 1980]

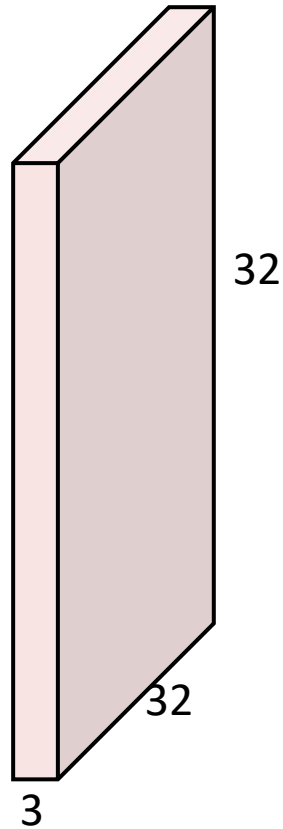
# Convolutional Layer

32x32x3 image



# Convolutional Layer

32x32x3 image

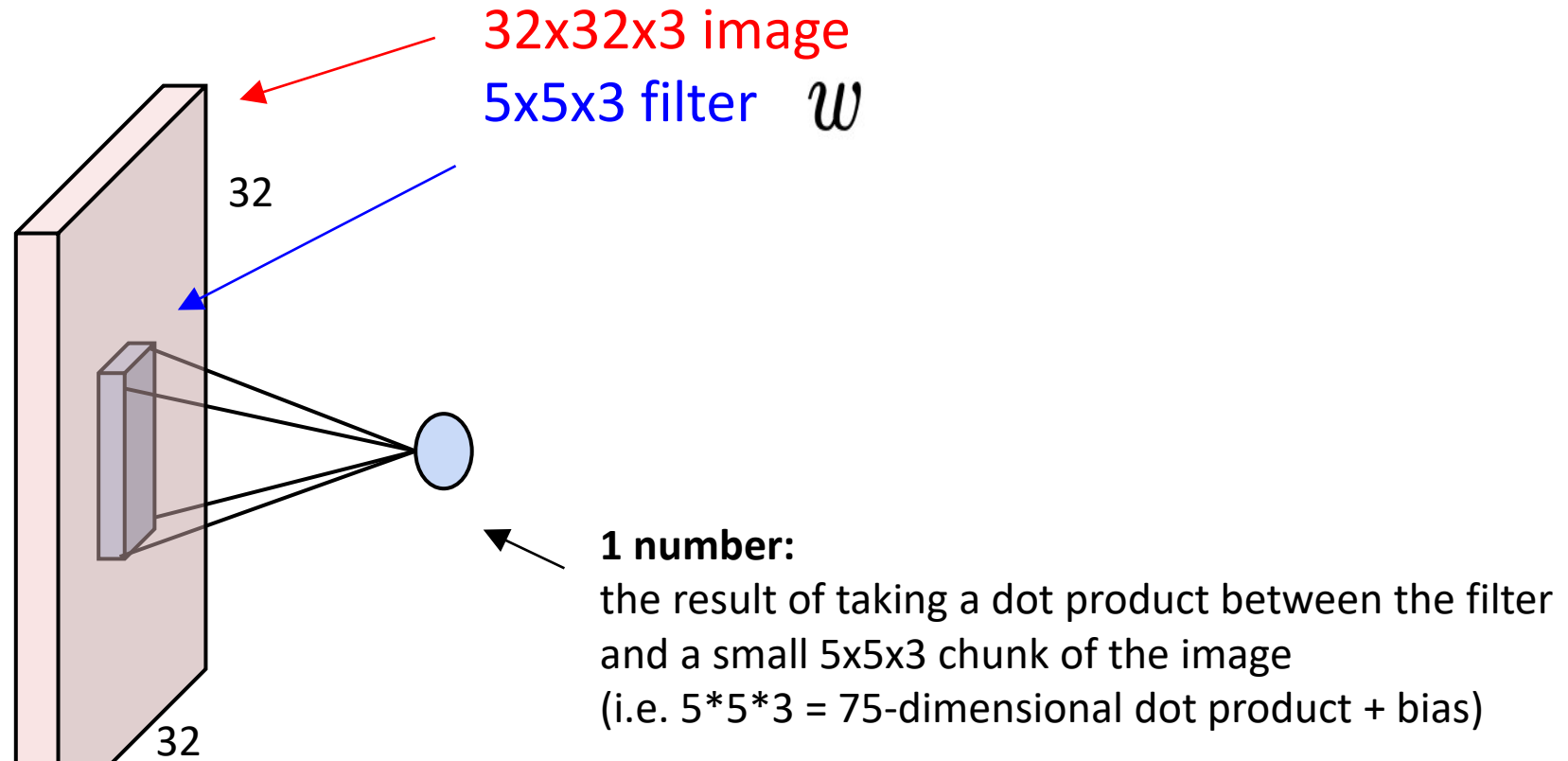


5x5x3 filter



**Convolve** the filter with the image  
i.e. “slide over the image spatially,  
computing dot products”

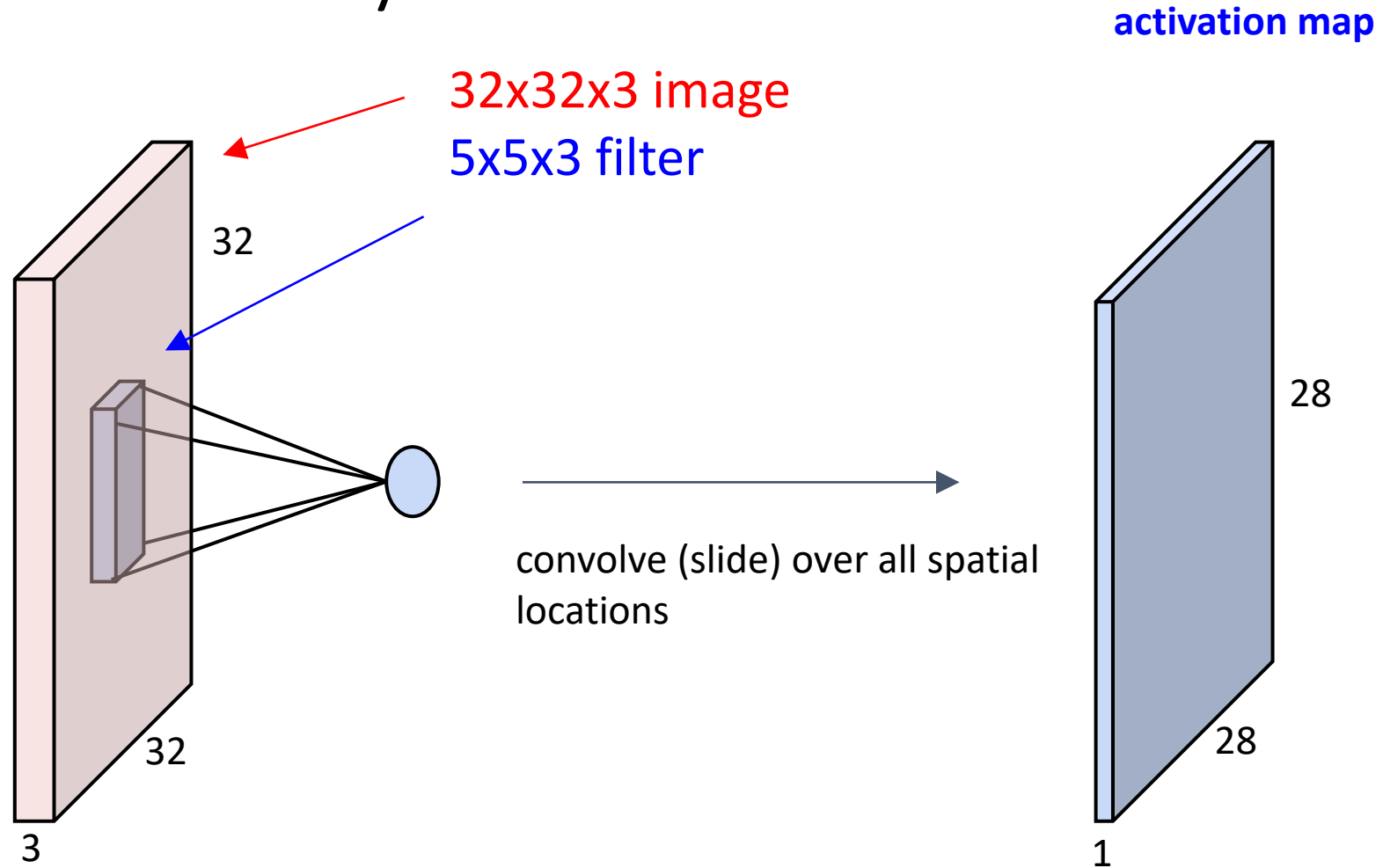
# Convolutional Layer



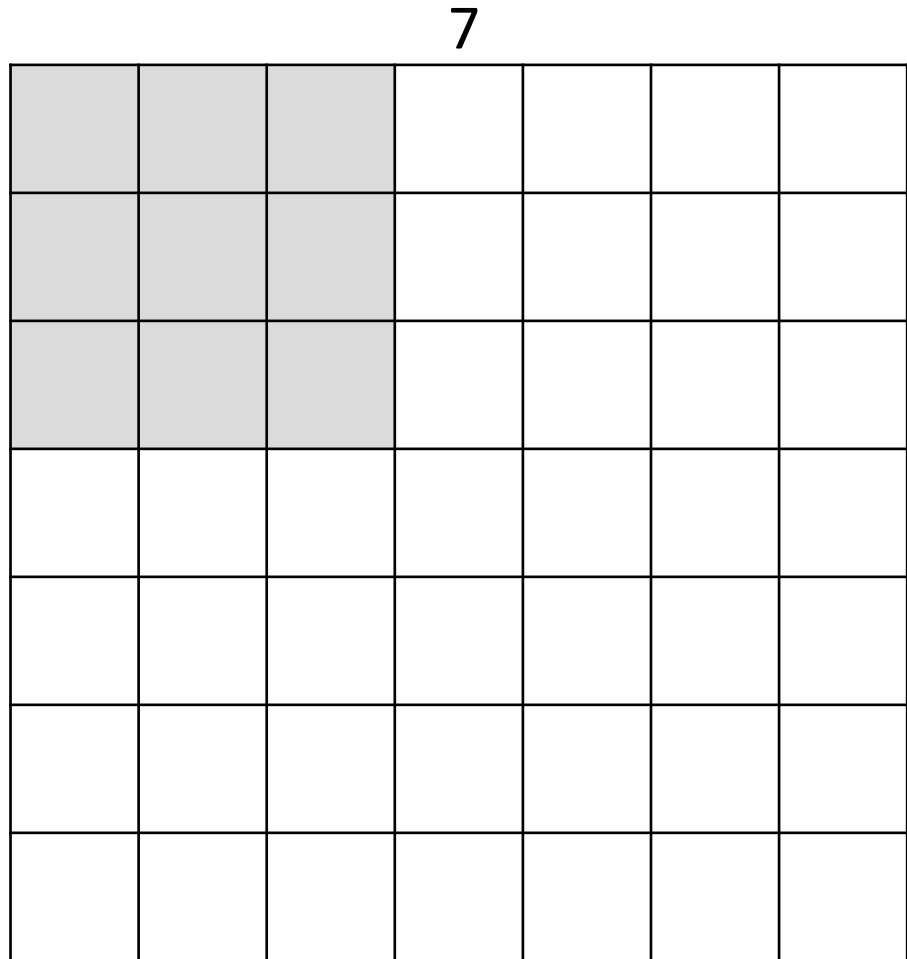
$$w^T x + b$$



# Convolutional Layer



# Convolutional Layer

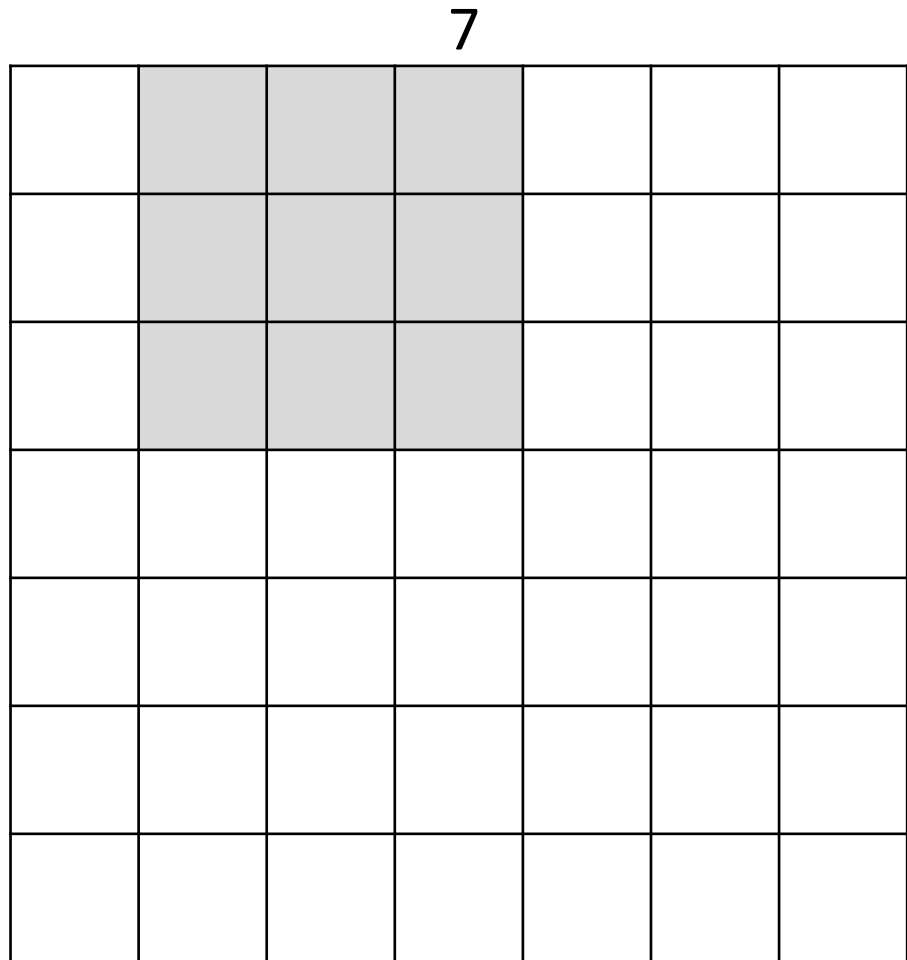


A closer look at spatial dimensions:

7x7 input (spatially)

assume 3x3 filter, with stride 1

# Convolutional Layer

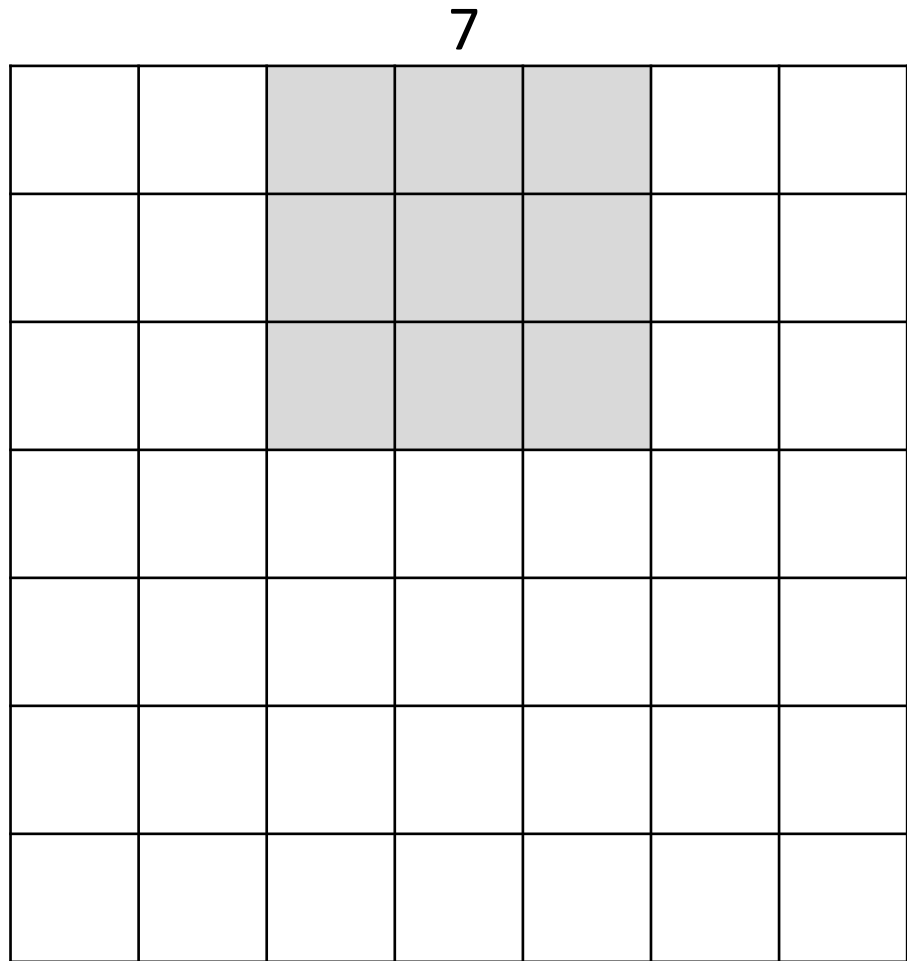


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# Convolutional Layer

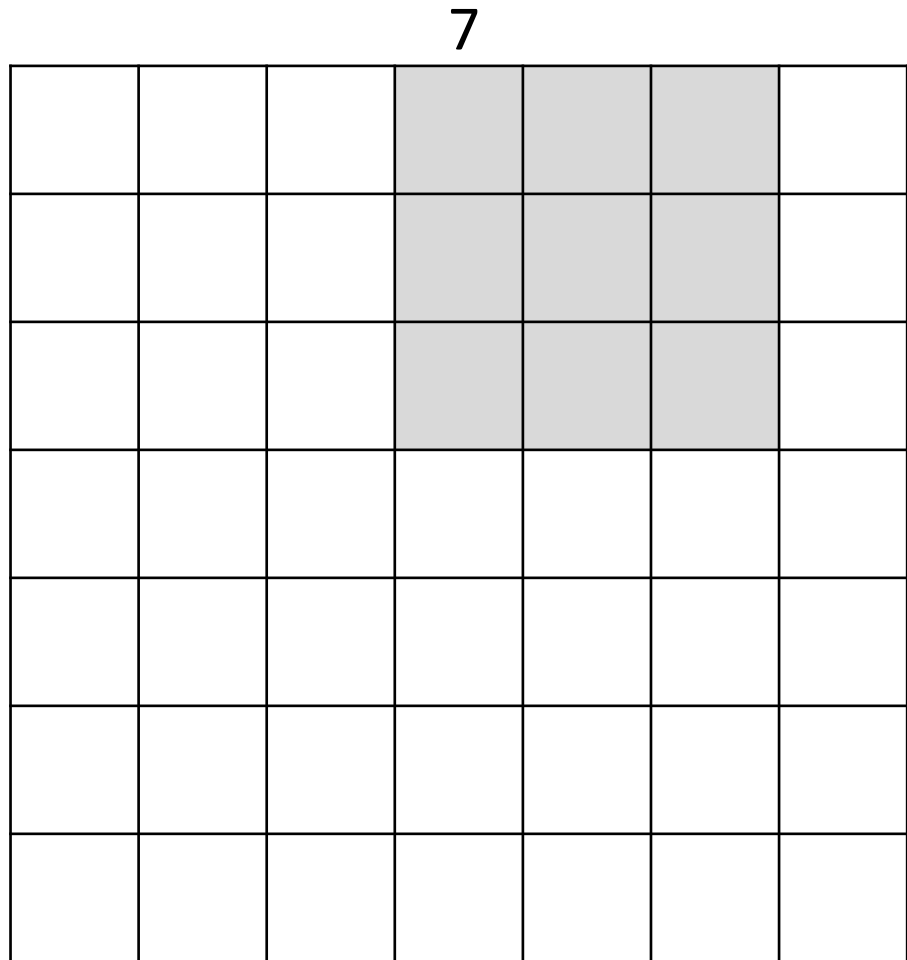


A closer look at spatial dimensions:

7x7 input (spatially)

assume 3x3 filter, with stride 1

# Convolutional Layer

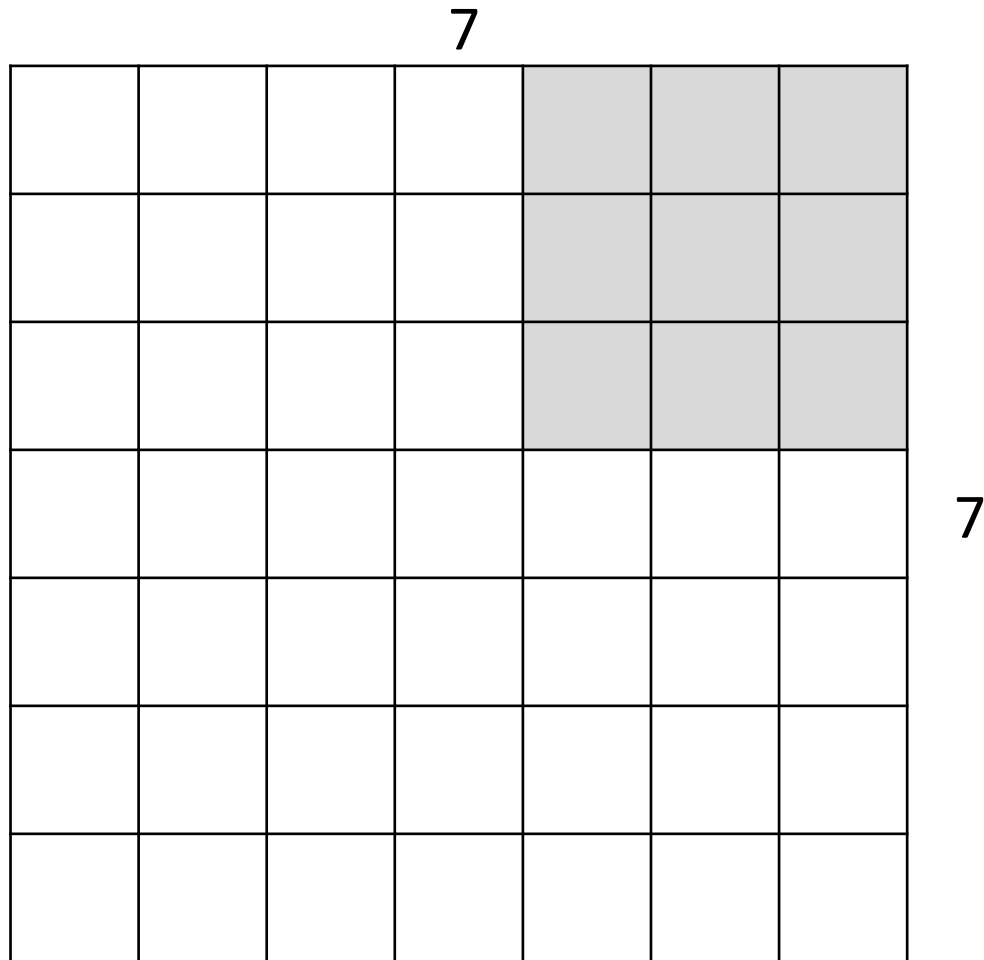


A closer look at spatial dimensions:

7x7 input (spatially)

assume 3x3 filter, with stride 1

# Convolutional Layer

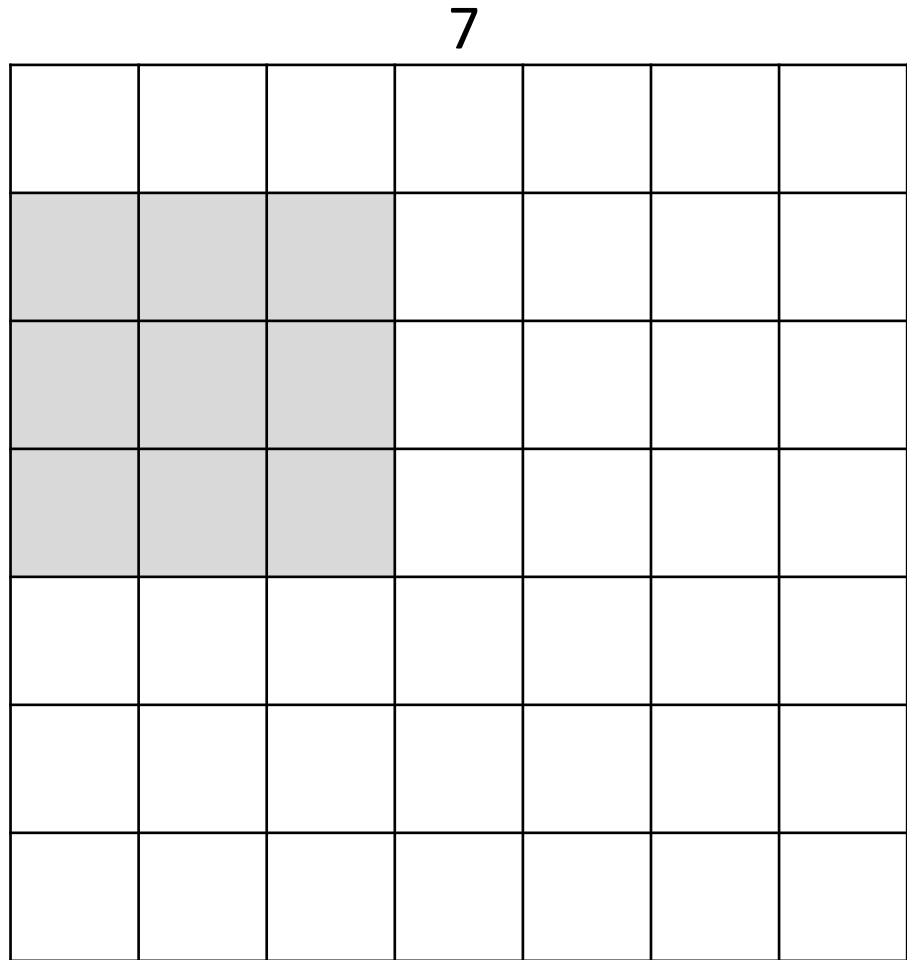


A closer look at spatial dimensions:

7x7 input (spatially)

assume 3x3 filter, with stride 1

# Convolutional Layer

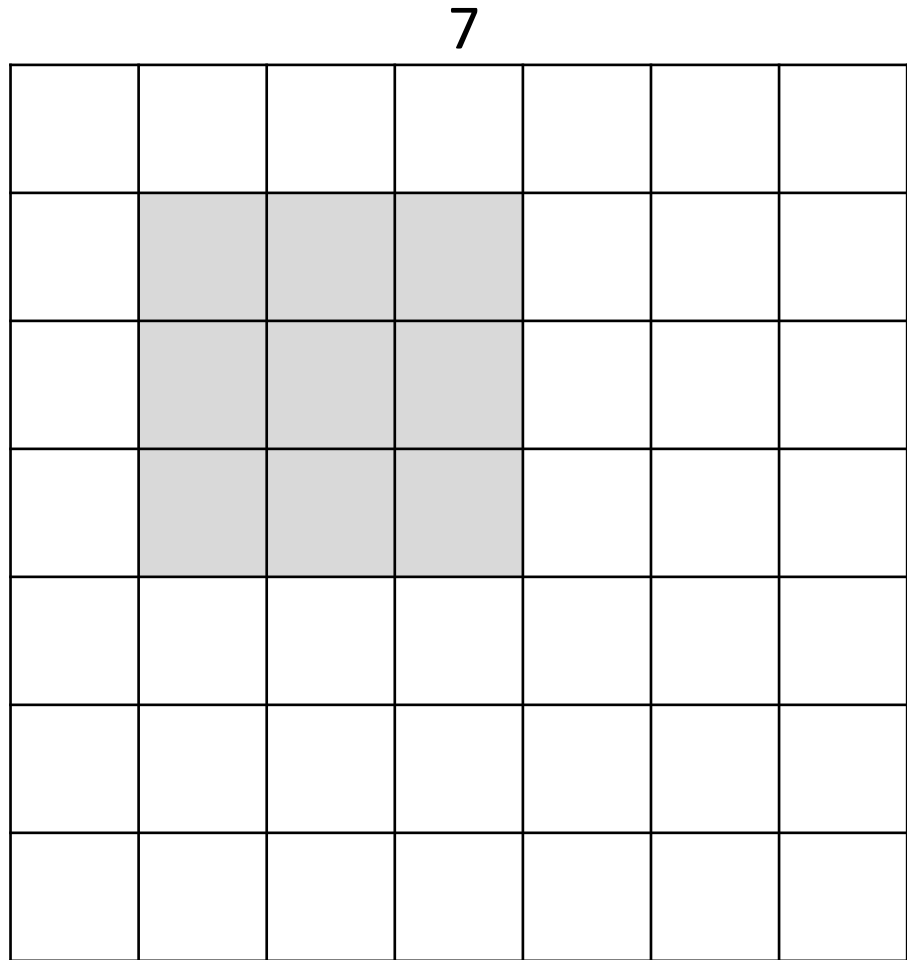


A closer look at spatial dimensions:

7x7 input (spatially)

assume 3x3 filter, with stride 1

# Convolutional Layer



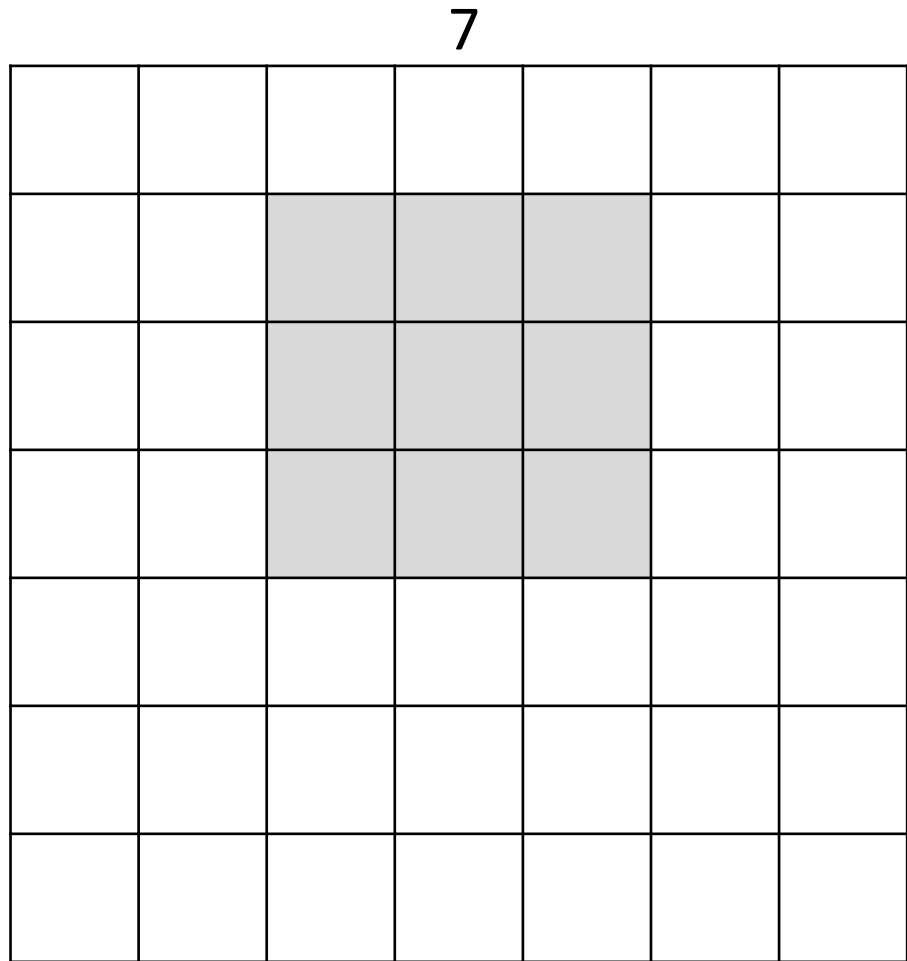
A closer look at spatial dimensions:

7x7 input (spatially)

assume 3x3 filter, with stride 1



# Convolutional Layer

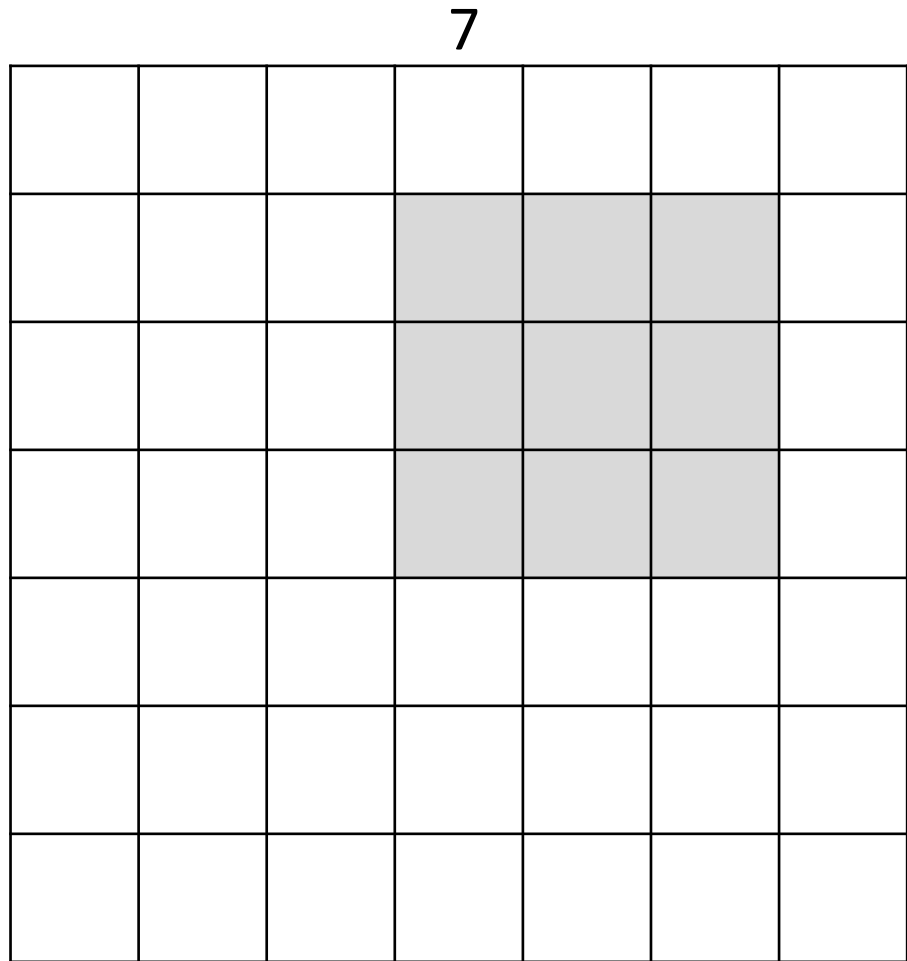


A closer look at spatial dimensions:

7x7 input (spatially)

assume 3x3 filter, with stride 1

# Convolutional Layer

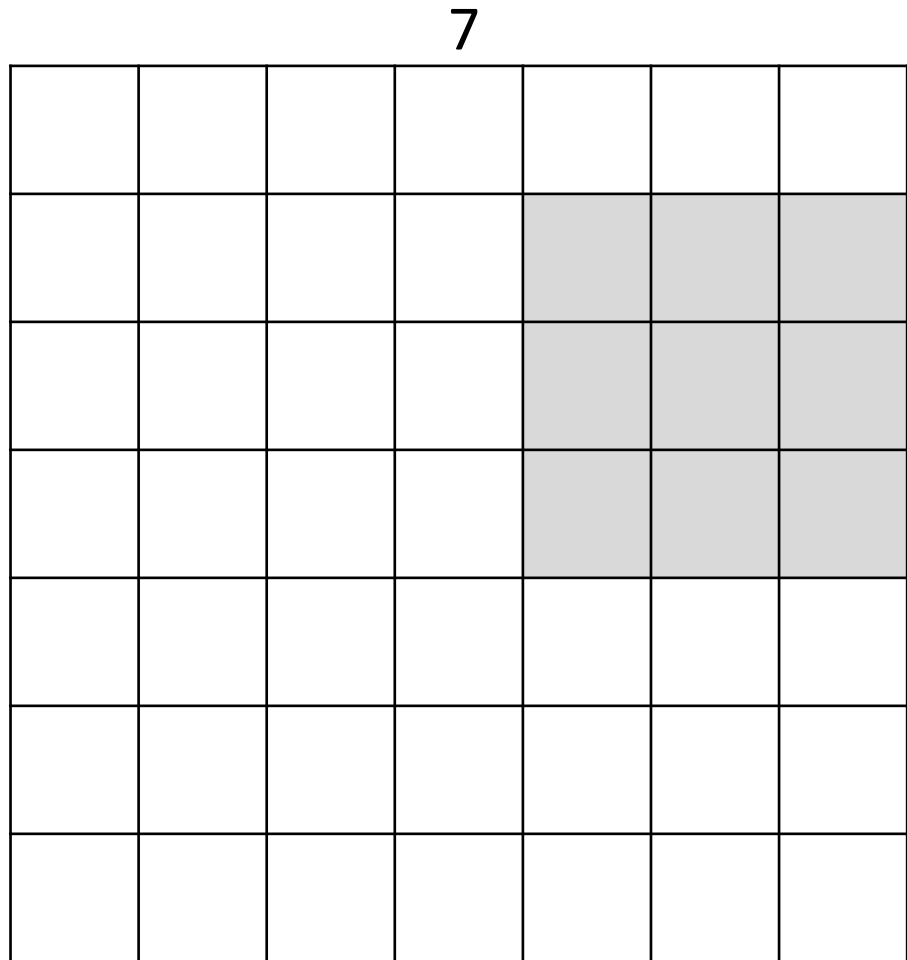


A closer look at spatial dimensions:

7x7 input (spatially)

assume 3x3 filter, with stride 1

# Convolutional Layer



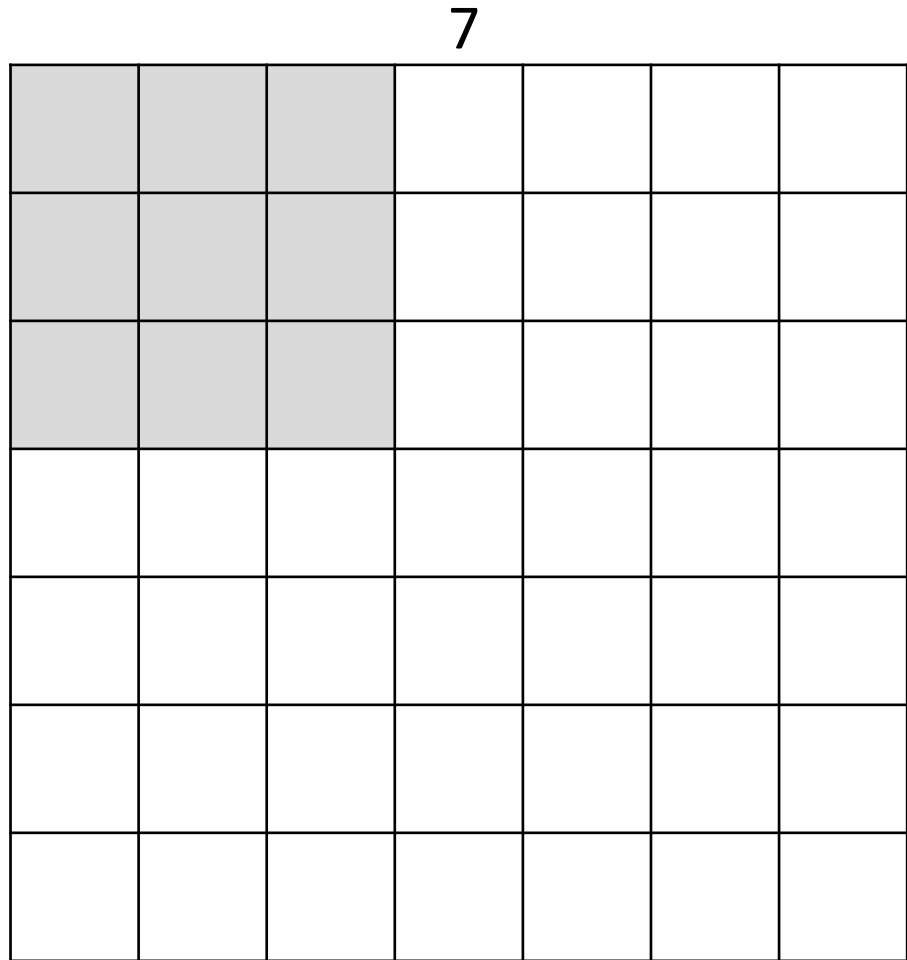
A closer look at spatial dimensions:

7x7 input (spatially)

assume 3x3 filter, with stride 1

**=> 5x5 output**

# Convolutional Layer

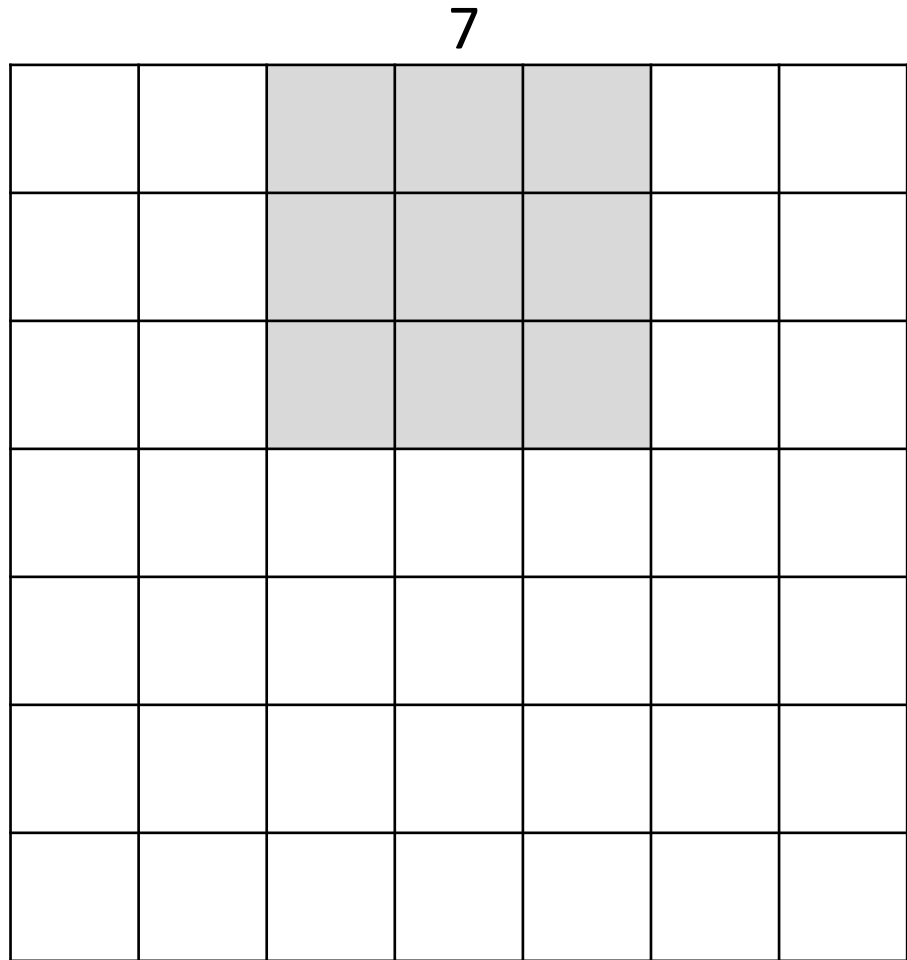


A closer look at spatial dimensions:

7x7 input (spatially)

assume 3x3 filter, **with stride 2**

# Convolutional Layer

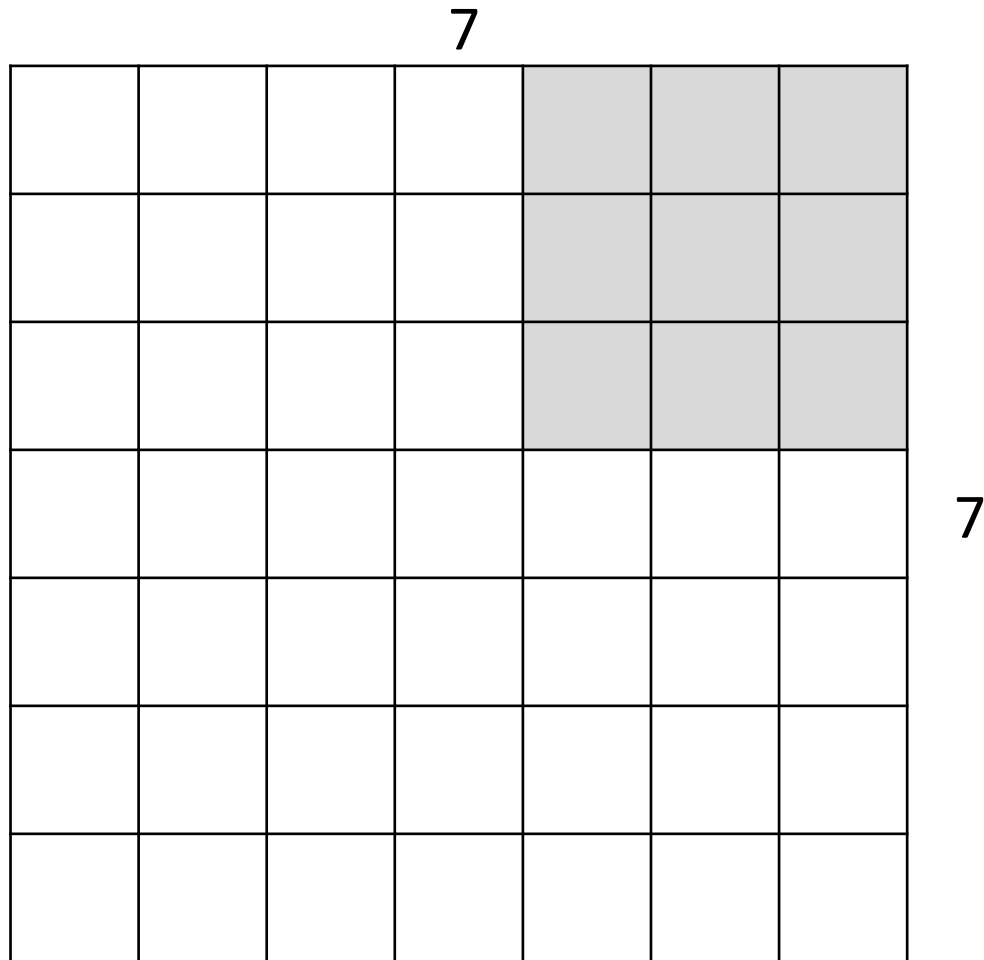


A closer look at spatial dimensions:

7x7 input (spatially)

assume 3x3 filter, **with stride 2**

# Convolutional Layer



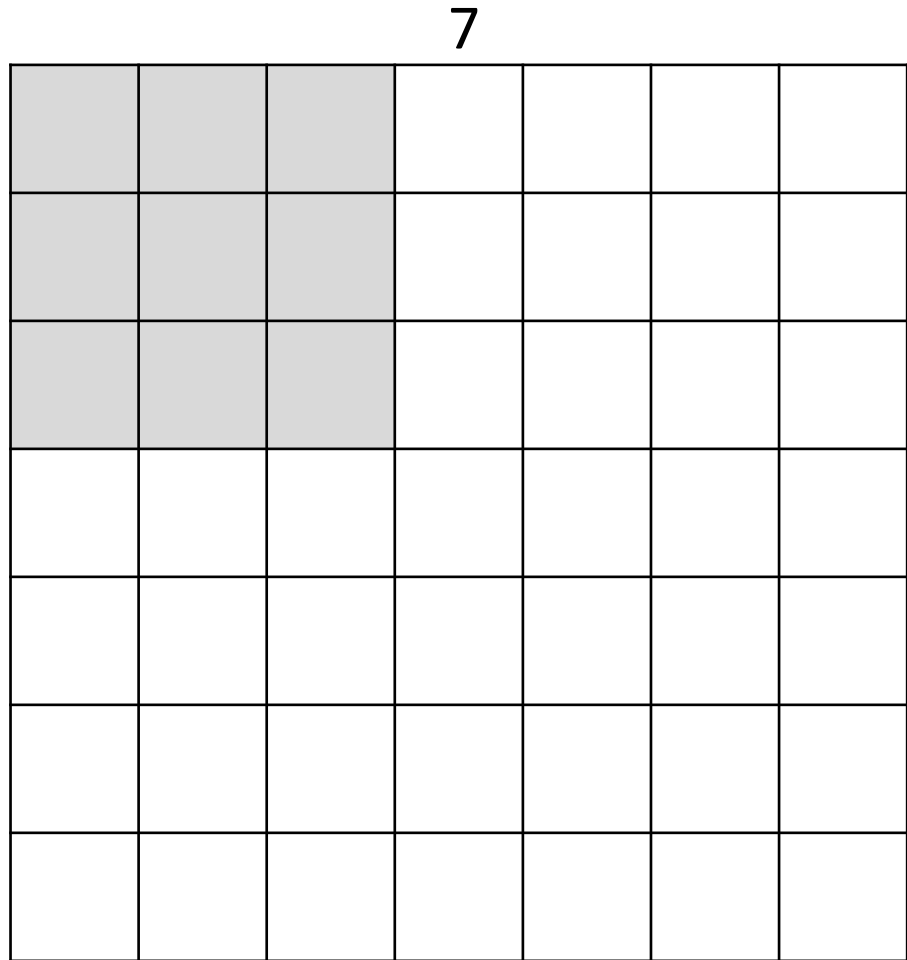
A closer look at spatial dimensions:

7x7 input (spatially)

assume 3x3 filter, **with stride 2**

=> **3x3 output!**

# Convolutional Layer

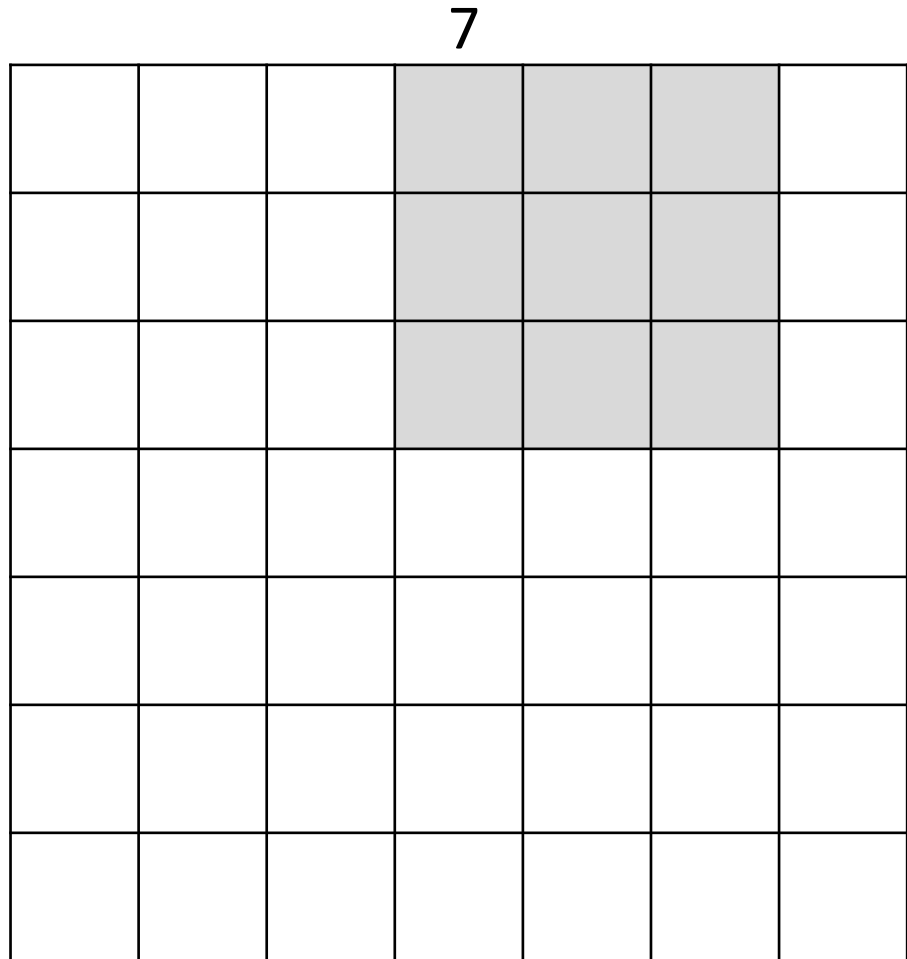


A closer look at spatial dimensions:

7x7 input (spatially)

assume 3x3 filter, **with stride 3**

# Convolutional Layer



A closer look at spatial dimensions:

7x7 input (spatially)

assume 3x3 filter, **with stride 3**

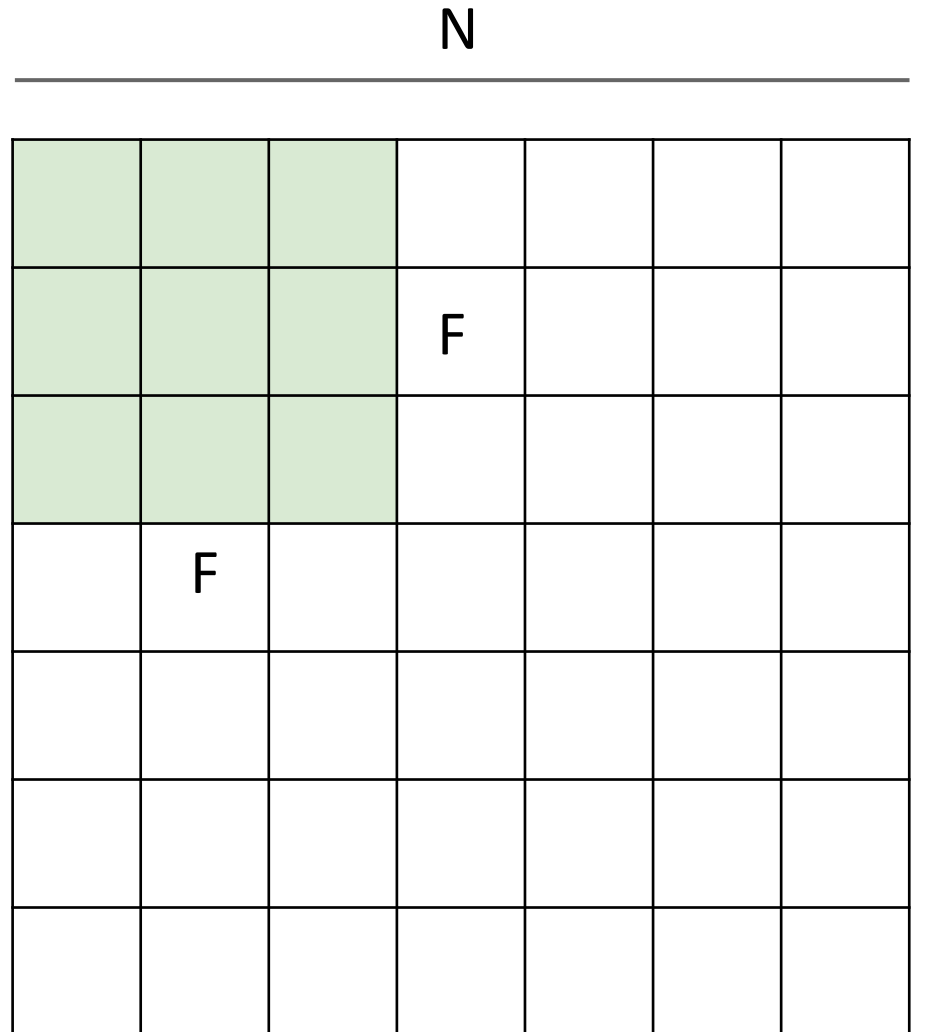
7

**doesn't fit!**

cannot apply 3x3 filter on 7x7  
input with stride 3.



# Convolutional Layer



Output size:  
 **$(N - F) / \text{stride} + 1$**

e.g.  $N = 7, F = 3$ :

stride 1  $\Rightarrow (7 - 3) / 1 + 1 = 5$

stride 2  $\Rightarrow (7 - 3) / 2 + 1 = 3$

stride 3  $\Rightarrow (7 - 3) / 3 + 1 = 2.33$

# Convolutional Layer

0	0	0	0	0	0			
0								
0								
0								
0								

In practice: Common to zero pad the border

e.g. input 7x7

**3x3** filter, applied with **stride 1**

**pad with 1 pixel** border => what is the output?

(recall:)

$(N - F) / \text{stride} + 1$

# Convolutional Layer

0	0	0	0	0	0			
0								
0								
0								
0								

In practice: Common to zero pad the border

e.g. input 7x7

**3x3** filter, applied with **stride 1**

**pad with 1 pixel** border => what is the output?

**7x7 output!**

# Convolutional Layer

0	0	0	0	0	0			
0								
0								
0								
0								

In practice: Common to zero pad the border

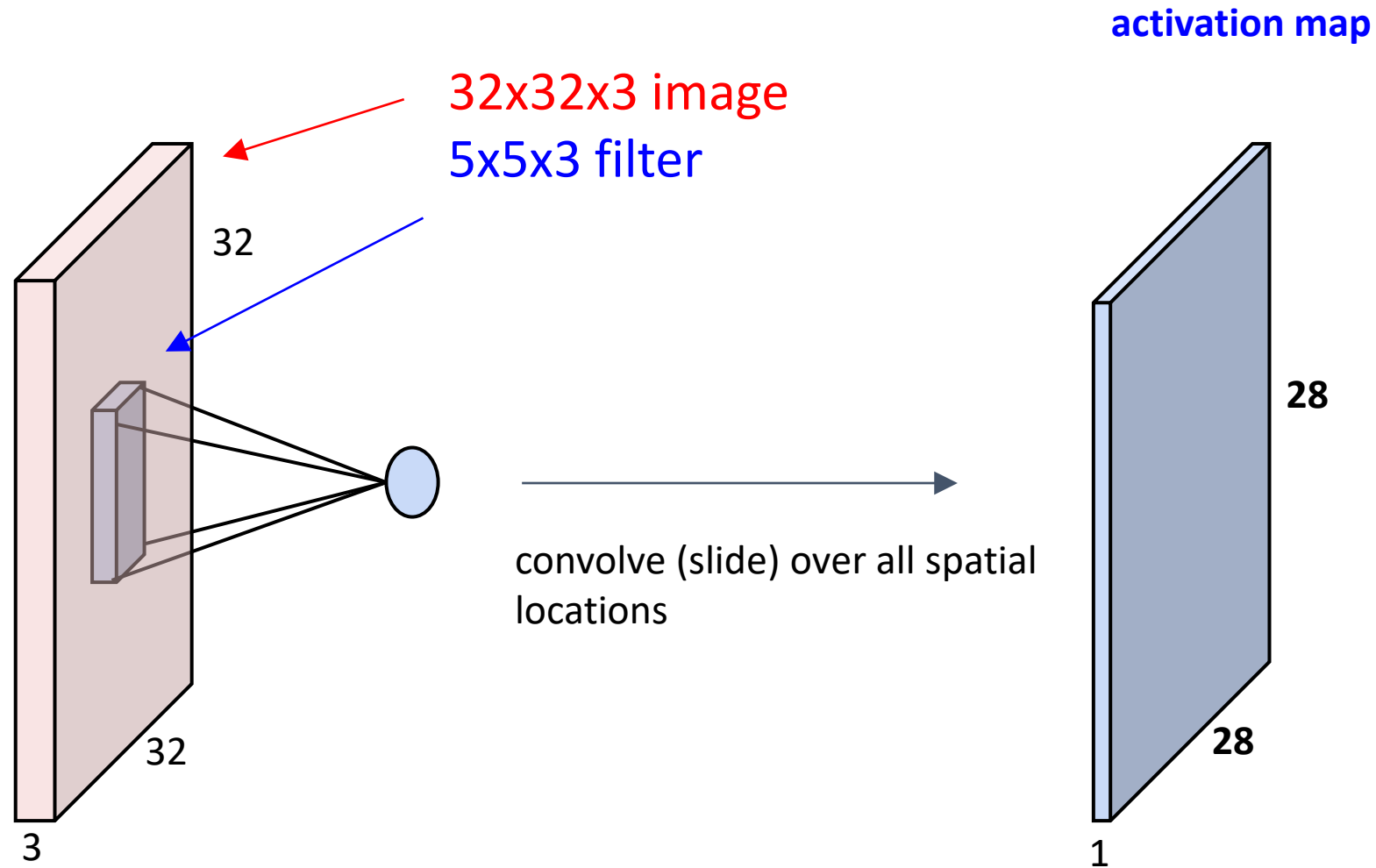
in general, common to see CONV layers with stride 1, filters of size  $F \times F$ , and zero-padding with  $(F-1)/2$ . (will preserve size spatially)

e.g.  $F = 3 \Rightarrow$  zero pad with 1

$F = 5 \Rightarrow$  zero pad with 2

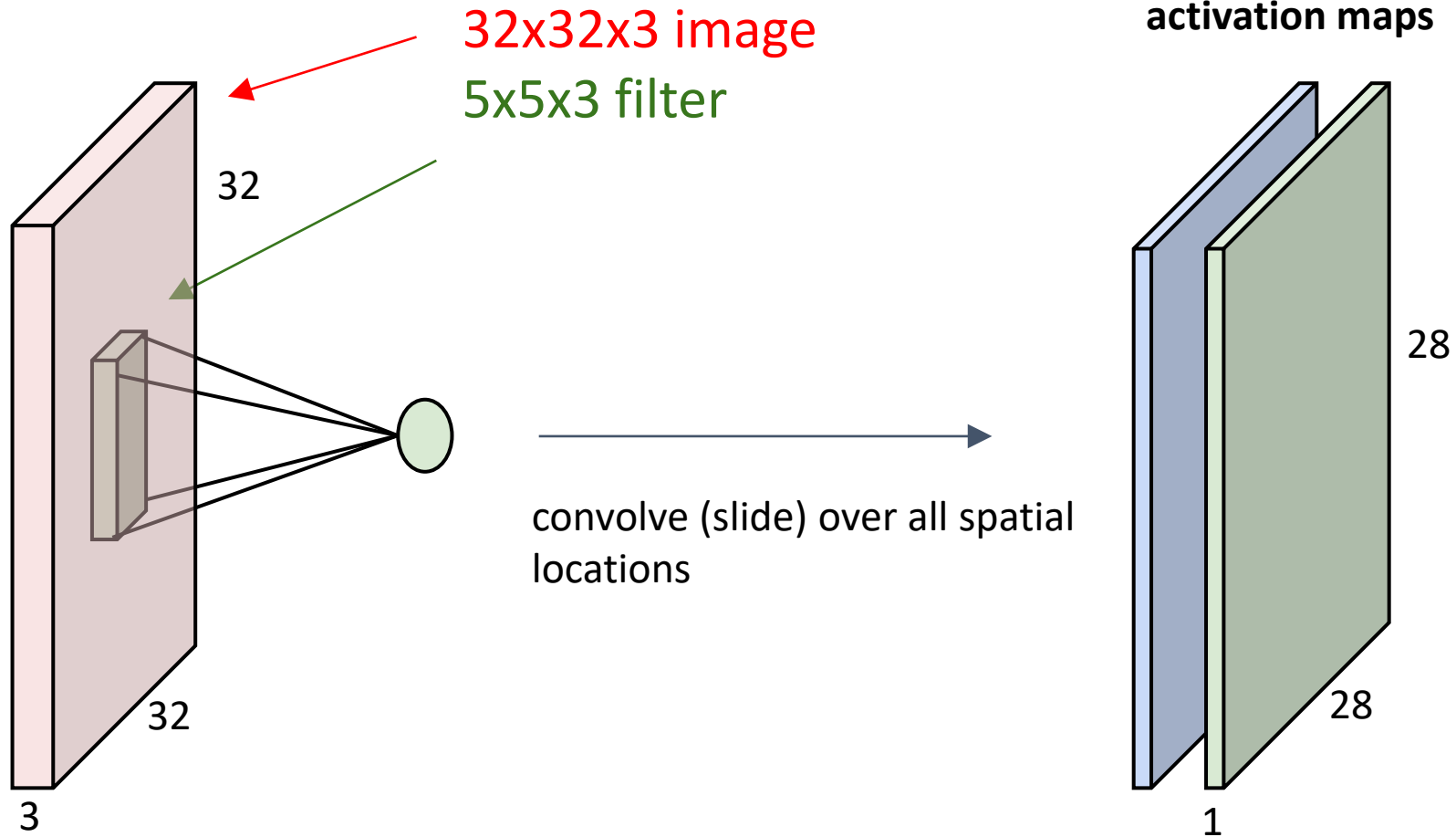
$F = 7 \Rightarrow$  zero pad with 3

# Convolutional Layer

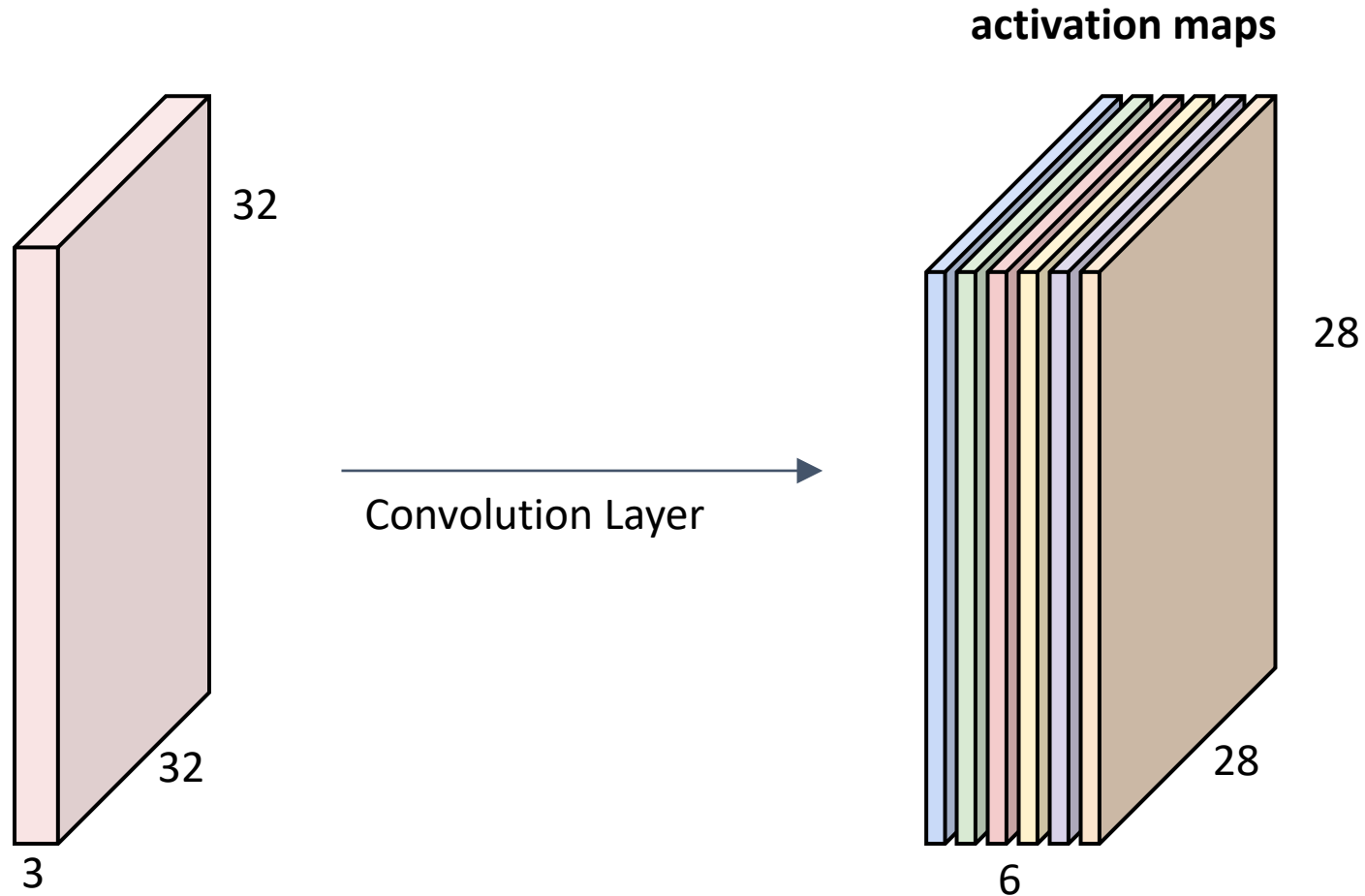


# Convolutional Layer

consider a second, **green** filter



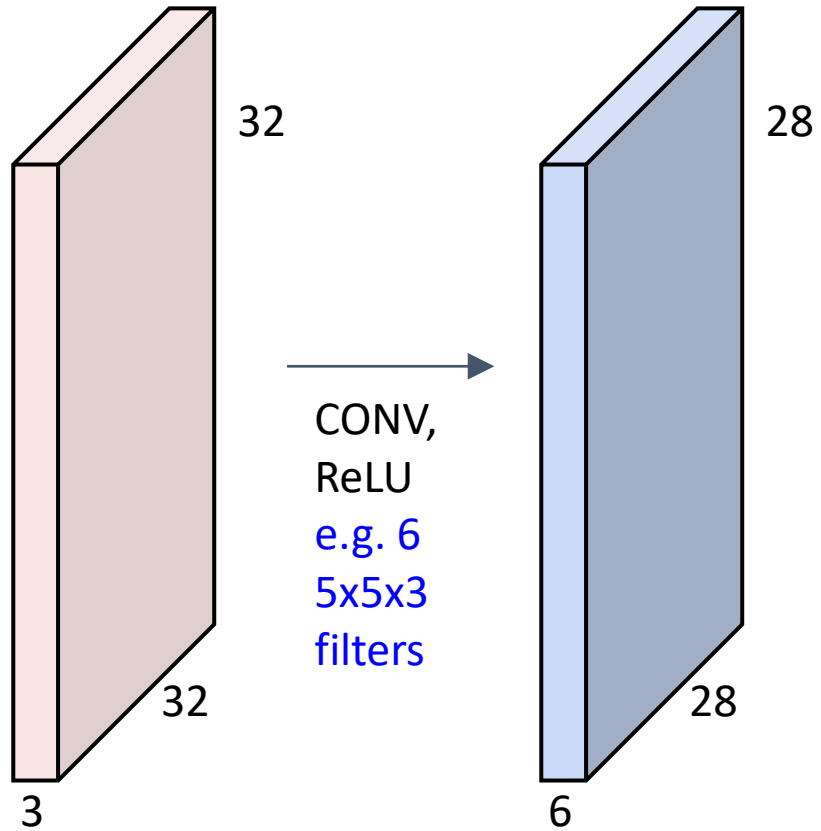
# Convolutional Layer



For example, if we had 6  
5x5 filters, we'll get 6  
separate activation maps

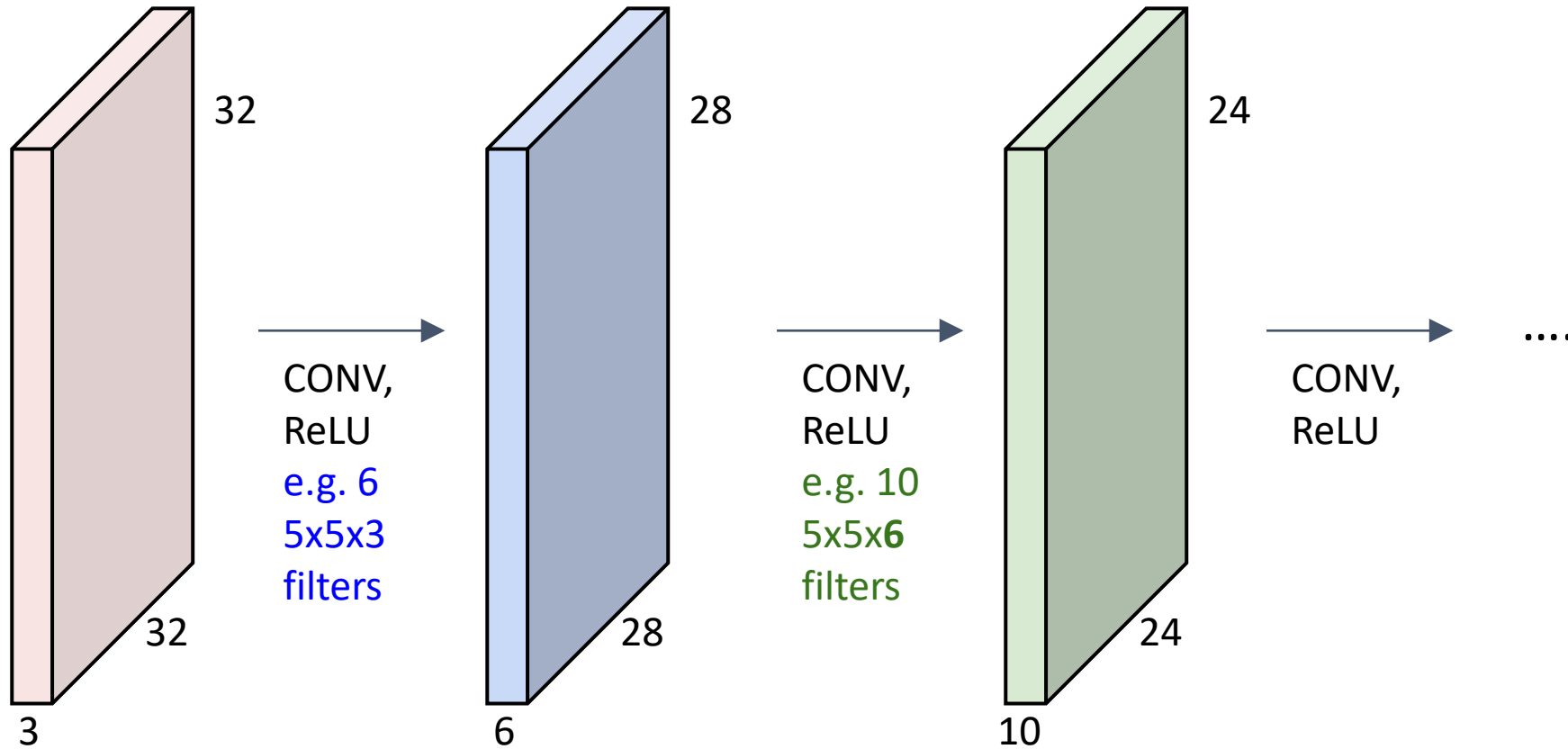
We stack these up to get a "new image" of size 28x28x6!

**Preview:** ConvNet is a sequence of Convolution Layers, interspersed with activation functions

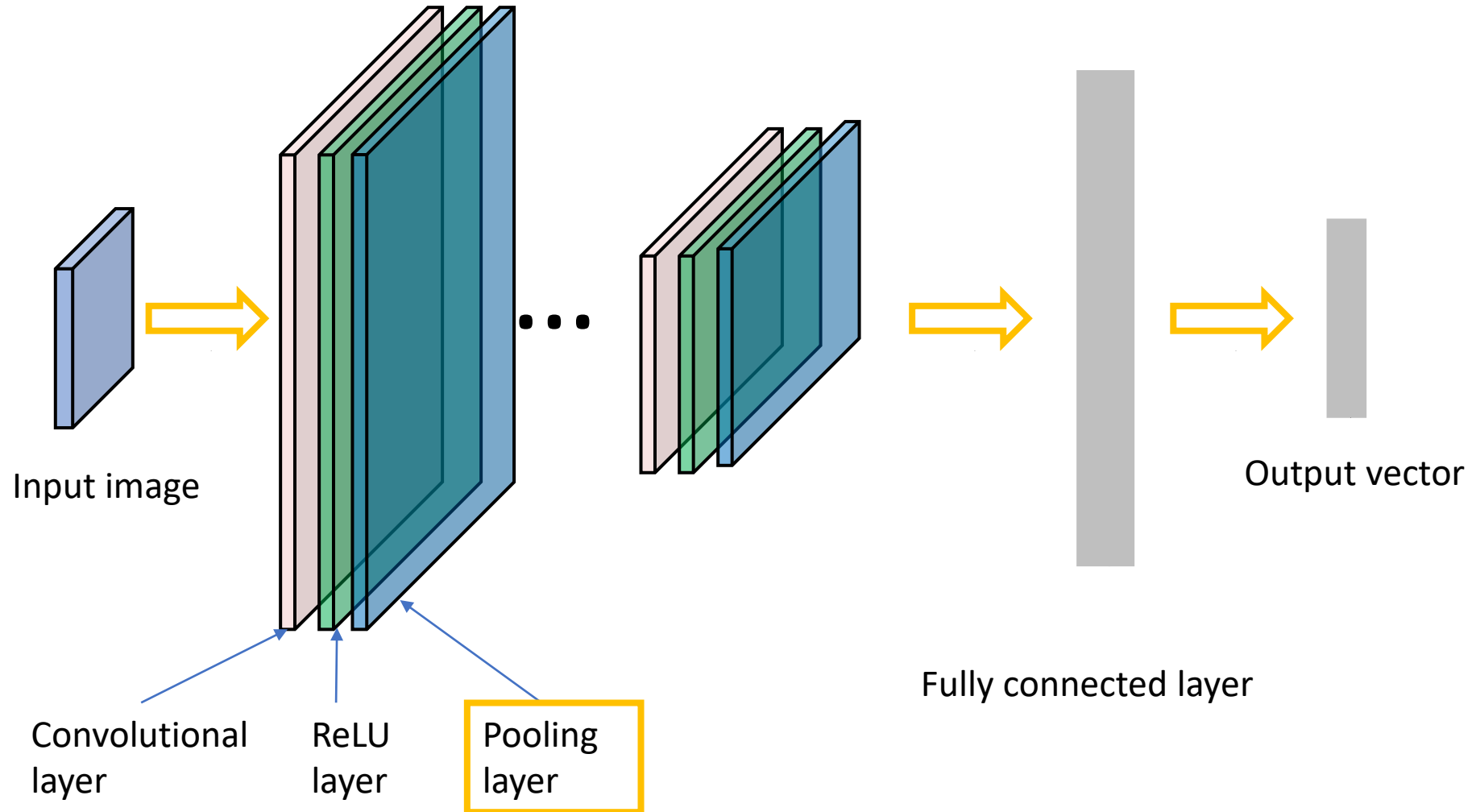




**Preview:** ConvNet is a sequence of Convolutional Layers, interspersed with activation functions

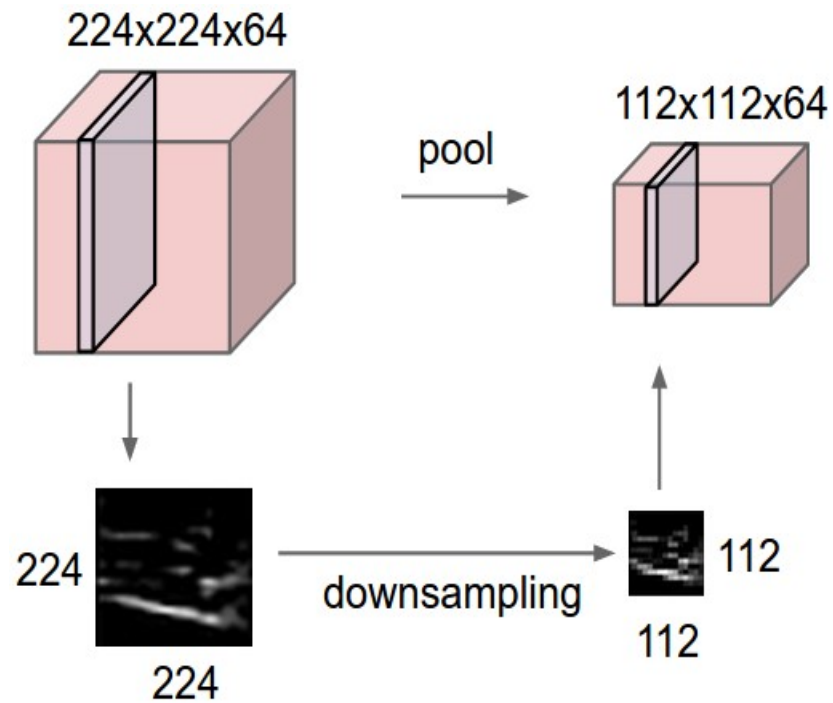


# Convolutional Neural Networks

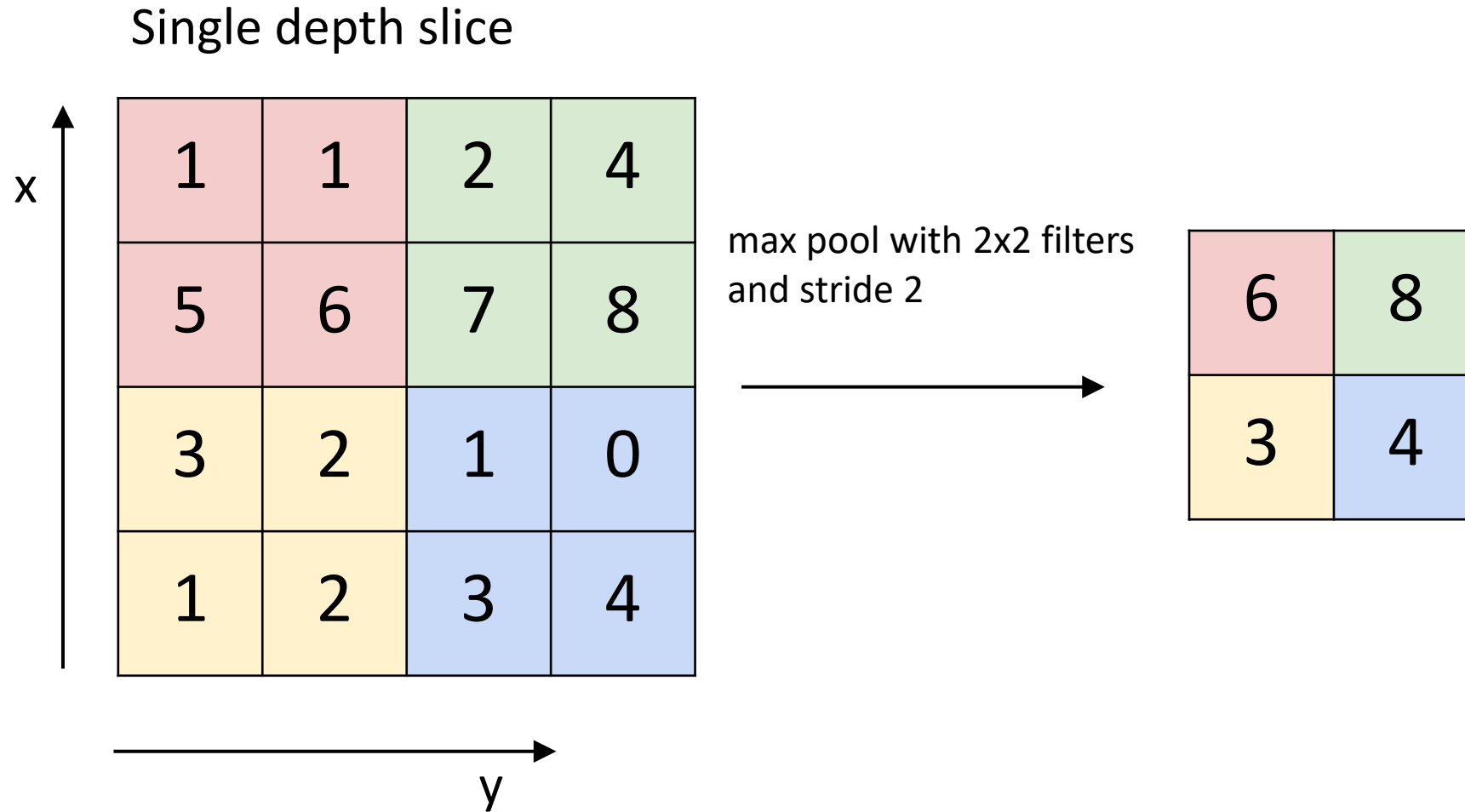


# Pooling Layer

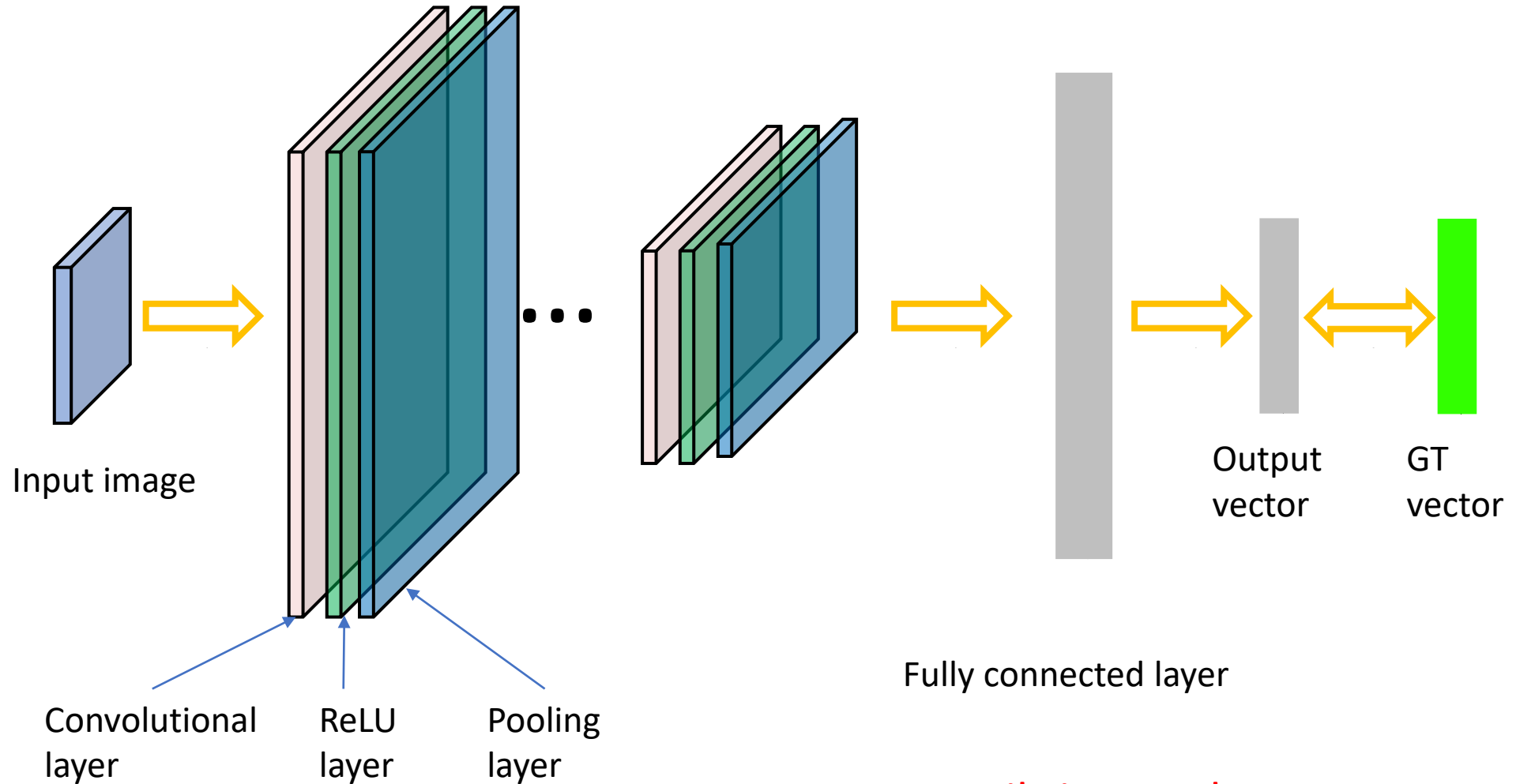
- makes the representations smaller and more manageable
- operates over each activation map independently:



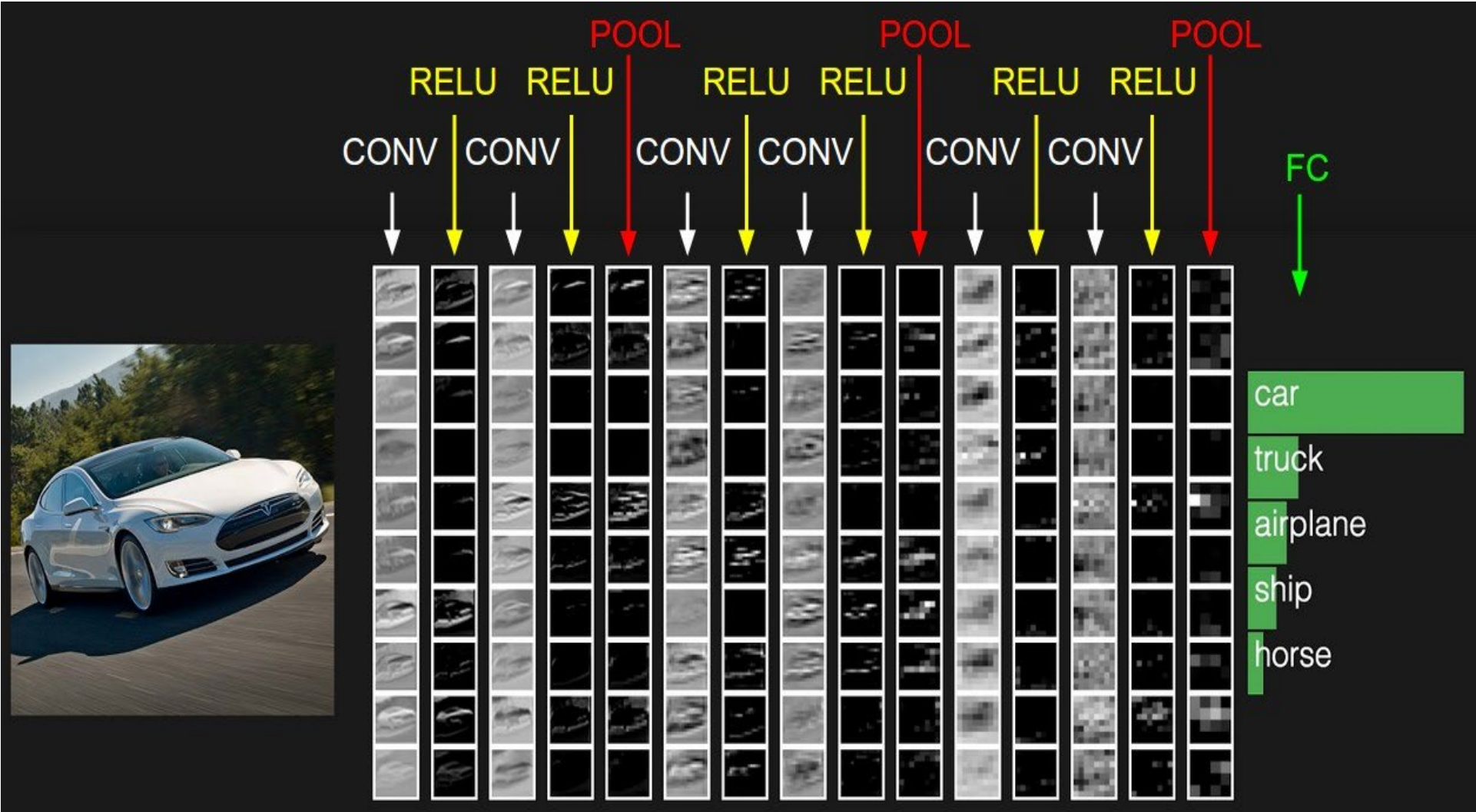
# Max Pooling Layer



# Training: back-propagate errors

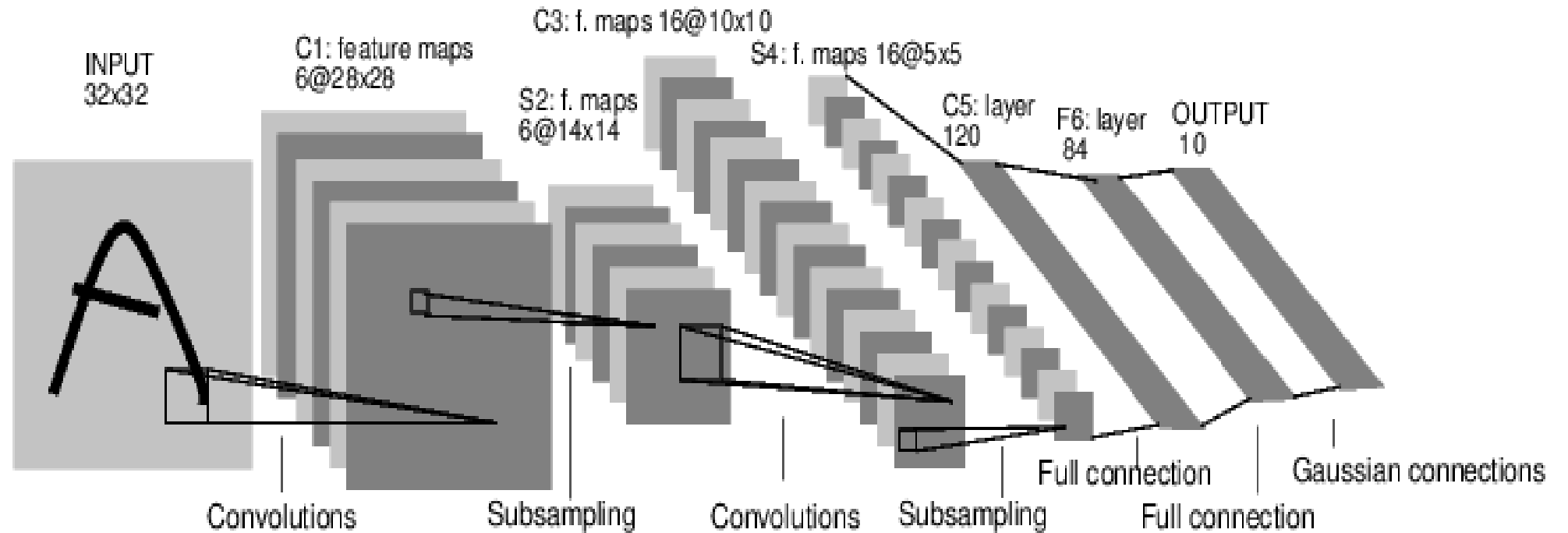


Details in next lecture



# Case Study: LeNet-5

[LeCun et al., 1998]



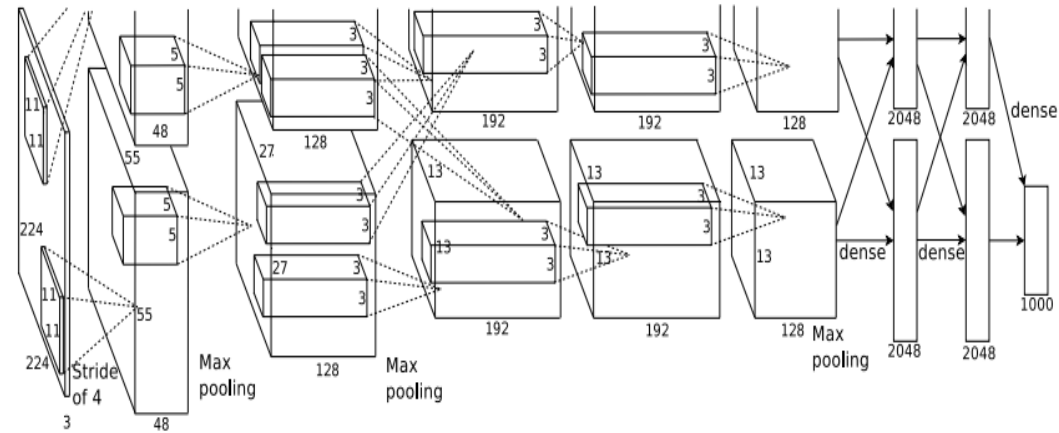
Conv filters were 5x5, applied at stride 1

Subsampling (Pooling) layers were 2x2 applied at stride 2

i.e. architecture is [CONV-POOL-CONV-POOL-CONV-FC]

# Case Study: AlexNet

[Krizhevsky et al. 2012]



Input: 227x227x3 images

**First layer (CONV1):** 96 11x11 filters applied at stride 4

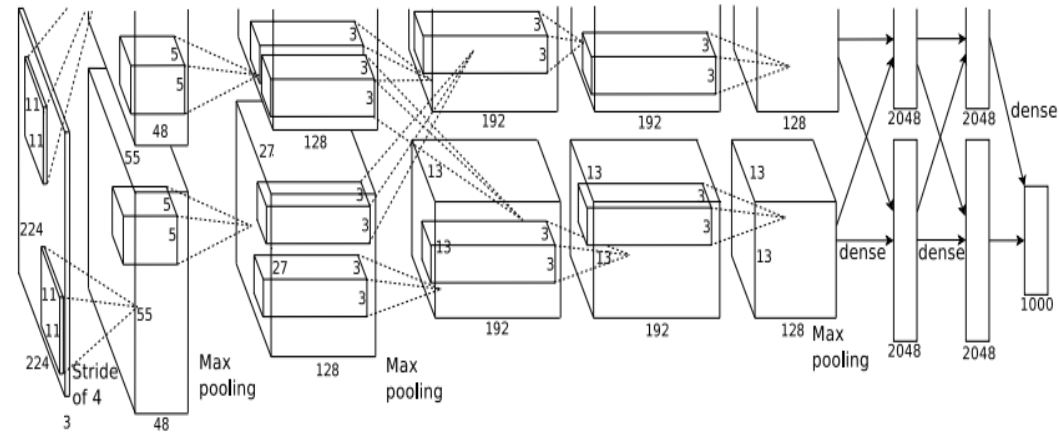
=>

Q: what is the output volume size? Hint:  $(227-11)/4+1 = 55$



# Case Study: AlexNet

[Krizhevsky et al. 2012]



Input: 227x227x3 images

**First layer (CONV1):** 96 11x11 filters applied at stride 4

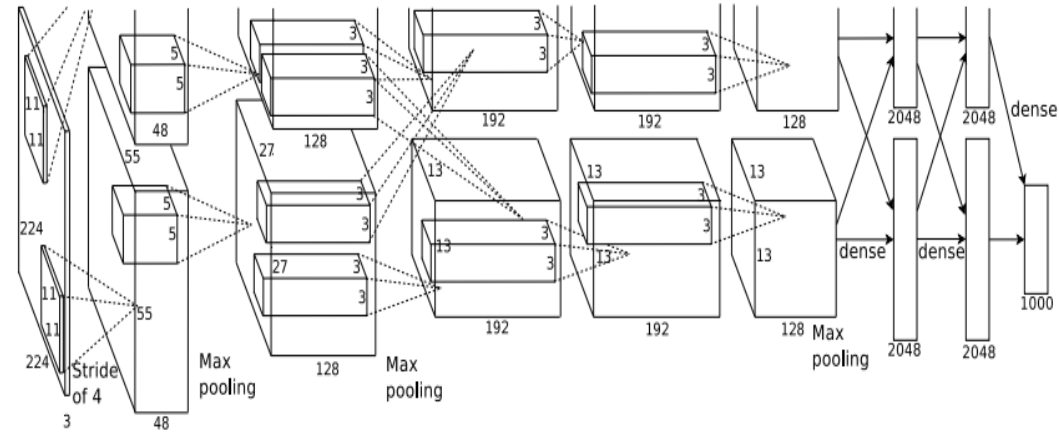
=>

Output volume **[55x55x96]**

Q: What is the total number of parameters in this layer?

# Case Study: AlexNet

[Krizhevsky et al. 2012]



Input: 227x227x3 images

**First layer (CONV1):** 96 11x11 filters applied at stride 4

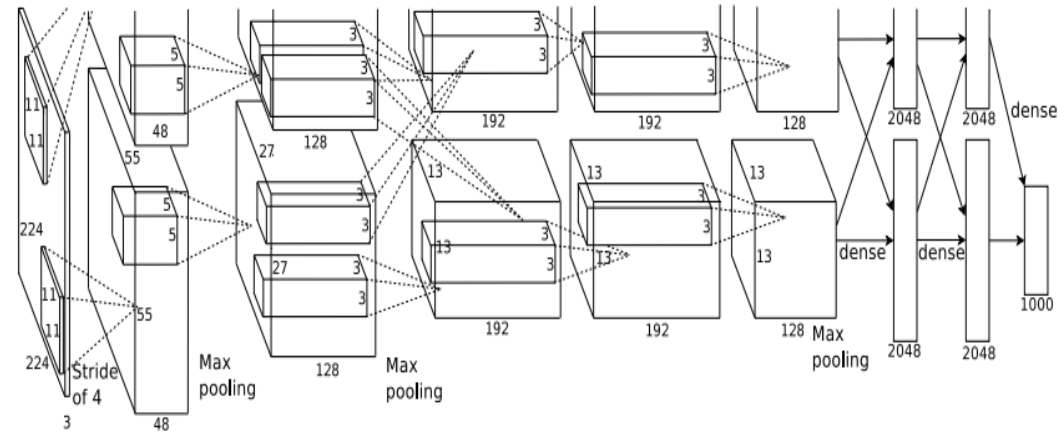
=>

Output volume [55x55x96]

Parameters:  $(11*11*3)*96 = 35K$

# Case Study: AlexNet

[Krizhevsky et al. 2012]



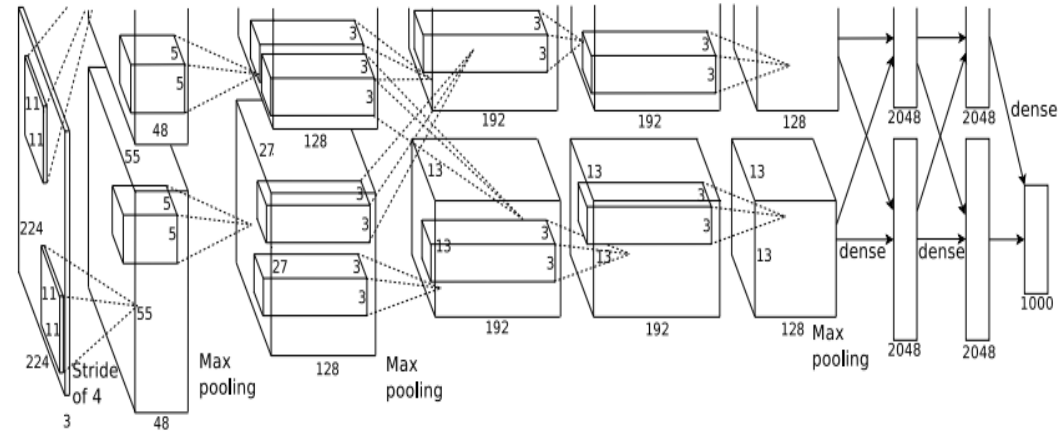
Input: 227x227x3 images  
After CONV1: 55x55x96

**Second layer (POOL1):** 3x3 filters applied at stride 2

Q: what is the output volume size? Hint:  $(55-3)/2+1 = 27$

# Case Study: AlexNet

[Krizhevsky et al. 2012]



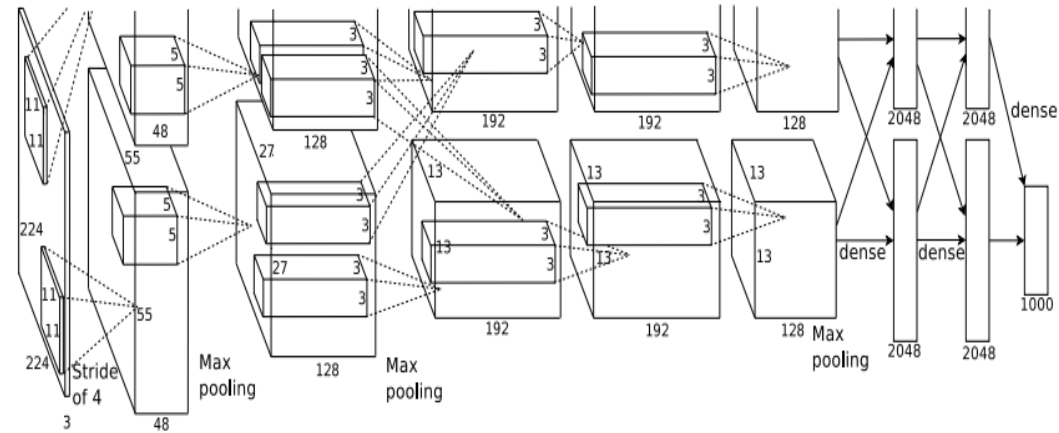
Input: 227x227x3 images  
After CONV1: 55x55x96

**Second layer (POOL1):** 3x3 filters applied at stride 2  
Output volume: 27x27x96

Q: what is the number of parameters in this layer?

# Case Study: AlexNet

[Krizhevsky et al. 2012]



Input: 227x227x3 images

After CONV1: 55x55x96

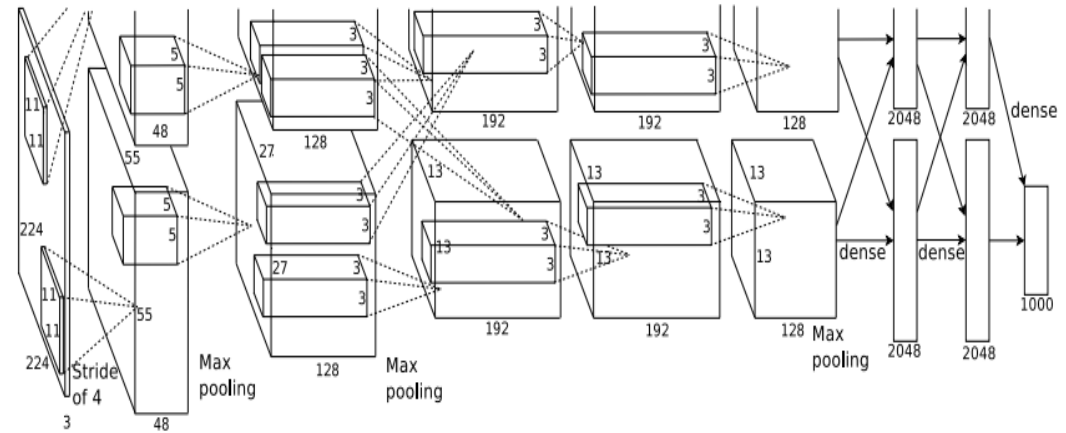
**Second layer (POOL1):** 3x3 filters applied at stride 2

Output volume: 27x27x96

Parameters: 0!

# Case Study: AlexNet

[Krizhevsky et al. 2012]



Input: 227x227x3 images

After CONV1: 55x55x96

After POOL1: 27x27x96

...

# Case Study: AlexNet

[Krizhevsky et al. 2012]

Full (simplified) AlexNet architecture:

[227x227x3] INPUT

[55x55x96] **CONV1**: 96 11x11 filters at stride 4, pad 0

[27x27x96] **MAX POOL1**: 3x3 filters at stride 2

[27x27x96] **NORM1**: Normalization layer

[27x27x256] **CONV2**: 256 5x5 filters at stride 1, pad 2

[13x13x256] **MAX POOL2**: 3x3 filters at stride 2

[13x13x256] **NORM2**: Normalization layer

[13x13x384] **CONV3**: 384 3x3 filters at stride 1, pad 1

[13x13x384] **CONV4**: 384 3x3 filters at stride 1, pad 1

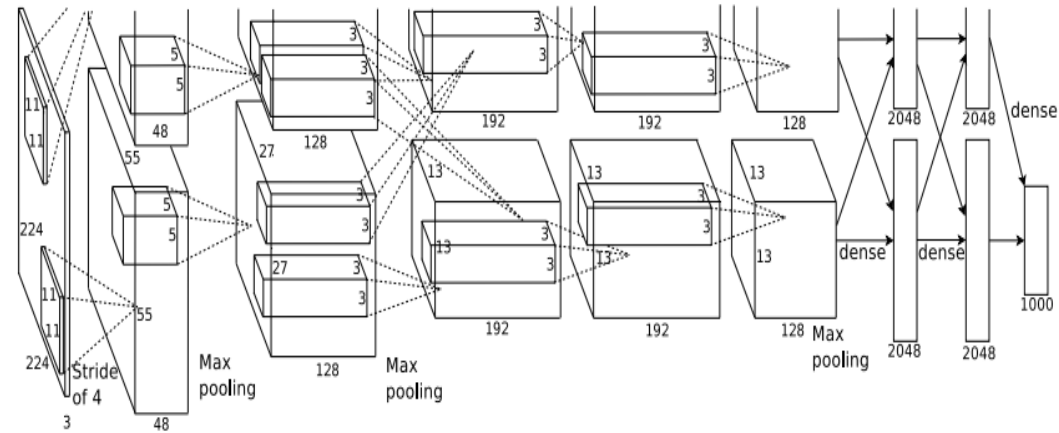
[13x13x256] **CONV5**: 256 3x3 filters at stride 1, pad 1

[6x6x256] **MAX POOL3**: 3x3 filters at stride 2

[4096] **FC6**: 4096 neurons

[4096] **FC7**: 4096 neurons

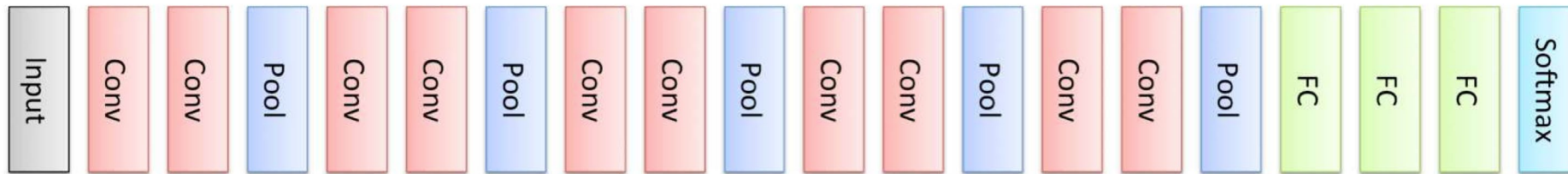
[1000] **FC8**: 1000 neurons (class scores)



# Case Study: VGGNet

[Simonyan and Zisserman, 2014]

## VGGNet



Only 3x3 CONV stride 1, pad 1  
and 2x2 MAX POOL stride 2

11.2% top 5 error in ILSVRC 2013

->

7.3% top 5 error



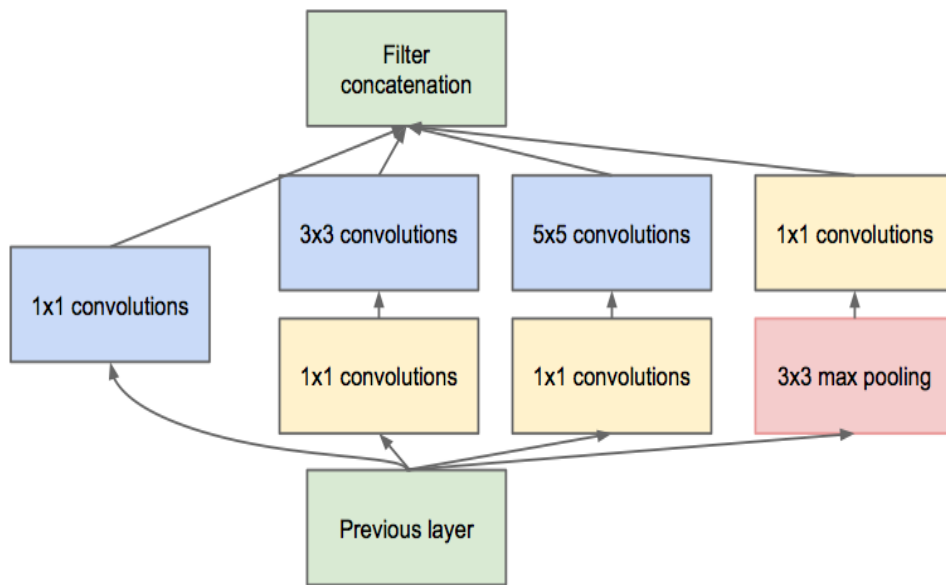
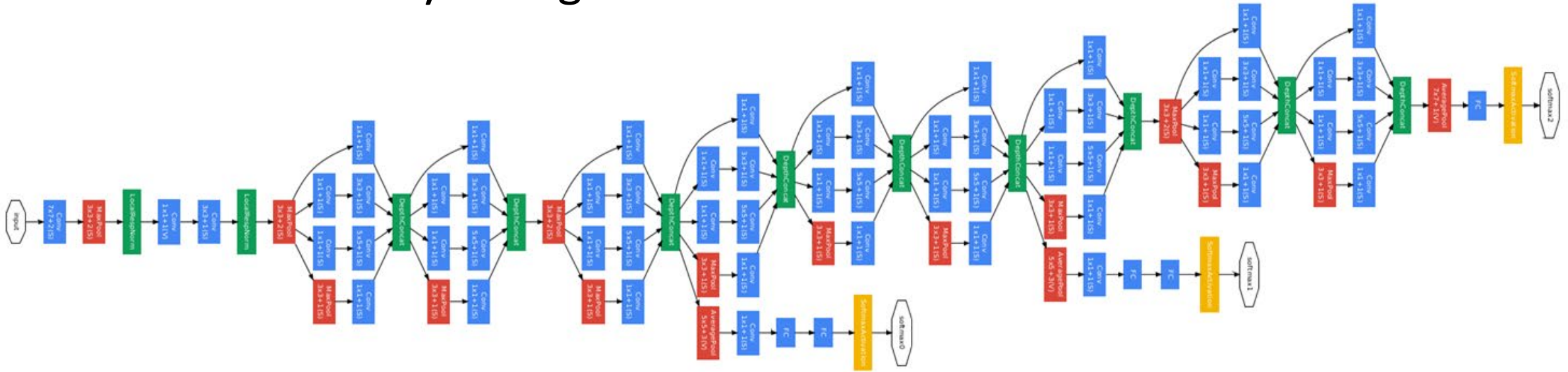
# Case Study: VGGNet

[Simonyan and Zisserman, 2014]

INPUT: [224x224x3] memory:  $224*224*3=150\text{K}$  params: 0  
CONV3-64: [224x224x64] memory:  $224*224*64=3.2\text{M}$  params:  $(3*3*3)*64 = 1,728$   
CONV3-64: [224x224x64] memory:  $224*224*64=3.2\text{M}$  params:  $(3*3*64)*64 = 36,864$   
POOL2: [112x112x64] memory:  $112*112*64=800\text{K}$  params: 0  
CONV3-128: [112x112x128] memory:  $112*112*128=1.6\text{M}$  params:  $(3*3*64)*128 = 73,728$   
CONV3-128: [112x112x128] memory:  $112*112*128=1.6\text{M}$  params:  $(3*3*128)*128 = 147,456$   
POOL2: [56x56x128] memory:  $56*56*128=400\text{K}$  params: 0  
CONV3-256: [56x56x256] memory:  $56*56*256=800\text{K}$  params:  $(3*3*128)*256 = 294,912$   
CONV3-256: [56x56x256] memory:  $56*56*256=800\text{K}$  params:  $(3*3*256)*256 = 589,824$   
CONV3-256: [56x56x256] memory:  $56*56*256=800\text{K}$  params:  $(3*3*256)*256 = 589,824$   
POOL2: [28x28x256] memory:  $28*28*256=200\text{K}$  params: 0  
CONV3-512: [28x28x512] memory:  $28*28*512=400\text{K}$  params:  $(3*3*256)*512 = 1,179,648$   
CONV3-512: [28x28x512] memory:  $28*28*512=400\text{K}$  params:  $(3*3*512)*512 = 2,359,296$   
CONV3-512: [28x28x512] memory:  $28*28*512=400\text{K}$  params:  $(3*3*512)*512 = 2,359,296$   
POOL2: [14x14x512] memory:  $14*14*512=100\text{K}$  params: 0  
CONV3-512: [14x14x512] memory:  $14*14*512=100\text{K}$  params:  $(3*3*512)*512 = 2,359,296$   
CONV3-512: [14x14x512] memory:  $14*14*512=100\text{K}$  params:  $(3*3*512)*512 = 2,359,296$   
CONV3-512: [14x14x512] memory:  $14*14*512=100\text{K}$  params:  $(3*3*512)*512 = 2,359,296$   
POOL2: [7x7x512] memory:  $7*7*512=25\text{K}$  params: 0  
FC: [1x1x4096] memory: 4096 params:  $7*7*512*4096 = 102,760,448$   
FC: [1x1x4096] memory: 4096 params:  $4096*4096 = 16,777,216$   
FC: [1x1x1000] memory: 1000 params:  $4096*1000 = 4,096,000$  (not counting biases)

# Case Study: GoogLeNet

[Szegedy et al., 2014]



Inception module

ILSVRC 2014 winner (6.7% top 5 error)

# Case Study: GoogLeNet

type	patch size/ stride	output size	depth	#1×1	#3×3 reduce	#3×3	#5×5 reduce	#5×5	pool proj	params	ops
convolution	7×7/2	112×112×64	1							2.7K	34M
max pool	3×3/2	56×56×64	0								
convolution	3×3/1	56×56×192	2		64	192				112K	360M
max pool	3×3/2	28×28×192	0								
inception (3a)		28×28×256	2	64	96	128	16	32	32	159K	128M
inception (3b)		28×28×480	2	128	128	192	32	96	64	380K	304M
max pool	3×3/2	14×14×480	0								
inception (4a)		14×14×512	2	192	96	208	16	48	64	364K	73M
inception (4b)		14×14×512	2	160	112	224	24	64	64	437K	88M
inception (4c)		14×14×512	2	128	128	256	24	64	64	463K	100M
inception (4d)		14×14×528	2	112	144	288	32	64	64	580K	119M
inception (4e)		14×14×832	2	256	160	320	32	128	128	840K	170M
max pool	3×3/2	7×7×832	0								
inception (5a)		7×7×832	2	256	160	320	32	128	128	1072K	54M
inception (5b)		7×7×1024	2	384	192	384	48	128	128	1388K	71M
avg pool	7×7/1	1×1×1024	0								
dropout (40%)		1×1×1024	0								
linear		1×1×1000	1							1000K	1M
softmax		1×1×1000	0								

Fun features:

- Only 5 million params!

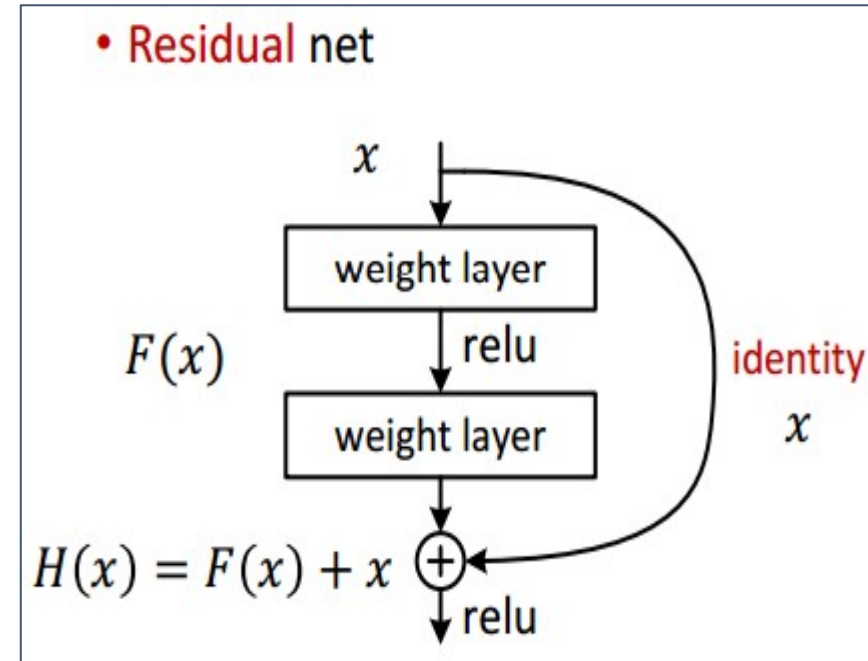
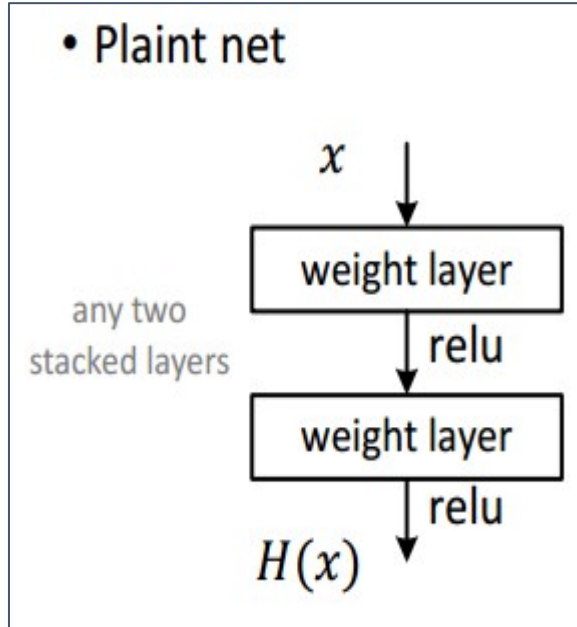
**Compared to AlexNet:**

- 12X less params
- 2x more compute
- 6.67% (vs. 16.4%)

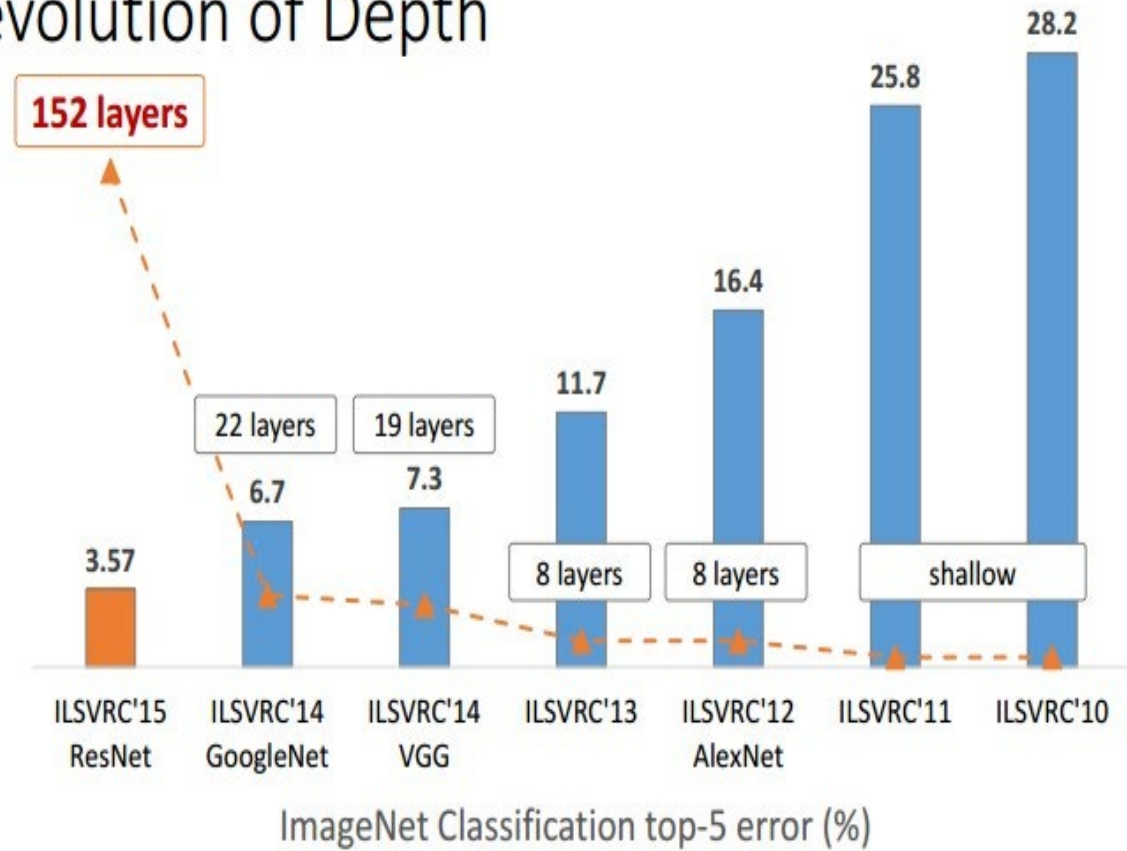


# Case Study: ResNet

[He et al., 2015]



# Revolution of Depth



Kaiming He, Xiangyu Zhang, Shaoqing Ren, & Jian Sun. "Deep Residual Learning for Image Recognition". arXiv 2015.



(slide from Kaiming He)



# Further Reading

- Stanford CS231n, lecture 5, Convolutional Neural Networks  
<http://cs231n.stanford.edu/schedule.html>
- Deep learning with PyTorch  
[https://pytorch.org/tutorials/beginner/deep\\_learning\\_60min\\_blitz.html](https://pytorch.org/tutorials/beginner/deep_learning_60min_blitz.html)
- AlexNet (2012):  
<https://papers.nips.cc/paper/2012/hash/c399862d3b9d6b76c8436e924a68c45b-Abstract.html>
- Vgg16 (2014): <https://arxiv.org/abs/1409.1556>
- GoogleNet (2014): <https://arxiv.org/abs/1409.4842>
- ResNet (2015): <https://arxiv.org/abs/1512.03385>