Epipolar Geometry and Stereo

CS 6384 Computer Vision

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Depth Perception

- Metric
  - The car is 10 meters away
- Ordinary
  - The tree is behind the car

Depth Cues

- Information for sensory stimulation that is relevant to depth perception

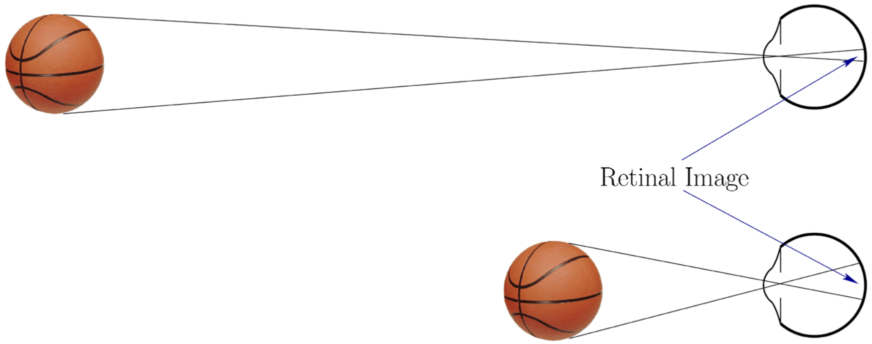
- Monocular cues: single eye

- Stereo cues: both eyes

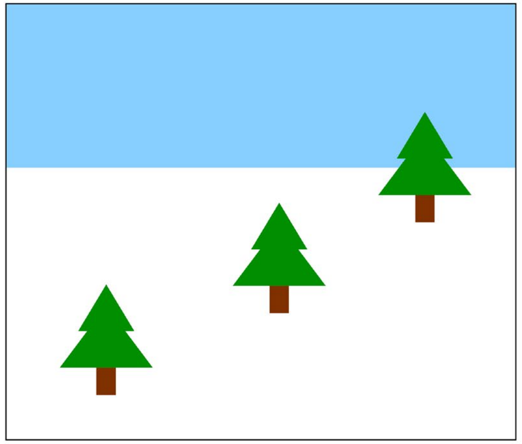
“Paris Street, Rainy Day,” Gustave Caillebotte, 1877. Art Institute of Chicago

- Texture of the bricks
- Perspective projection
- Etc.

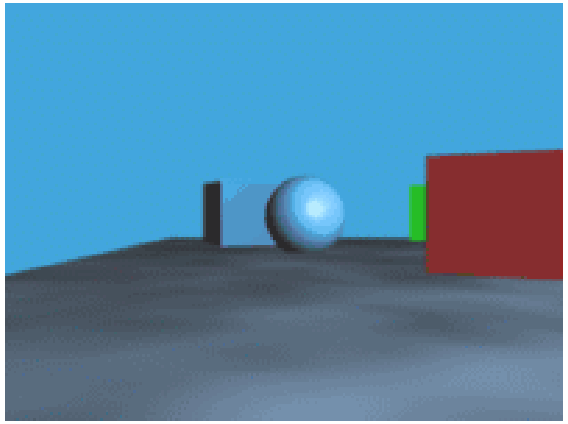
Monocular Depth Cues



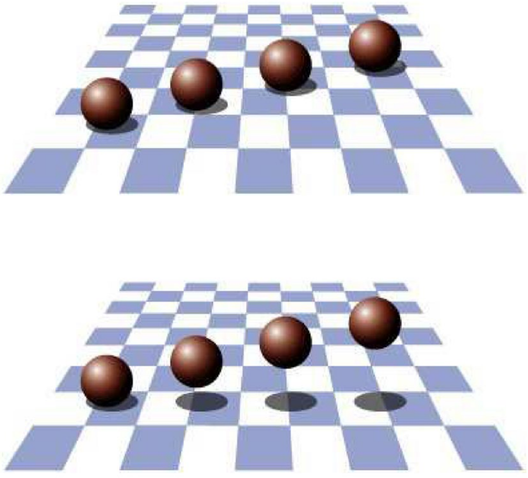
Retina image size



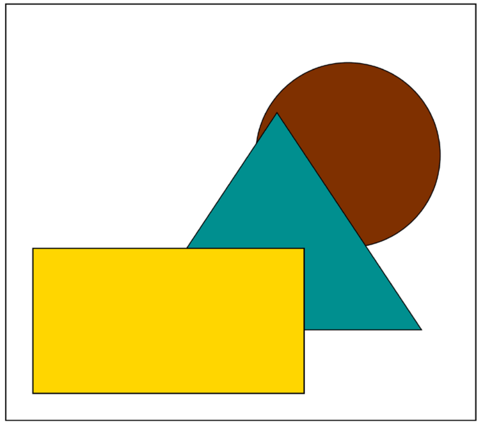
Height in visual field



Motion parallax (relative difference in speed)
Further objects move slower



Shadow



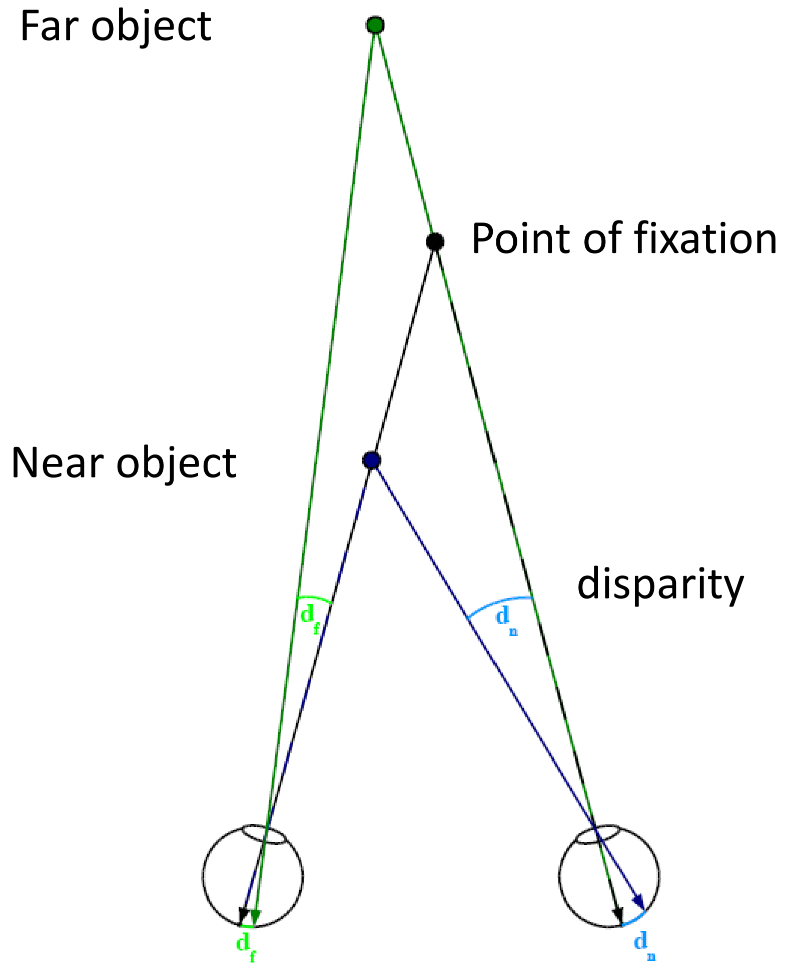
Occlusions

Image blur

Atmospheric cue
further away because it has lower contrast

Stereo Depth Cues

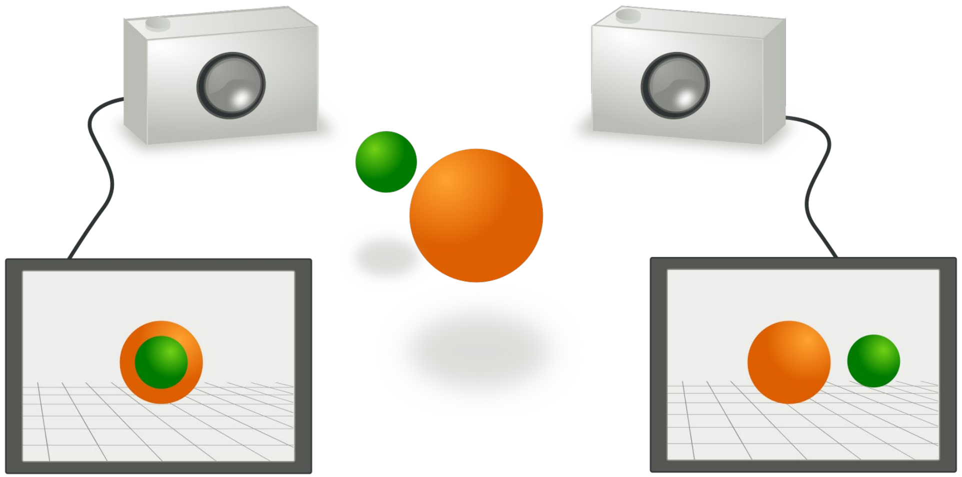
- Binocular disparity
  - Each eye provides a different viewpoint, which results in different images on the retina



From Wikipedia

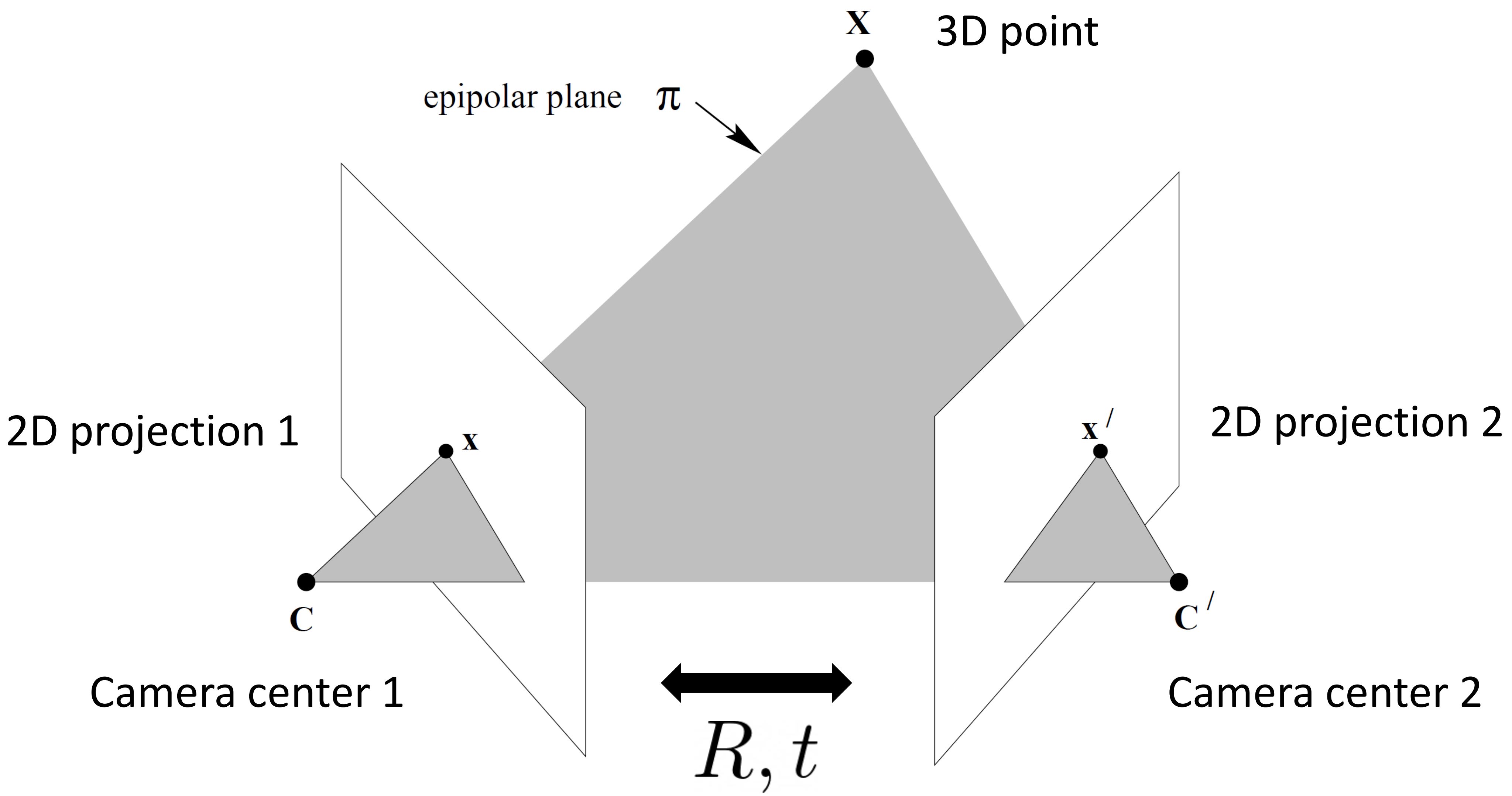
Epipolar Geometry

- The geometry of stereo vision
  - Given 2D images of two views
  - What is the relationship between pixels of the images?
  - Can we recover the 3D structure of the world from the 2D images?

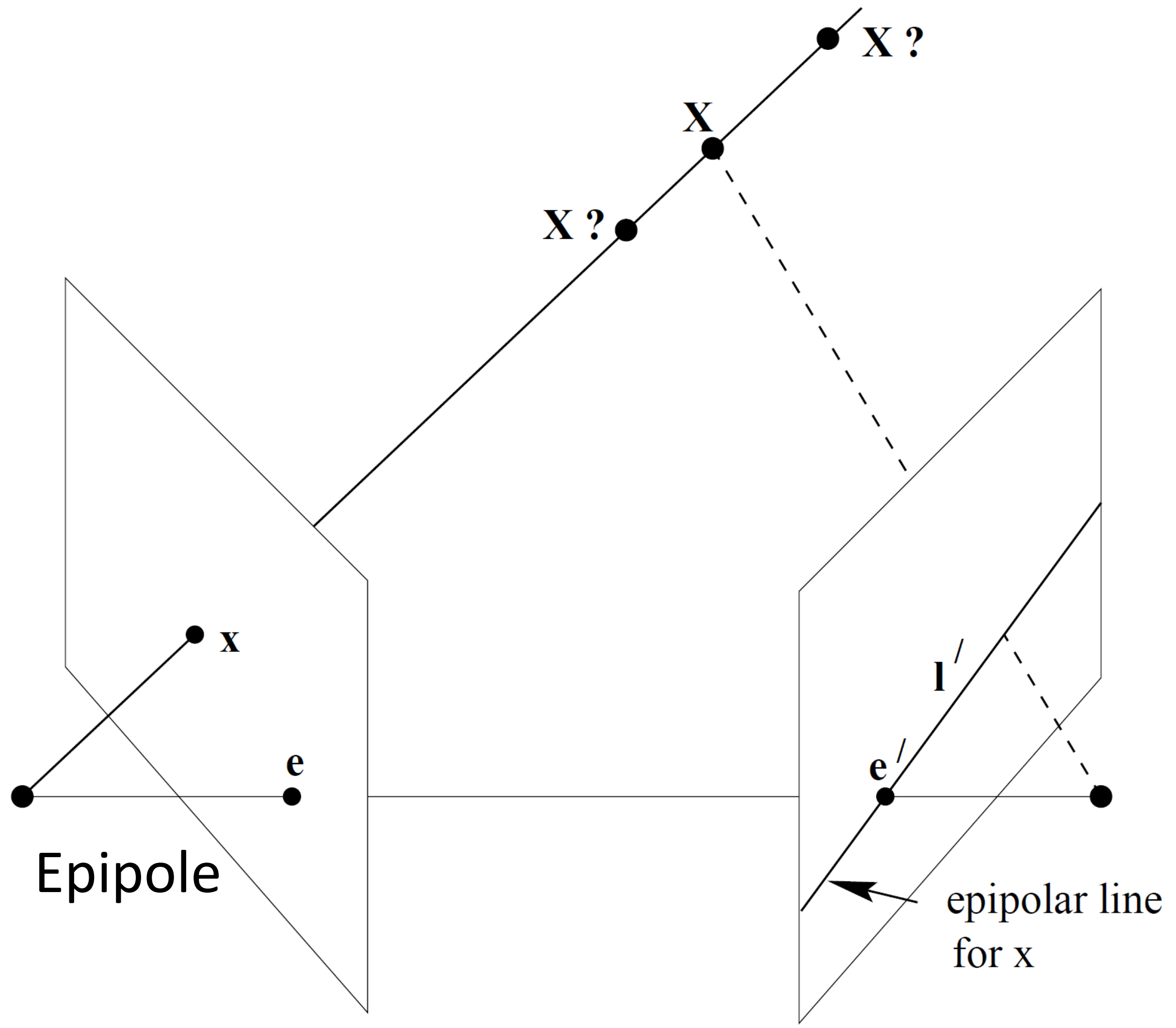


Wikipedia

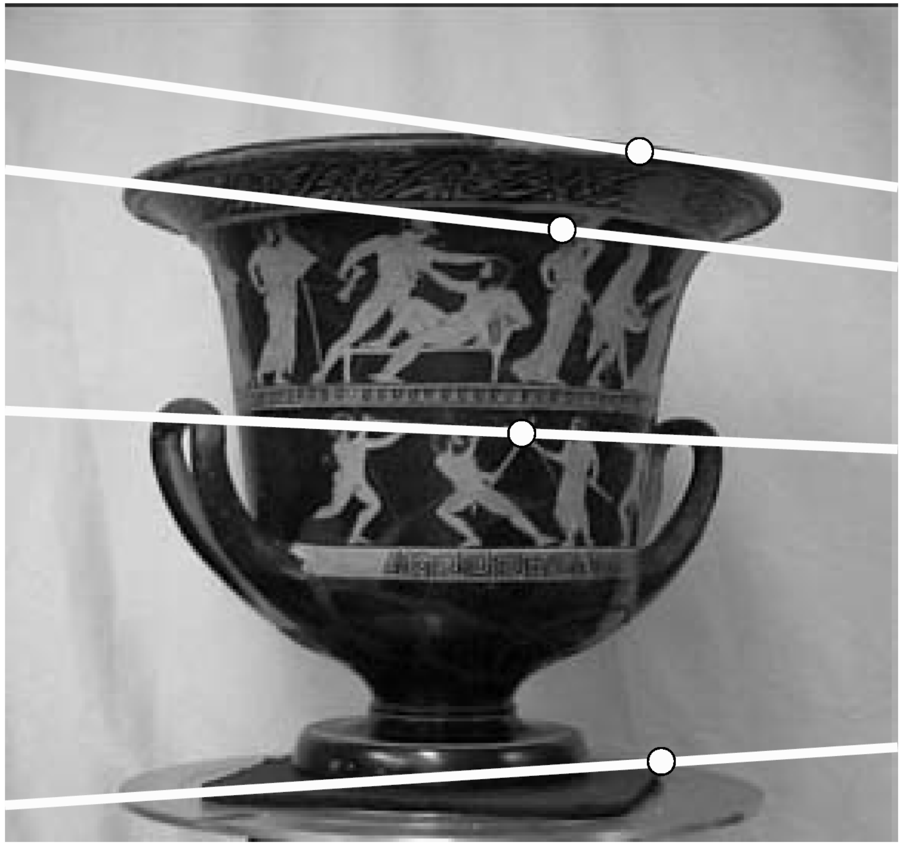
Epipolar Geometry



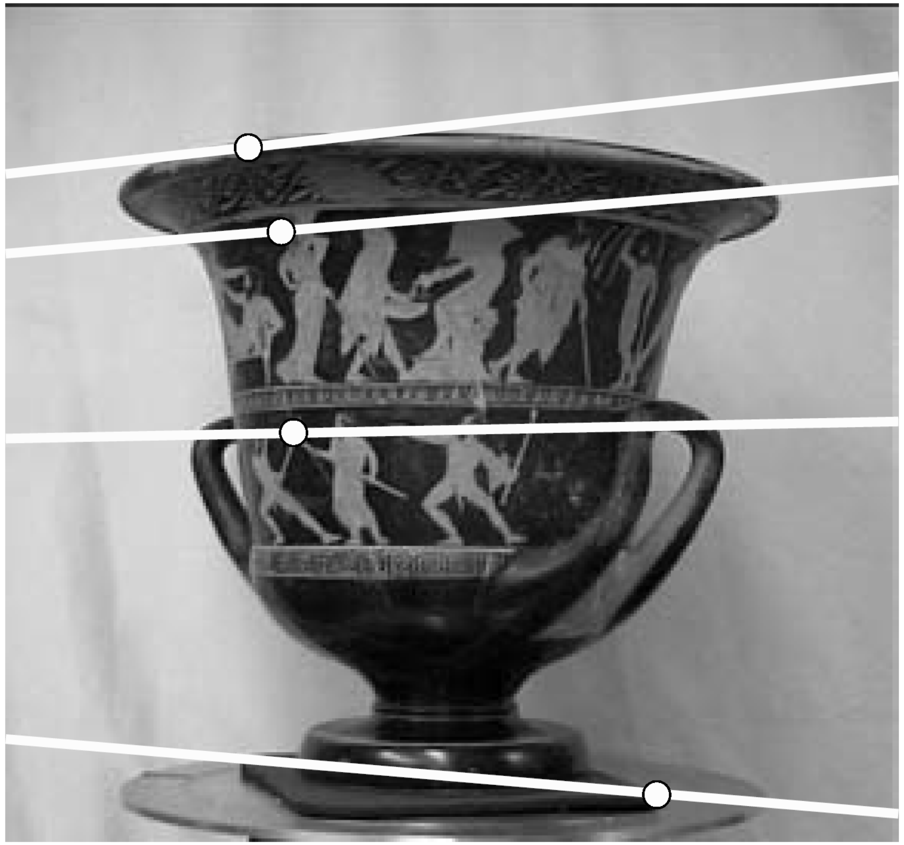
Epipolar Geometry



Epipolar Geometry



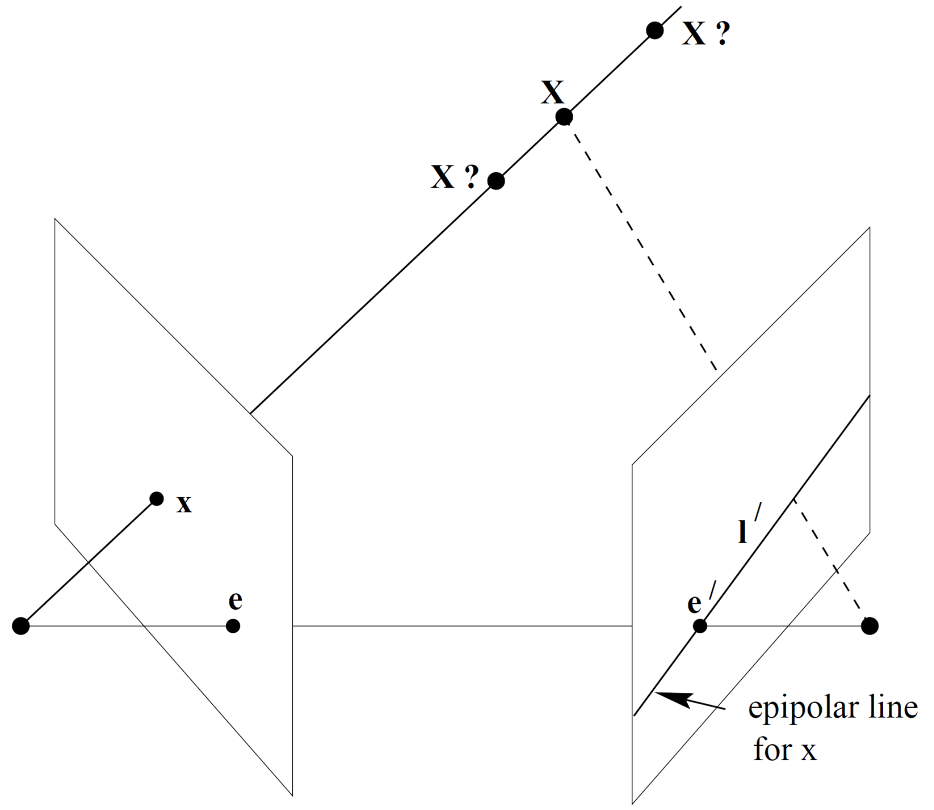
Epipolar lines



Rotation and Translation
between two views

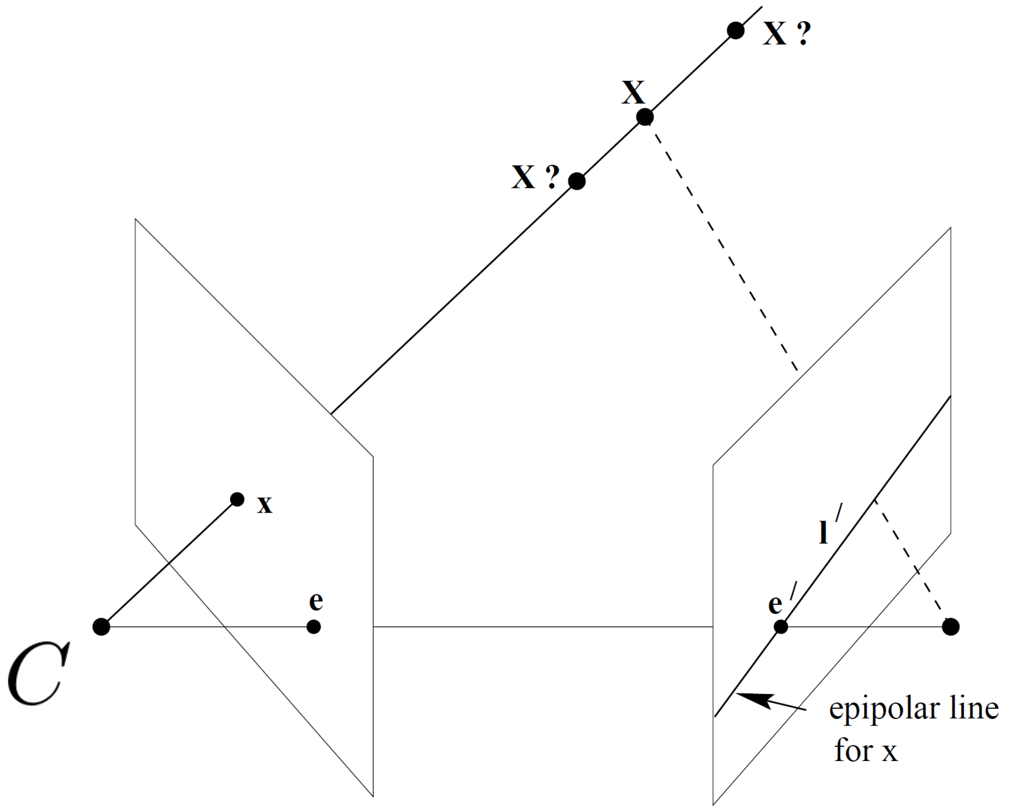
Epipolar Geometry

- What is the mapping for a point in one image to its epipolar line?



$$\mathbf{x} \mapsto \mathbf{l}'$$

Fundamental Matrix



- Recall camera projection

$$P = K[R|\mathbf{t}]$$

$$\mathbf{x} = P\mathbf{X}$$ Homogeneous coordinates

- Backprojection

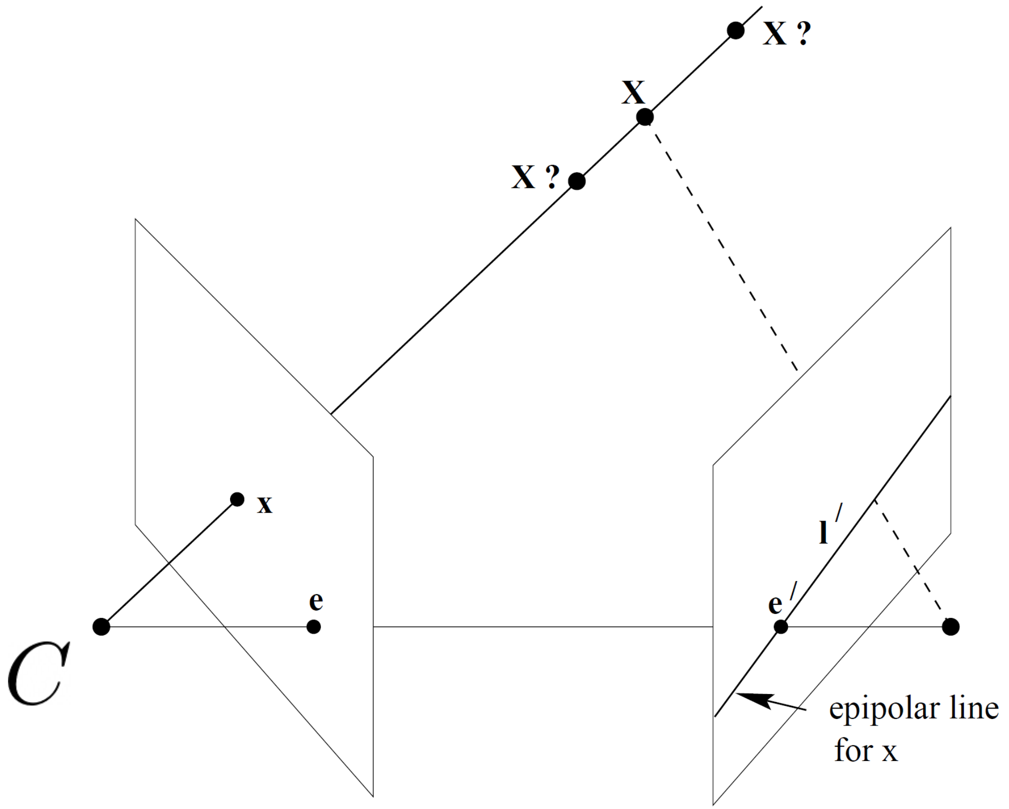
$P^{+}\mathbf{x}$ and C are two points on the ray

P^{+} is the pseudo-inverse of P, $PP^{+} = I$

$$P^{+} = P^{T}(PP^{T})^{-1}$$

$$\mathbf{X}(\lambda) = (1 - \lambda)P^{+}\mathbf{x} + \lambda C$$

Fundamental Matrix



$P^{+}\mathbf{x}$ and C are two points on the ray

- Project to the other image

$P'P^{+}\mathbf{x}$ and $P'C$

- Epipolar line

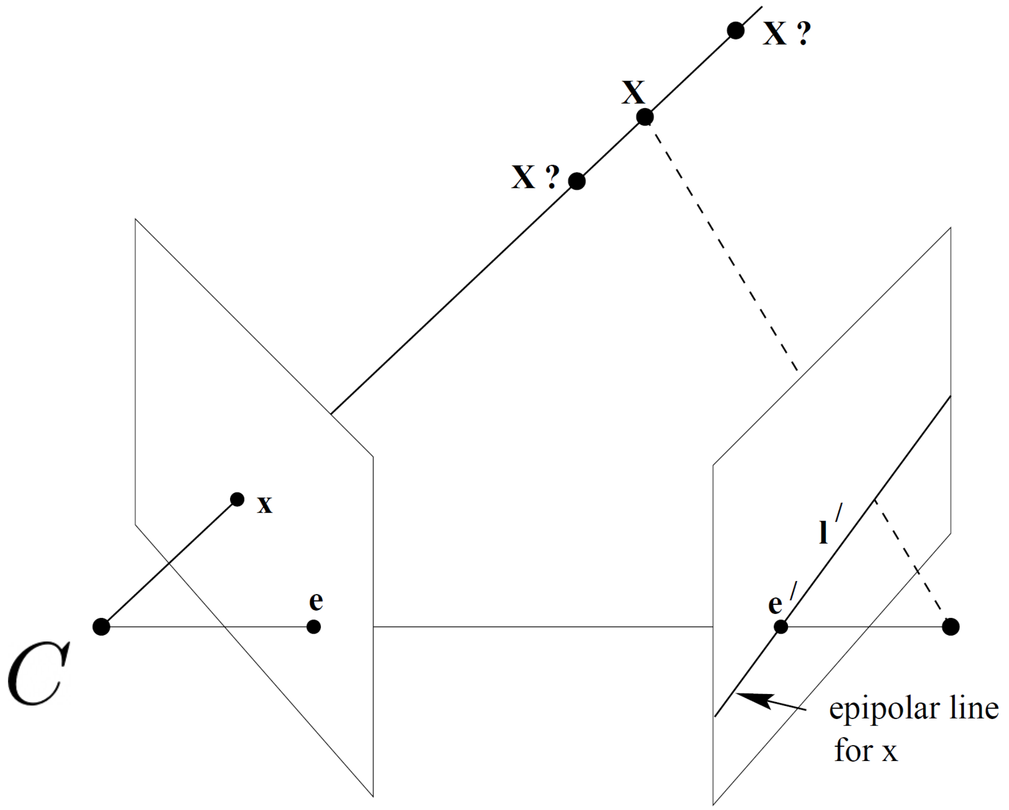
$$\mathbf{l}' = (P'C) \times (P'P^{+}\mathbf{x})$$

Epipole $\mathbf{e}' = (P'C)$

$$\mathbf{l}' = [\mathbf{e}']_{\times}(P'P^{+}\mathbf{x})$$

Cross product matrix

Fundamental Matrix



- Epipolar line

$$\mathbf{l}' = [\mathbf{e}']_\times (P' P^+ \mathbf{x}) = F\mathbf{x}$$

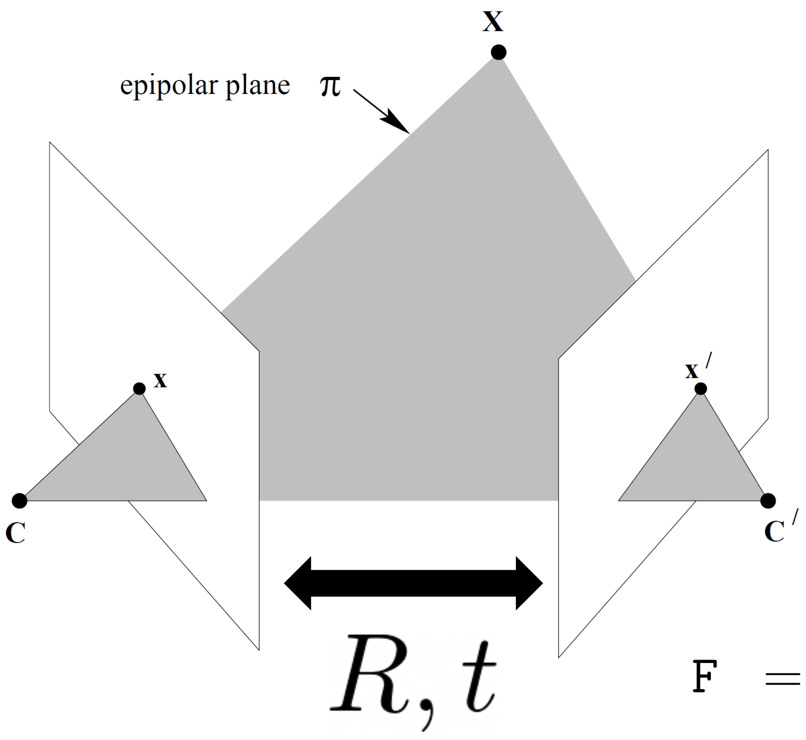
- Fundamental matrix

$$F = [\mathbf{e}']_\times P' P^+$$

3x3

$$\mathbf{l}' = F\mathbf{x}$$

Fundamental Matrix



$$F = [\mathbf{e}']_\times P' P^+ \qquad \mathbf{e}' = (P'C)$$

$$\mathtt{P} = \mathtt{K}[\mathtt{I} \mid \mathbf{0}] \qquad \mathtt{P}' = \mathtt{K}'[\mathtt{R} \mid \mathbf{t}]$$

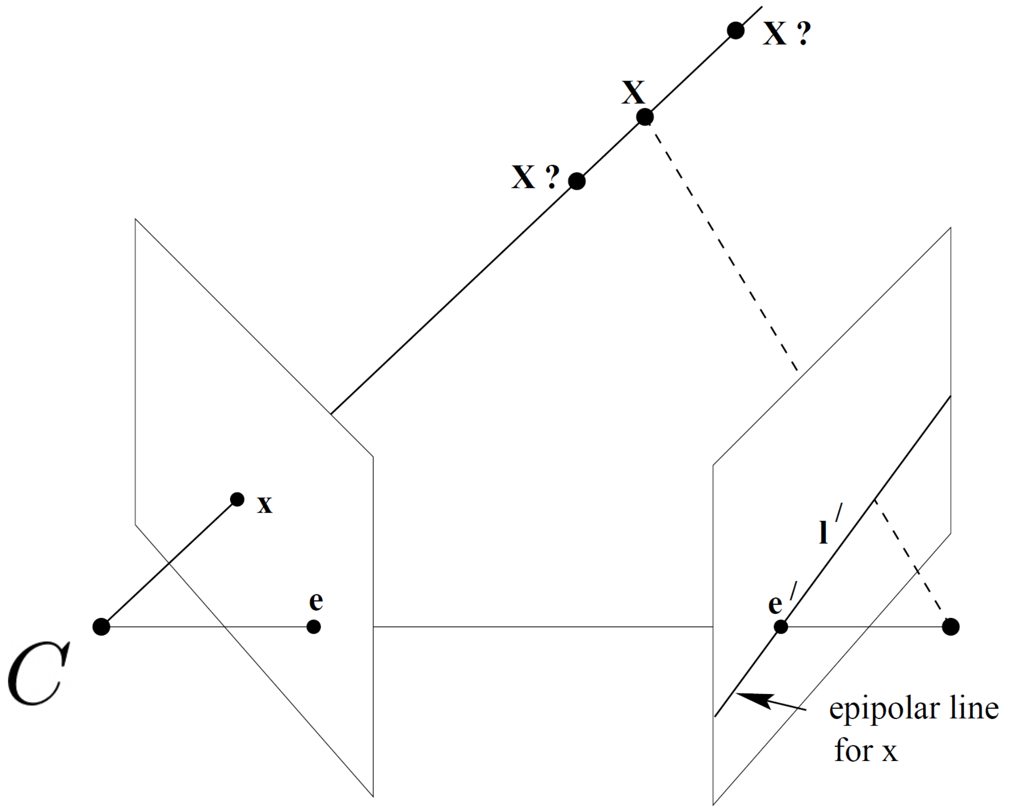
$$\mathtt{P}^+ = \begin{bmatrix} \mathtt{K}^{-1} \\ \mathbf{0}^\mathsf{T} \end{bmatrix} \qquad \mathbf{C} = \begin{pmatrix} \mathbf{0} \\ 1 \end{pmatrix}$$

$$\begin{aligned} \mathtt{F} &= [\mathtt{P}'\mathbf{C}]_\times \mathtt{P}'\mathtt{P}^+ \\ &= [\mathtt{K}'\mathbf{t}]_\times \mathtt{K}'\mathtt{R}\mathtt{K}^{-1} = \mathtt{K}'^{-\mathsf{T}}[\mathbf{t}]_\times \mathtt{R}\mathtt{K}^{-1} = \mathtt{K}'^{-\mathsf{T}}\mathtt{R}[\mathtt{R}^\mathsf{T}\mathbf{t}]_\times \mathtt{K}^{-1} = \mathtt{K}'^{-\mathsf{T}}\mathtt{R}\mathtt{K}^\mathsf{T}[\mathtt{K}\mathtt{R}^\mathsf{T}\mathbf{t}]_\times \end{aligned}$$

$$\mathbf{e} = \mathtt{P}\begin{pmatrix} -\mathtt{R}^\mathsf{T}\mathbf{t} \\ 1 \end{pmatrix} = \mathtt{K}\mathtt{R}^\mathsf{T}\mathbf{t} \qquad \mathbf{e}' = \mathtt{P}'\begin{pmatrix} \mathbf{0} \\ 1 \end{pmatrix} = \mathtt{K}'\mathbf{t}$$

$$\mathtt{F} = [\mathbf{e}']_\times \mathtt{K}'\mathtt{R}\mathtt{K}^{-1} = \mathtt{K}'^{-\mathsf{T}}[\mathbf{t}]_\times \mathtt{R}\mathtt{K}^{-1} = \mathtt{K}'^{-\mathsf{T}}\mathtt{R}[\mathtt{R}^\mathsf{T}\mathbf{t}]_\times \mathtt{K}^{-1} = \mathtt{K}'^{-\mathsf{T}}\mathtt{R}\mathtt{K}^\mathsf{T}[\mathbf{e}]_\times$$

Properties of Fundamental Matrix



$\mathbf{x}'$ is on the epiploar line $\mathbf{l}' = F\mathbf{x}$

$$\mathbf{x}'^T F \mathbf{x} = 0$$

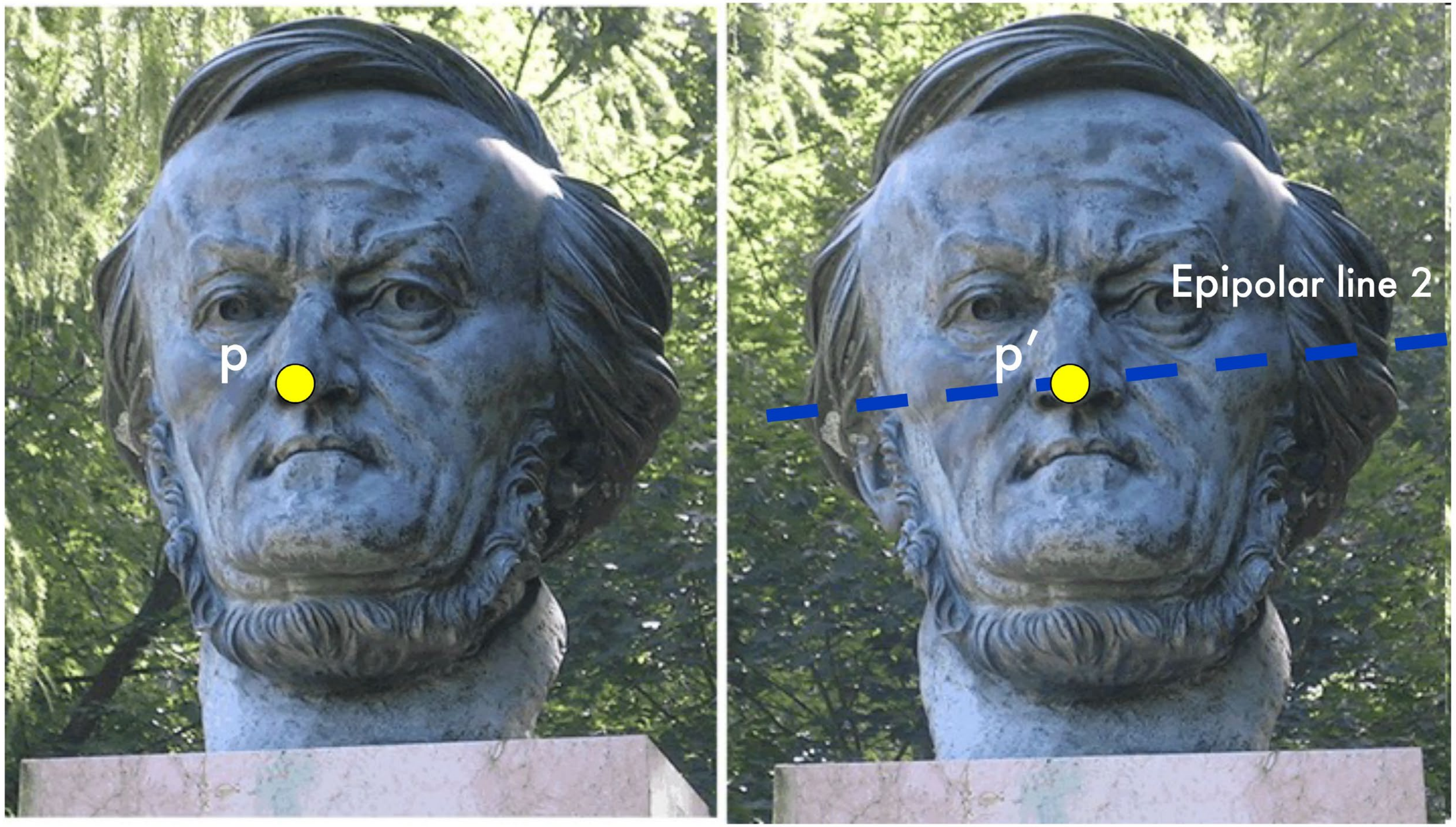
- Transpose: if F is the fundamental matrix of (P, P'), then F^T is the fundamental matrix of (P', P)
- Epipolar line: $\mathbf{l}' = F\mathbf{x}$ $\quad \mathbf{l} = F^T\mathbf{x}'$
- Epipole: $\mathbf{e}'^T\mathbf{F} = \mathbf{0} \quad \mathbf{F}\mathbf{e} = \mathbf{0}$

$$\mathbf{e}'^T(\mathbf{F}\mathbf{x}) = (\mathbf{e}'^T\mathbf{F})\mathbf{x} = 0 \text{ for all } \mathbf{x}$$

- 7 degrees of freedom

$$\det \mathbf{F} = 0$$

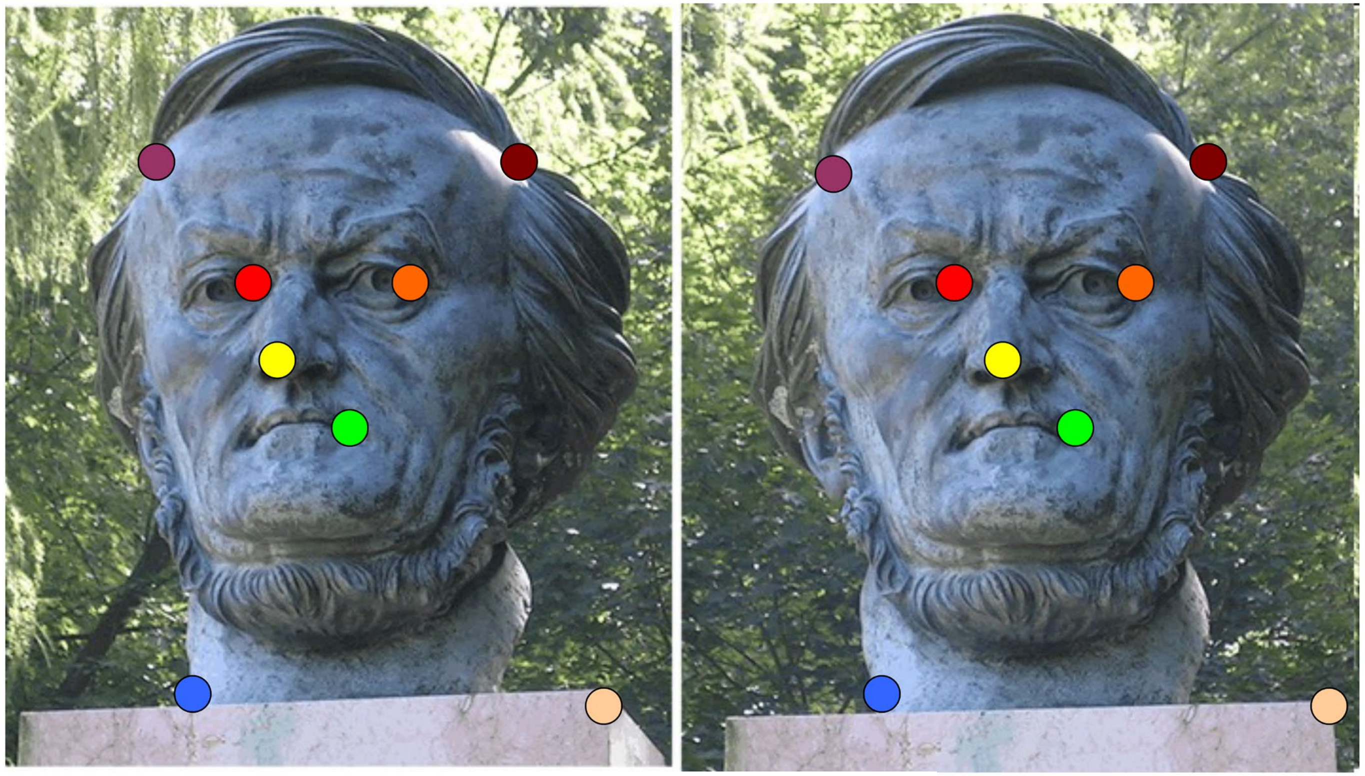
Why the Fundamental Matrix is Useful?



$$\mathbf{l}' = F\mathbf{p}$$

Estimating the Fundamental Matrix

- The 8-point algorithm



$$\mathbf{x}'^{\mathsf{T}}\mathbf{F}\mathbf{x} = 0$$

Estimating the Fundamental Matrix

$$\mathbf{x}'^{\mathsf{T}}\mathbf{F}\mathbf{x} = 0 \qquad \mathbf{x} = (x, y, 1)^{\mathsf{T}} \qquad \mathbf{x}' = (x', y', 1)^{\mathsf{T}}$$

$$x'xf_{11} + x'yf_{12} + x'f_{13} + y'xf_{21} + y'yf_{22} + y'f_{23} + xf_{31} + yf_{32} + f_{33} = 0$$

$$(x'x, x'y, x', y'x, y'y, y', x, y, 1)\,\mathbf{f} = 0$$

n correspondences

$$\mathtt{A}\mathbf{f} = \begin{bmatrix} x'_1x_1 & x'_1y_1 & x'_1 & y'_1x_1 & y'_1y_1 & y'_1 & x_1 & y_1 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \\ x'_nx_n & x'_ny_n & x'_n & y'_nx_n & y'_ny_n & y'_n & x_n & y_n & 1 \end{bmatrix}\mathbf{f} = \mathbf{0}$$

Linear System

$$\underset{n \times 9}{A}\underset{9 \times 1}{\mathbf{f}} = 0$$

- Find non-zero solutions
- If f is a solution, k×f is also a solution for $k \in \mathcal{R}$
- If the rank of A is 8, unique solution (up to scale)
- Otherwise, we can seek a solution $||\mathbf{f}|| = 1$

$$\min ||A\mathbf{f}||$$

Subject to $||\mathbf{f}|| = 1$

Solution: $A = UDV^T$ SVD decomposition of A

$n \times 9 \quad 9 \times 9 \quad 9 \times 9$

f is the last column of V

A5.3 in HZ

Estimating the Fundamental Matrix

- The singularity constraint $\det \mathbf{F} = 0$

$$\min \|F - F'\|$$

Subject to $\det F' = 0$

$$F = UDV^T$$

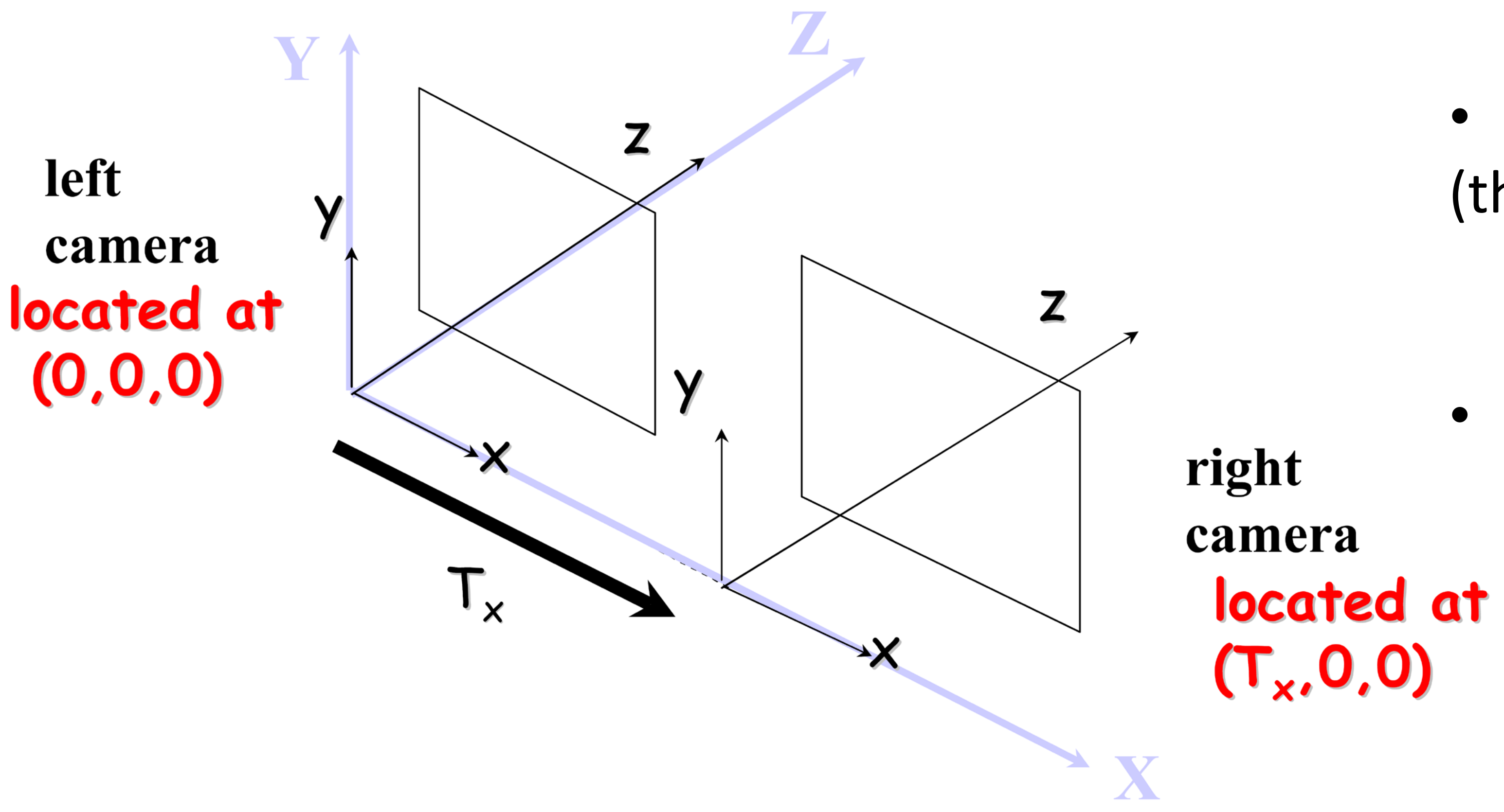
$$\mathtt{D} = \mathrm{diag}(r, s, t)$$

$$r \geq s \geq t.$$

Solution:

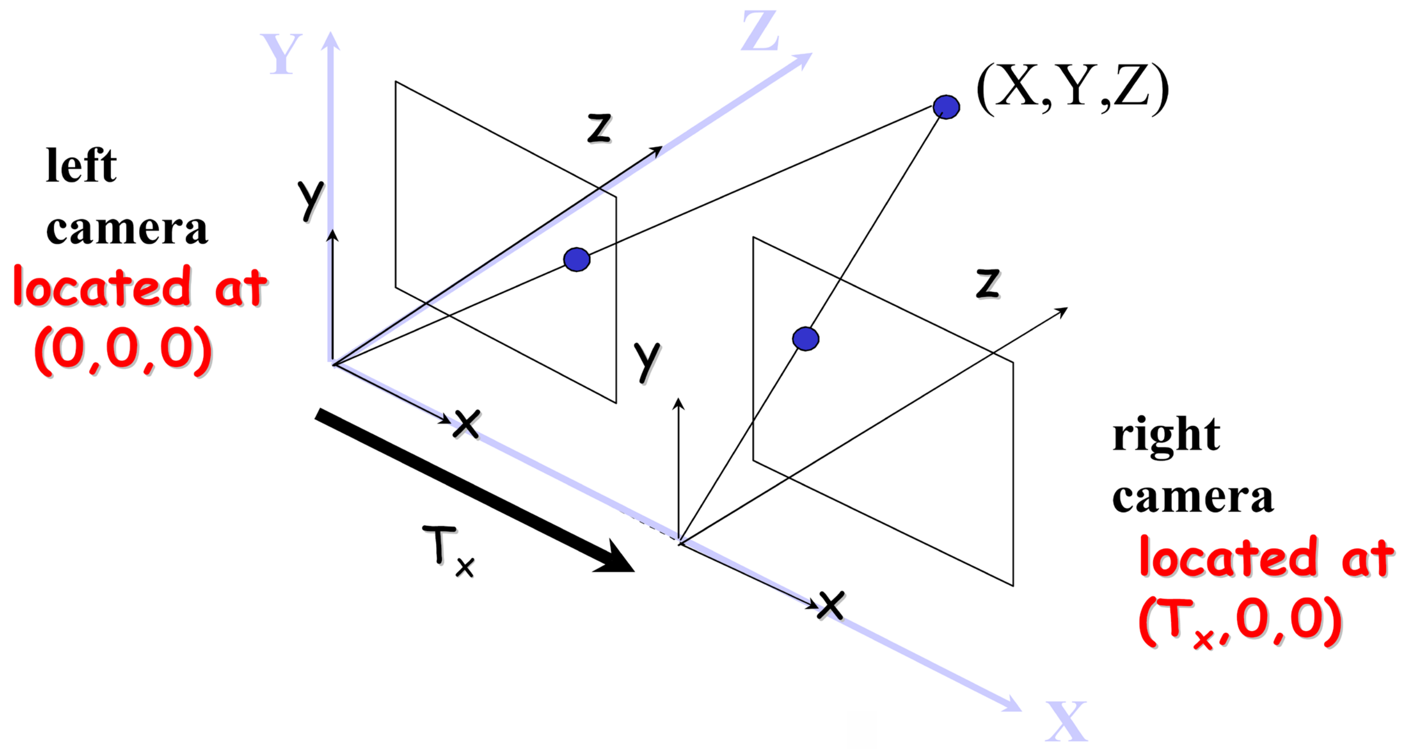
$$\mathtt{F}' = \mathtt{U}\mathrm{diag}(r, s, 0)\mathtt{V}^\mathsf{T}$$

Special Case: A Stereo System



- The right camera is shifted by T_x (the stereo baseline)
- The camera intrinsics are the same

Special Case: A Stereo System



- Left camera

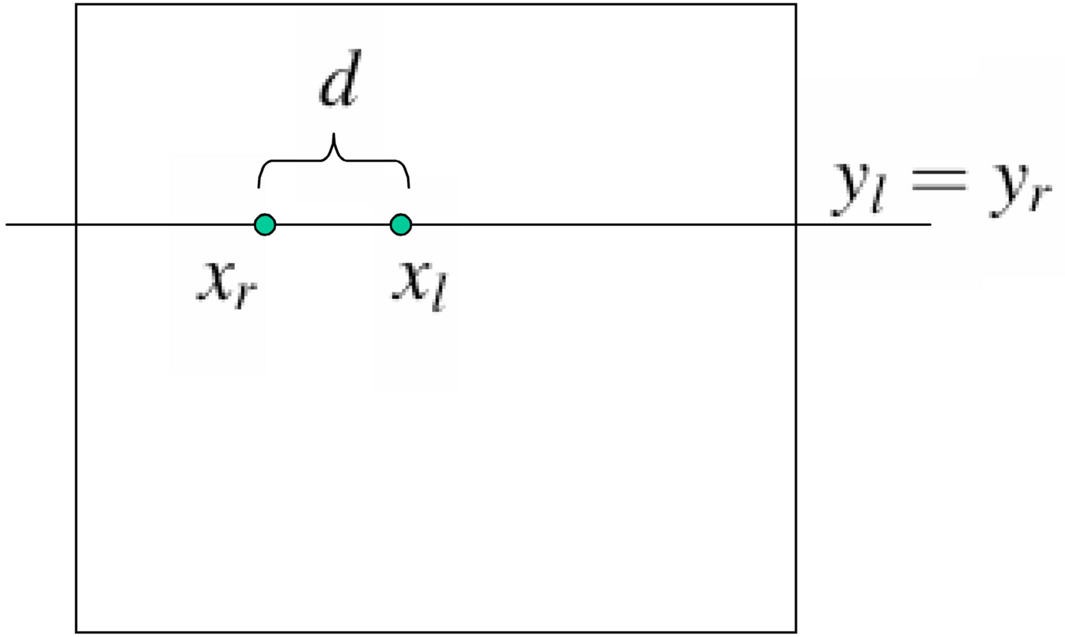
$$x_l = f\frac{X}{Z} + p_x \qquad y_l = f\frac{Y}{Z} + p_y$$

- Right camera

$$x_r = f\frac{X - T_x}{Z} + p_x$$

$$y_r = f\frac{Y}{Z} + p_y$$

Stereo Disparity



- Disparity

$$\begin{aligned} d &= x_l - x_r \\ &= \left(f\frac{X}{Z} + p_x\right) - \left(f\frac{X - T_x}{Z} + p_x\right) \\ &= f\frac{T_x}{Z} \end{aligned}$$

- Depth

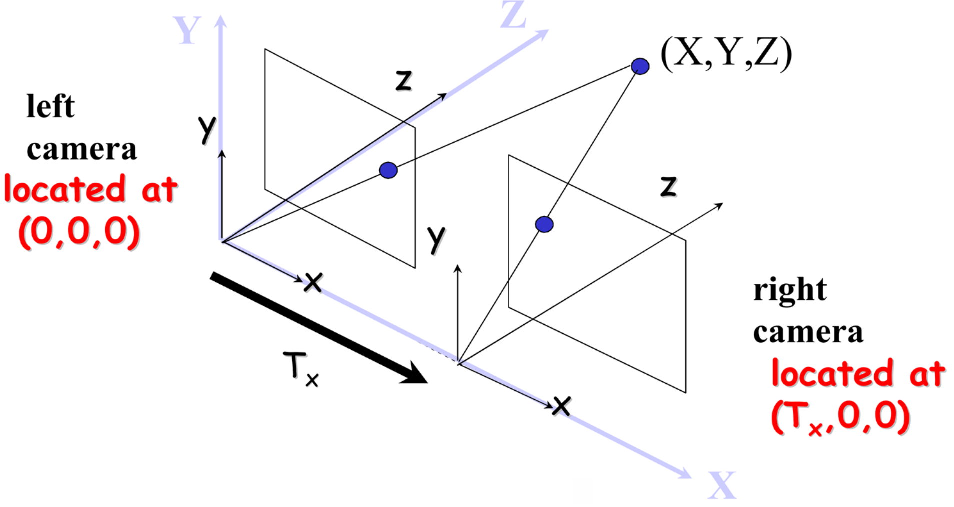
$$Z = f\frac{T_x}{d}$$

Baseline (T_x)

Disparity (d)

Recall motion parallax: near objects move faster (large disparity)

Special Case: A Stereo System



$$\mathtt{P} = \mathtt{K}[\mathtt{I} \mid \mathbf{0}] \qquad \mathtt{P}' = \mathtt{K}[\mathtt{I} \mid \mathbf{t}]$$

$$\mathtt{F} = [\mathbf{e}']_\times \mathtt{K}'\mathtt{R}\mathtt{K}^{-1} = \mathtt{K}'^{-\mathsf{T}}[\mathbf{t}]_\times \mathtt{R}\mathtt{K}^{-1} = \mathtt{K}'^{-\mathsf{T}}\mathtt{R}[\mathtt{R}^\mathsf{T}\mathbf{t}]_\times \mathtt{K}^{-1} = \mathtt{K}'^{-\mathsf{T}}\mathtt{R}\mathtt{K}^\mathsf{T}[\mathbf{e}]_\times$$

$$\mathbf{F} = [\mathbf{e}']_\times \mathbf{K}\mathbf{K}^{-1} = [\mathbf{e}']_\times$$

$$\mathbf{e}' = (P'C) \qquad \mathbf{C} = \begin{pmatrix} \mathbf{0} \\ 1 \end{pmatrix}$$

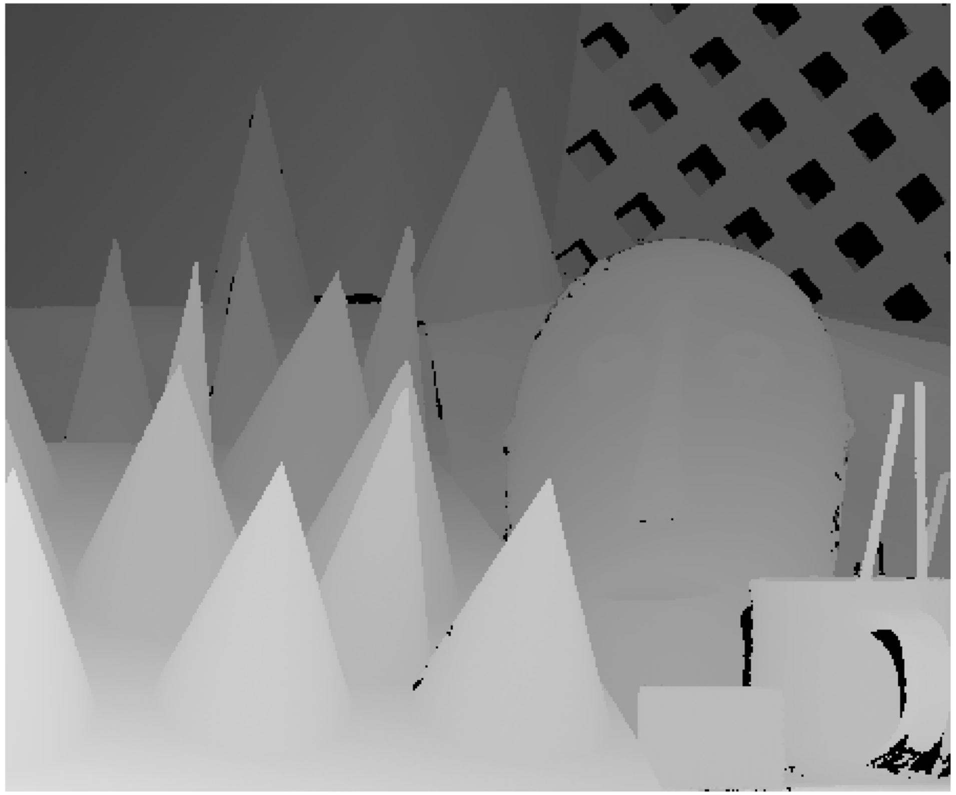
$$K = \begin{bmatrix} f_x & 0 & p_x \\ 0 & f_y & p_y \\ 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{e}' = \begin{bmatrix} f_x T_x \\ 0 \\ 0 \end{bmatrix} \qquad F = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -f_x T_x \\ 0 & f_x T_x & 0 \end{bmatrix}$$

$$\mathbf{x}'^T F \mathbf{x} = 0$$

$$y = y'$$

Stereo Example

Disparity values (0-64)

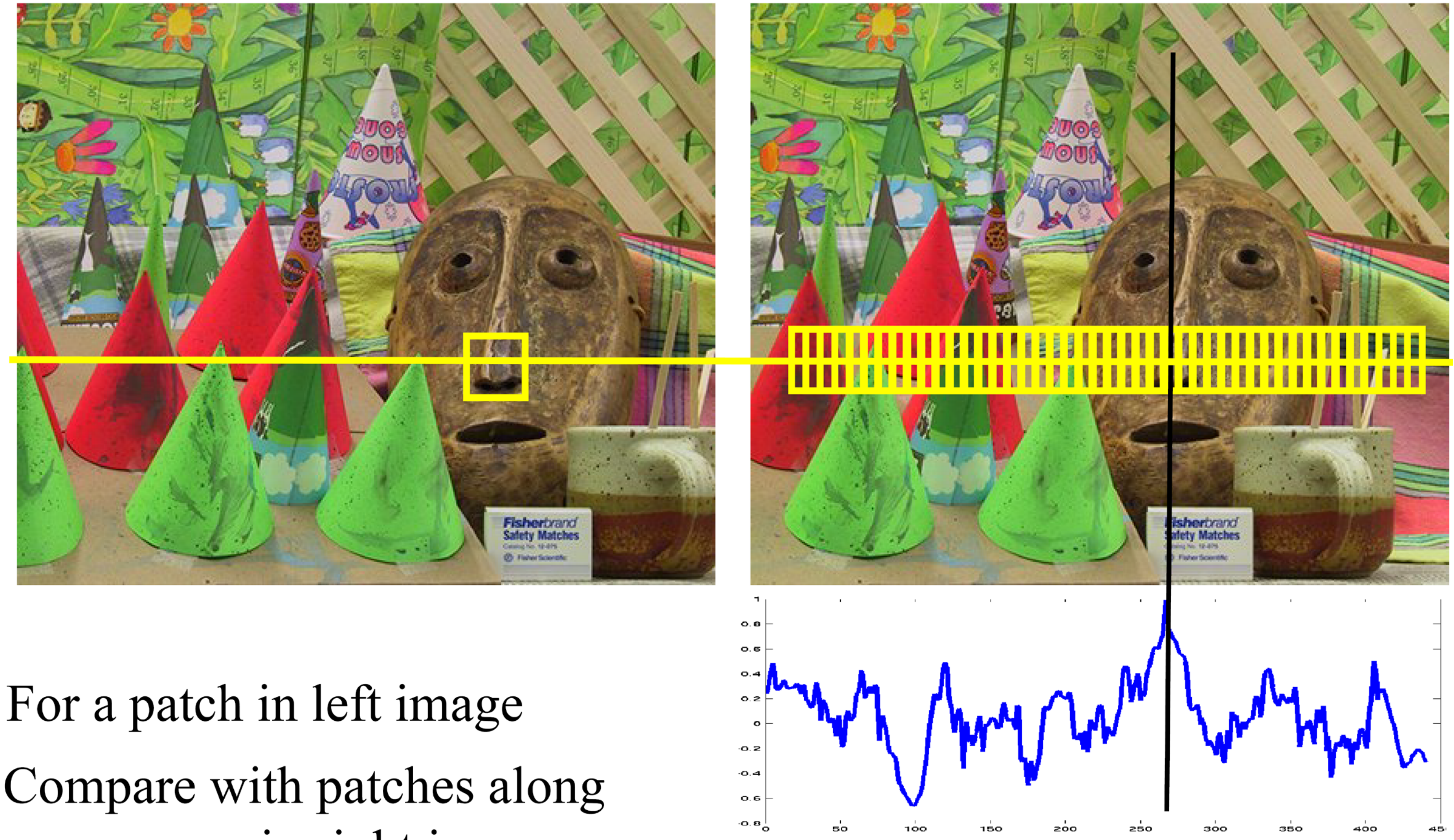


Note how disparity is larger (brighter) for closer surfaces.

$$d = f\frac{T_x}{Z}$$

Computing Disparity

Left Image   Right Image



Match Score Values

For a patch in left image

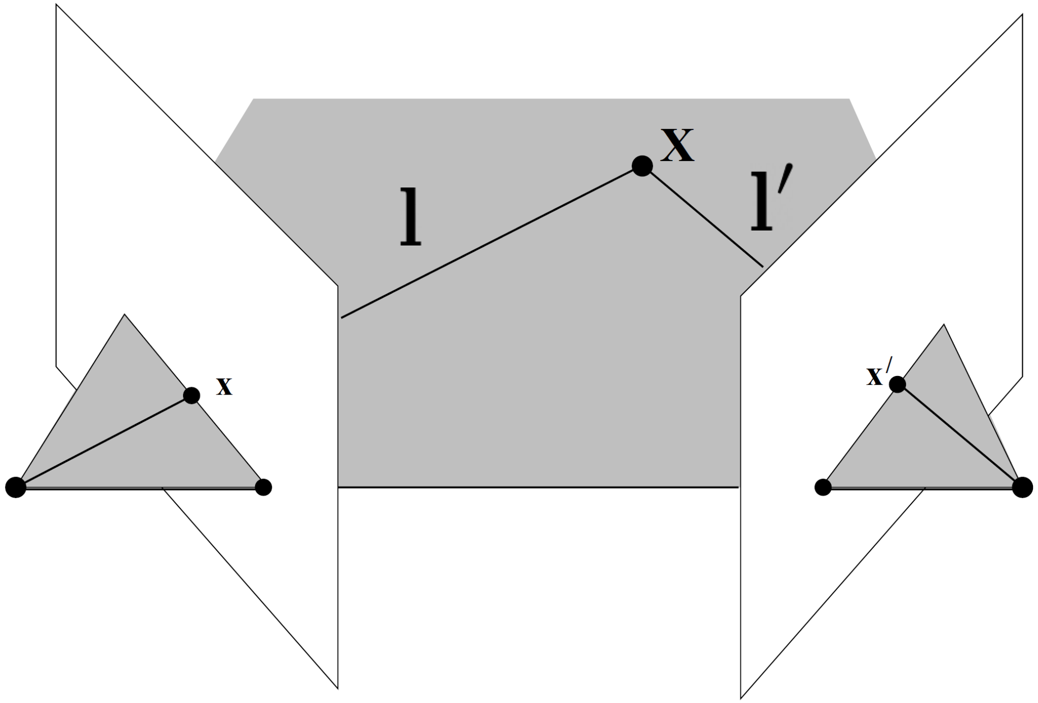
Compare with patches along same row in right image

- Eipipolar lines are horizontal lines in stereo
- For general cases, we can find correspondences on eipipolar lines
- Depth from disparity

$$Z = f\frac{T_x}{d}$$

Triangulation

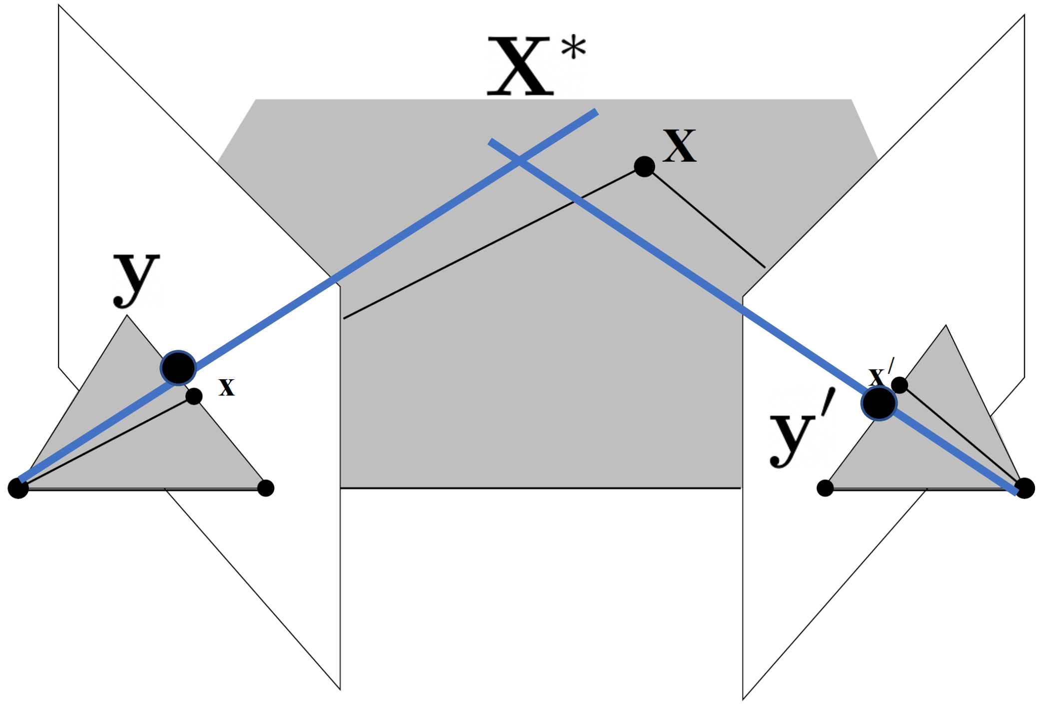
- Compute the 3D point given image correspondences



Intersection of two backprojected lines

$$\mathbf{X} = \mathbf{l} \times \mathbf{l}'$$

Triangulation



- In practice, we find the correspondences $\mathbf{y}$ $\mathbf{y}'$
- The backprojected lines may not intersect
- Find X* that minimizes

$$d(\mathbf{y}, P\mathbf{X}^*) + d(\mathbf{y}', P'\mathbf{X}^*)$$

Projection matrix

Further Reading

- Stanford CS231A: Computer Vision, From 3D Reconstruction to Recognition, Lecture 5 https://web.stanford.edu/class/cs231a/syllabus.html

- Multiview Geometry in Computer Vision, Richard Hartley and Andrew Zisserman, Chapter 9, Epipolar Geometry and Fundamental Matrix