# Visual Perception: Depth Perception

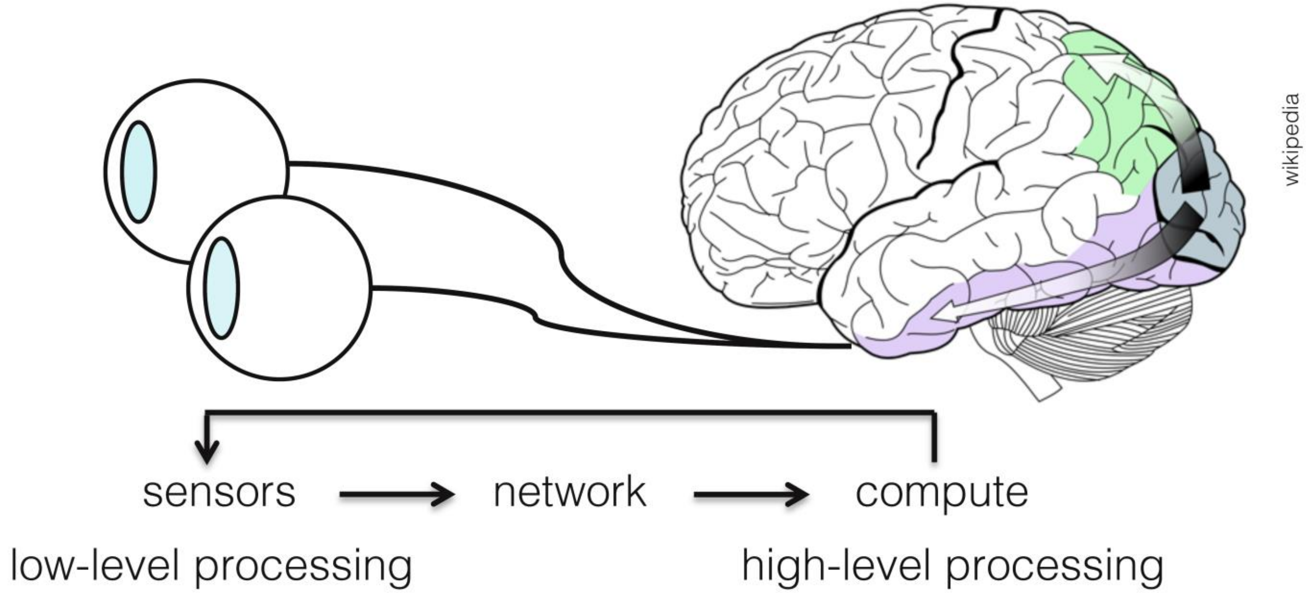
CS 6334 Virtual Reality

Professor Yu Xiang

The University of Texas at Dallas

# Visual Perception

- How humans perceive or interpret the real world using vision?




- We need to understand visual perception to achieve visual unawareness in VR systems

# Depth Perception

- Metric
  - The car is 10 meters away
- Ordinary
  - The tree is behind the car

# Depth Cues

- Information for sensory stimulation that is relevant to depth perception

- Monocular cues: single eye

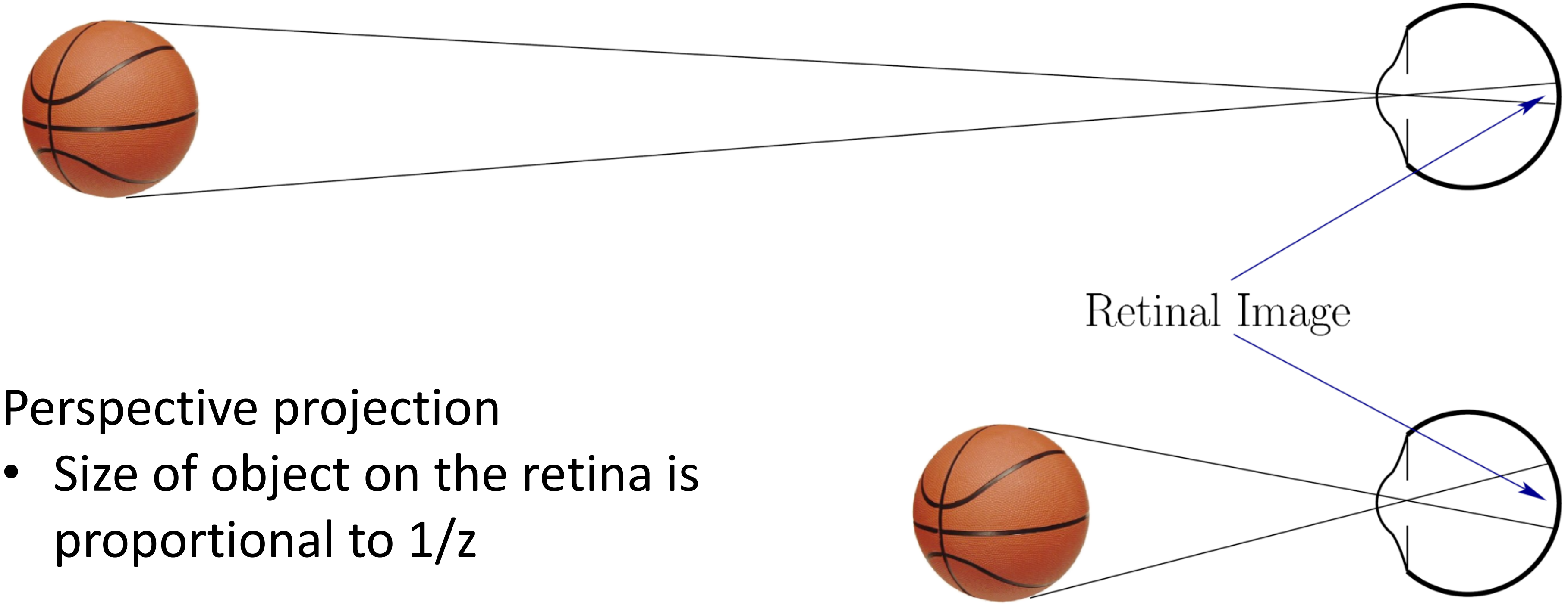
- Stereo cues: both eyes

"Paris Street, Rainy Day," Gustave Caillebotte, 1877. Art Institute of Chicago

- Texture of the bricks
- Perspective projection
- Etc.

# Monocular Depth Cues

- Retinal image size

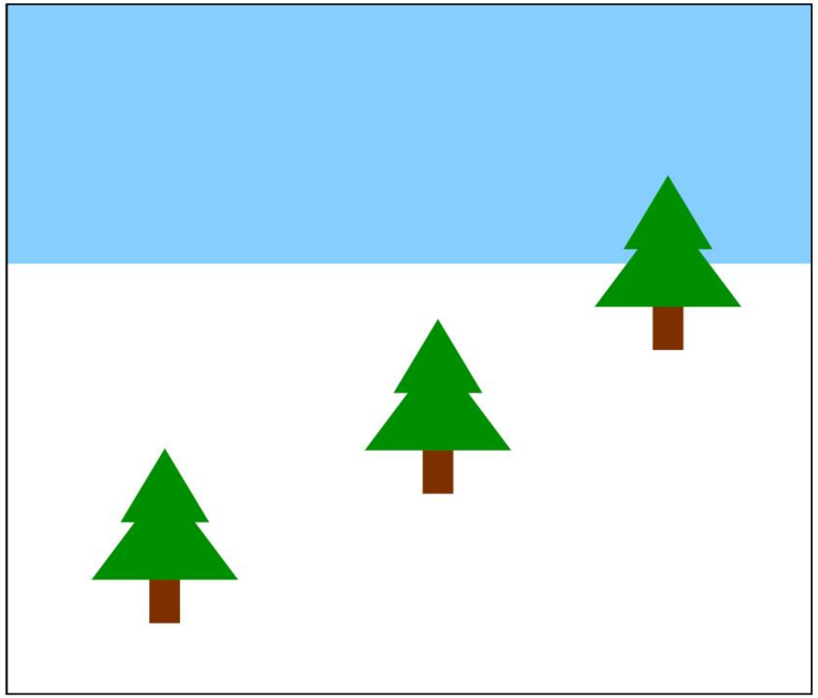



Perspective projection

- Size of object on the retina is proportional to 1/z

# Monocular Depth Cues

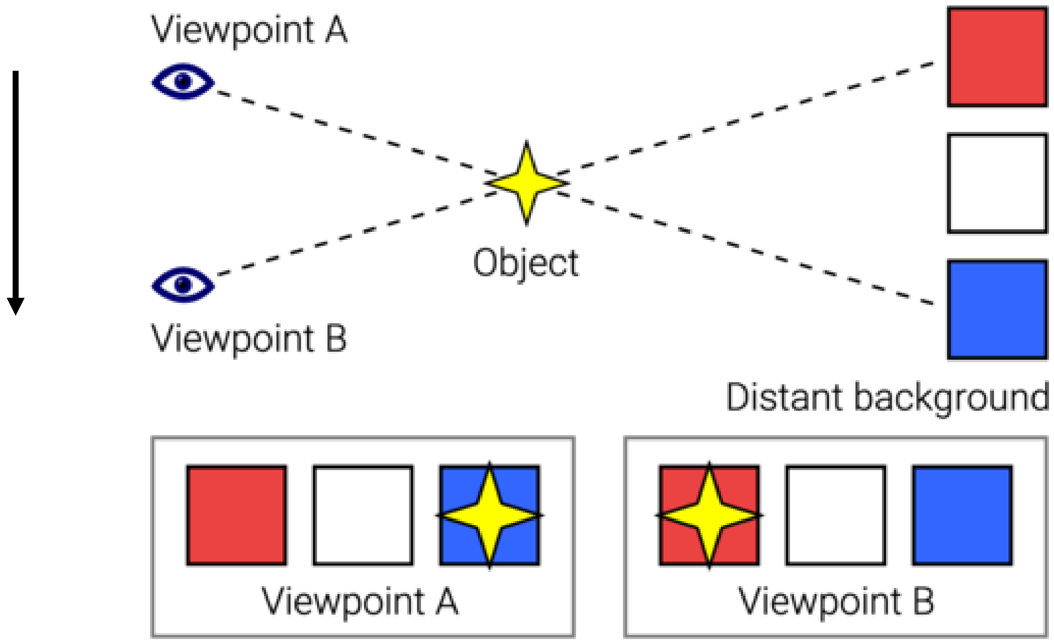
- Height in visual field
  - The closer to the horizon, the further the perceived distance



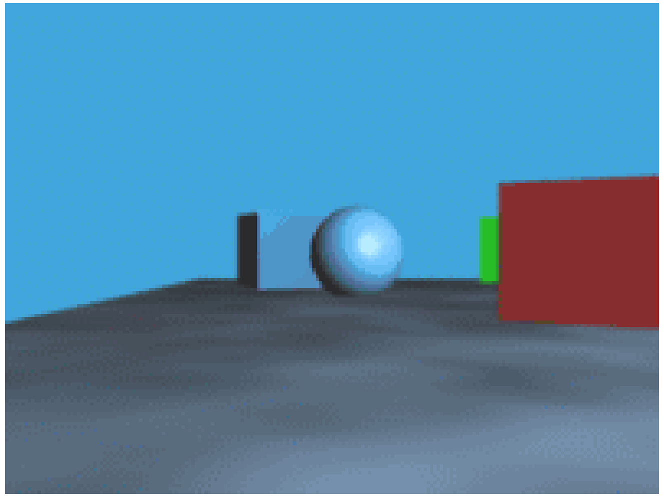
size constancy scaling

# Monocular Depth Cues

- Motion parallax
  - Parallax: relative difference in speed

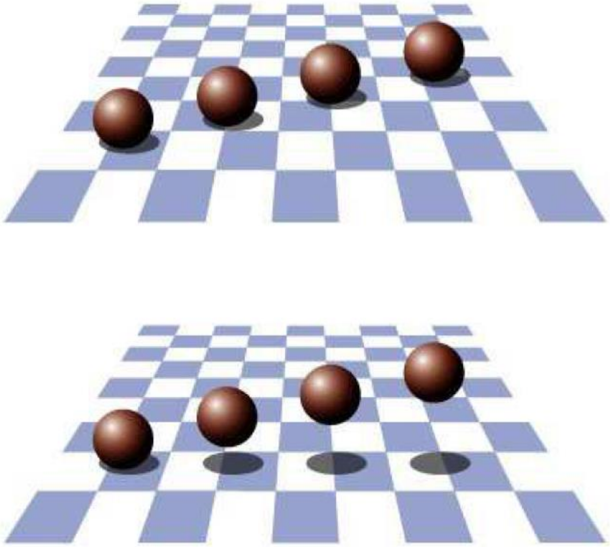



Closer objects have larger image displacements than further objects

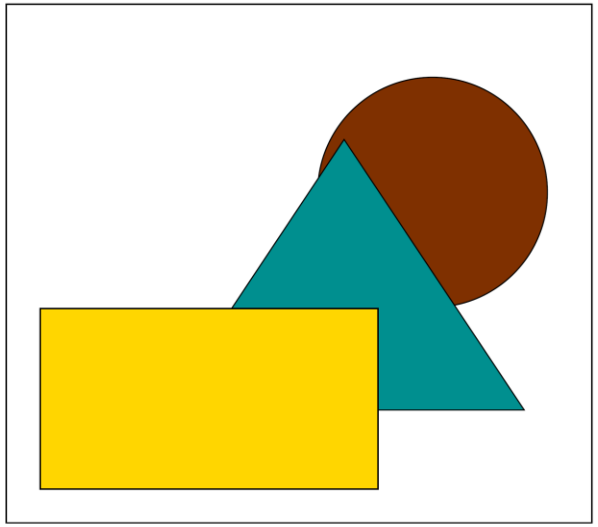


Further objects move slower

# Monocular Depth Cues



Shadow



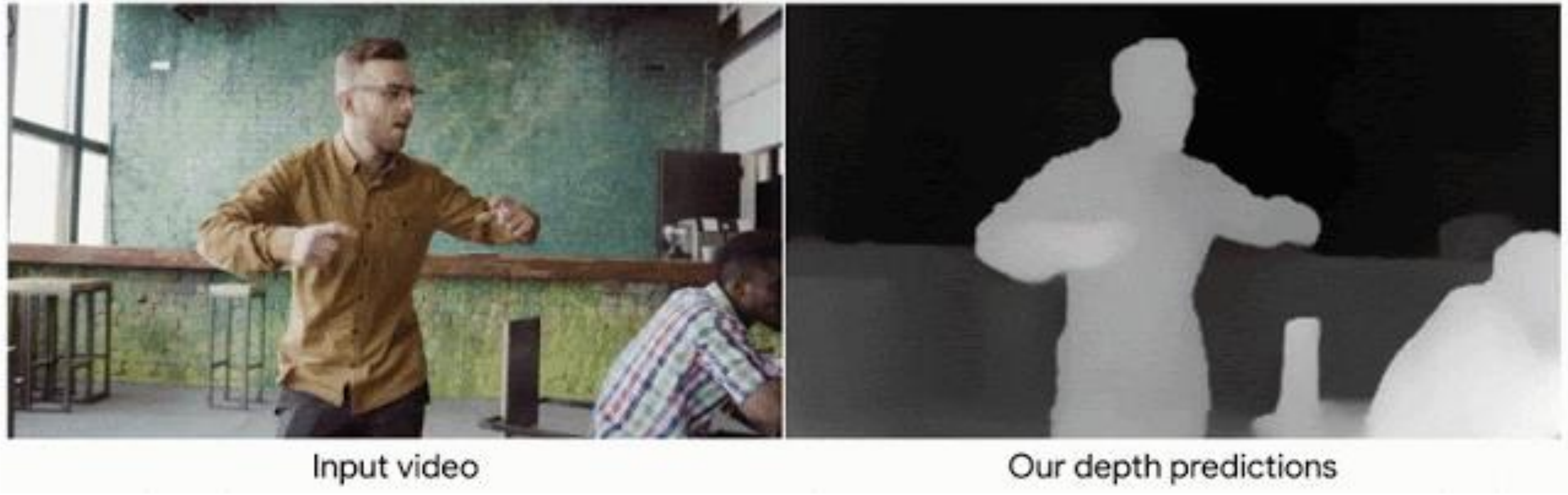
Occlusions

Image blur

Atmospheric cue

further away because it has lower contrast

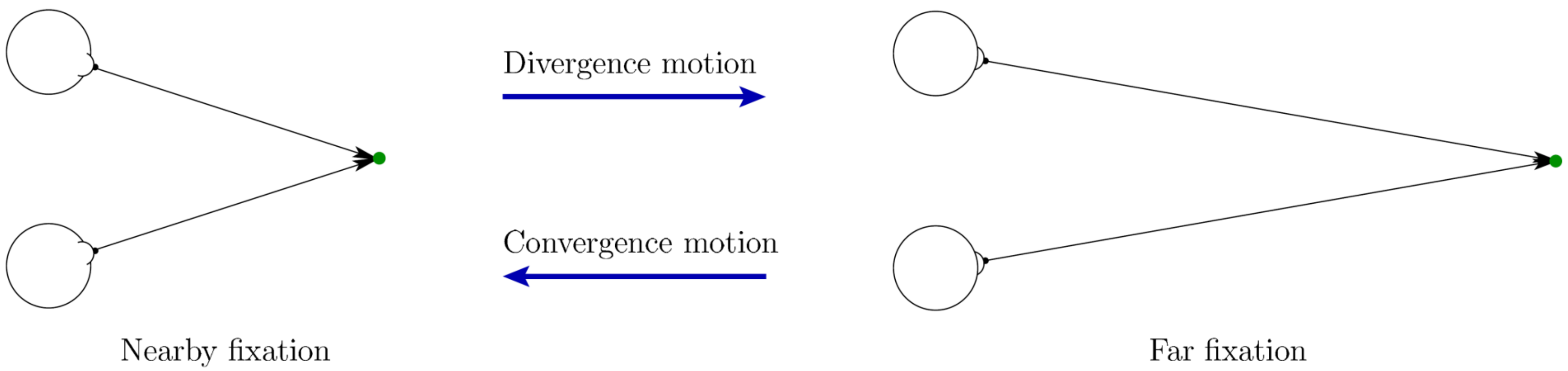
# Monocular Depth Estimation




https://heartbeat.fritz.ai/research-guide-for-depth-estimation-with-deep-learning-1a02a439b834

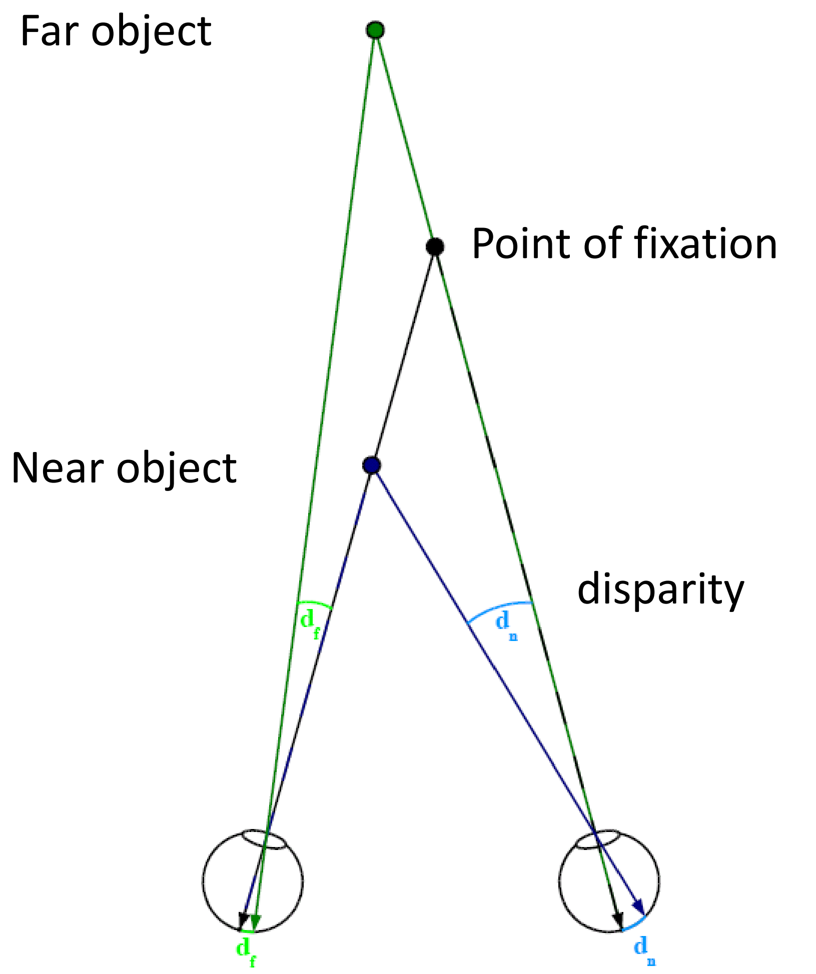
# Stereo Depth Cues

- Vergence motion
  - Signals from motor control of the eye muscles



# Stereo Depth Cues

- Binocular disparity
  - Each eye provides a different viewpoint, which results in different images on the retina




From Wikipedia

# Geometry of Stereo Vision

- Basics: points and lines
- Homogeneous representation of lines

A line in a 2D plane $ax + by + c = 0$ $(a, b, c)^T$

$k(a, b, c)^T$ represents the same line for nonzero k

A point lies on the line $\mathbf{x}^T\mathbf{l} = 0$ $\mathbf{x} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$ $\mathbf{l} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$

# Points and Lines

- Intersection of lines

$$\mathbf{l} = (a, b, c)^T \qquad \mathbf{l}' = (a', b', c')^T$$

The intersection is $\mathbf{x} = \mathbf{l} \times \mathbf{l}'$ (vector cross product)

$$\mathbf{l} \cdot (\mathbf{l} \times \mathbf{l}') = \mathbf{l}' \cdot (\mathbf{l} \times \mathbf{l}') = 0$$

$$\mathbf{l}^T \mathbf{x} = \mathbf{l}'^T \mathbf{x} = 0$$

# Points and Lines

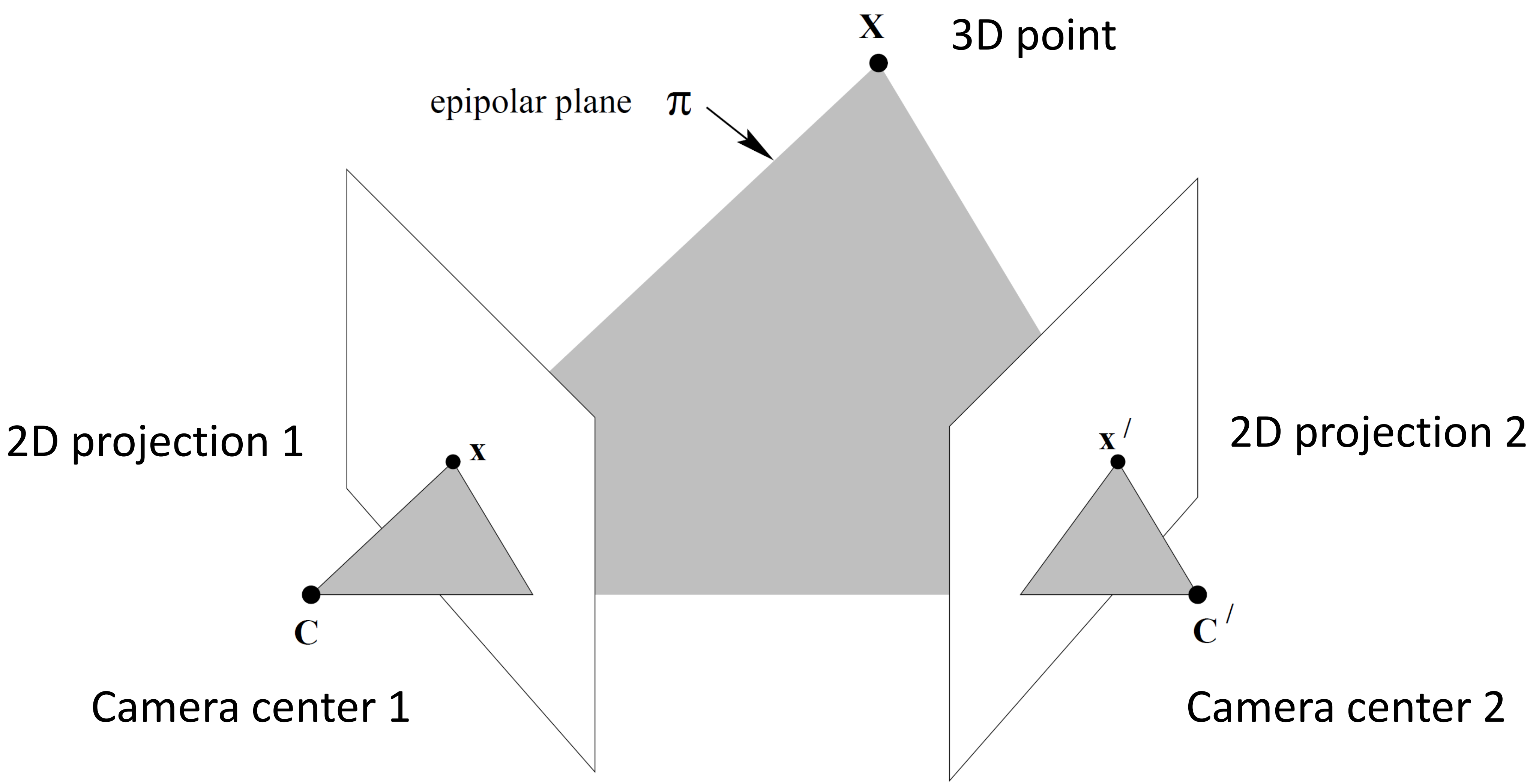
- Line joining points

$$\mathbf{l} = \mathbf{x} \times \mathbf{x}'$$

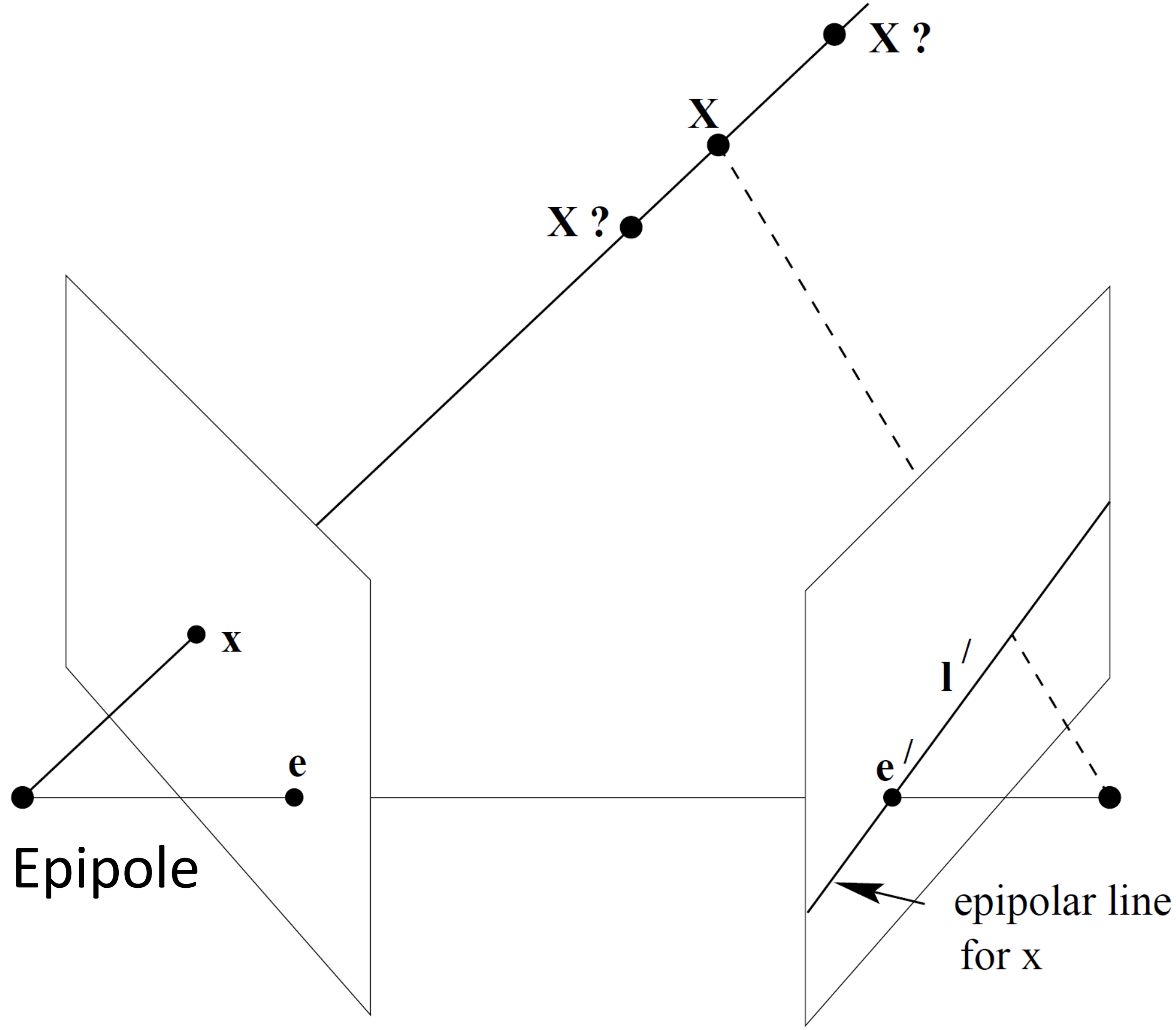
$$\mathbf{x} \cdot (\mathbf{x} \times \mathbf{x}') = \mathbf{x}' \cdot (\mathbf{x} \times \mathbf{x}') = 0$$

$$\mathbf{x}^T \mathbf{l} = \mathbf{x}'^T \mathbf{l} = 0$$

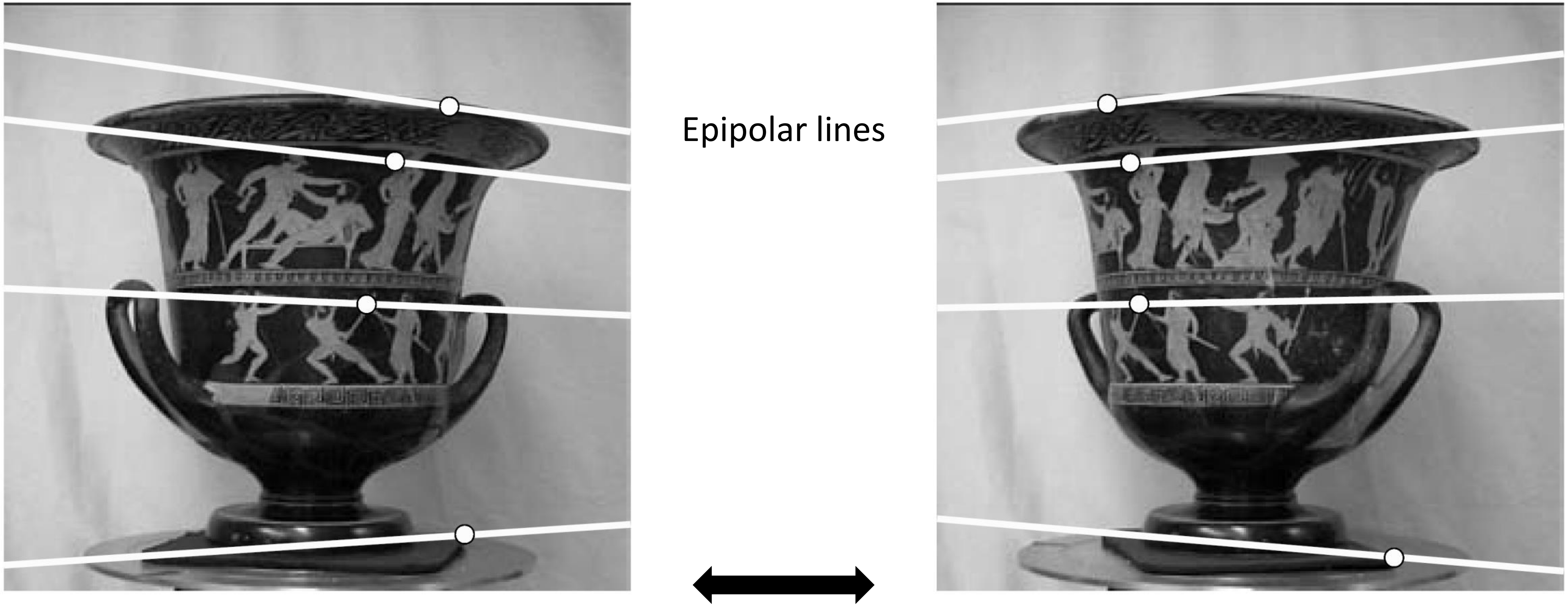
# Epipolar Geometry



# Epipolar Geometry



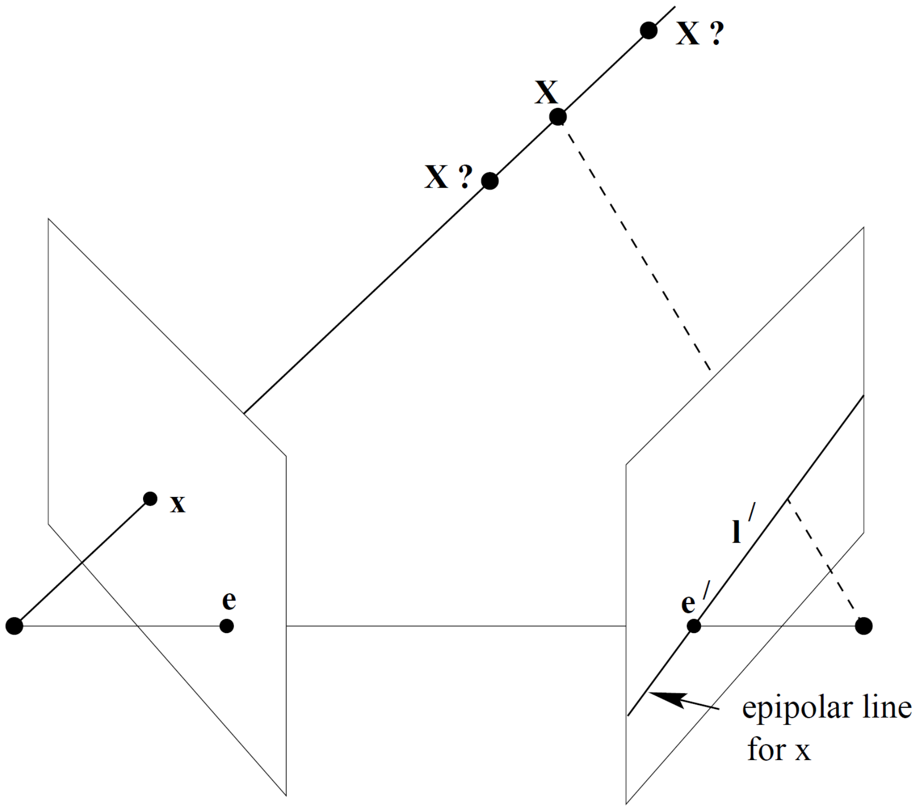
# Epipolar Geometry




Rotation and Translation
between two views

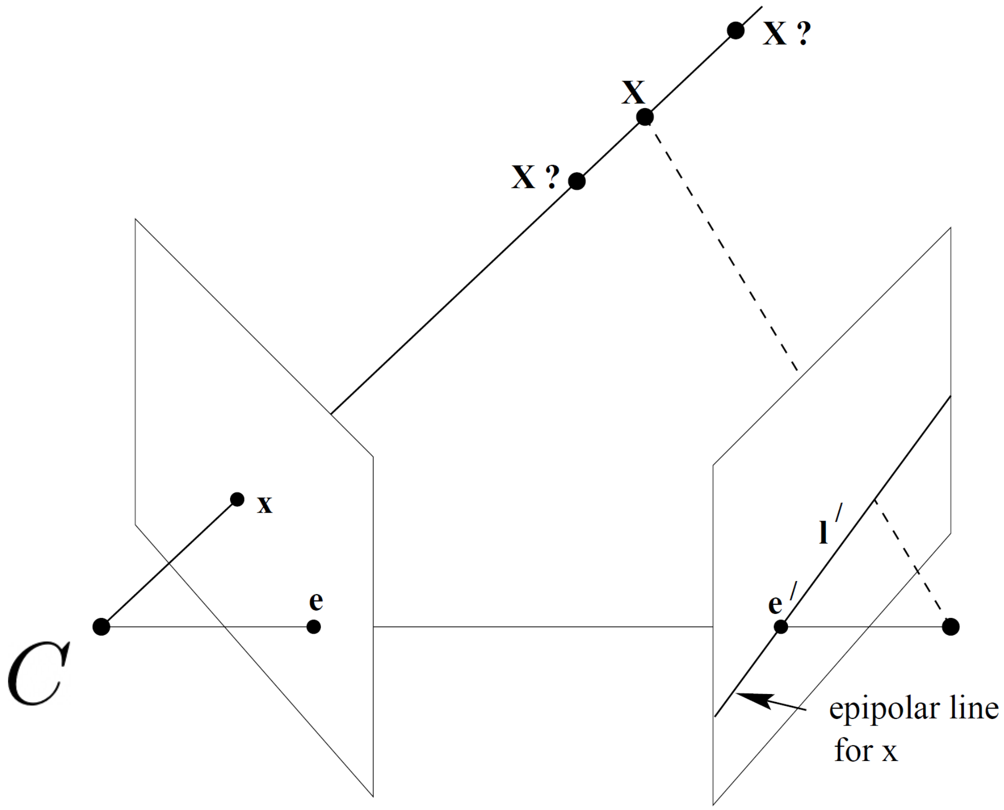
# Epipolar Geometry

- What is the mapping for a point in one image to its epipolar line?




$$\mathbf{x} \mapsto \mathbf{l}'$$

# Fundamental Matrix




- Recall camera projection

$$P = K[R|\mathbf{t}]$$

$$\mathbf{x} = P\mathbf{X}$$ Homogeneous coordinates

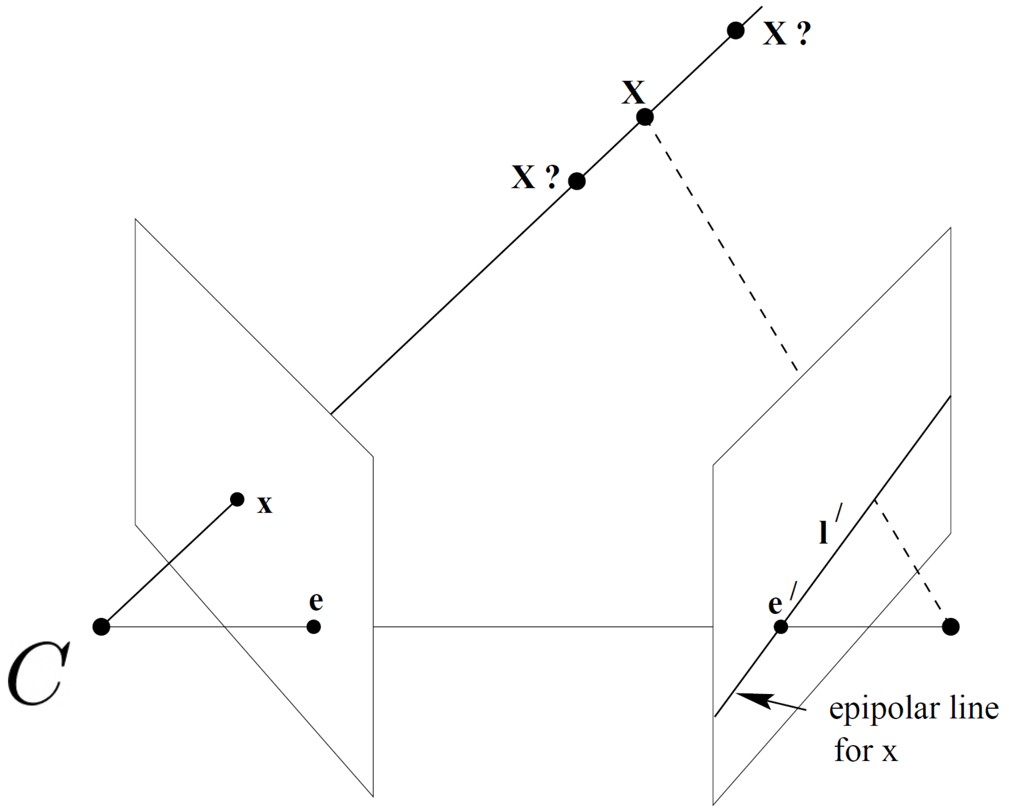
- Backprojection

$$\mathbf{X}(\lambda) = \mathrm{P}^{+}\mathbf{x} + \lambda\mathbf{C}$$

$P^{+}$ is the pseudo-inverse of $P$, $PP^{+} = I$

$P^{+}\mathbf{x}$ and $C$ are two points on the ray

# Fundamental Matrix




- Project to the other image

$P^{+}\mathbf{x}$ and $C$ are two points on the ray

$P'P^{+}\mathbf{x}$ and $P'C$

- Epipolar line

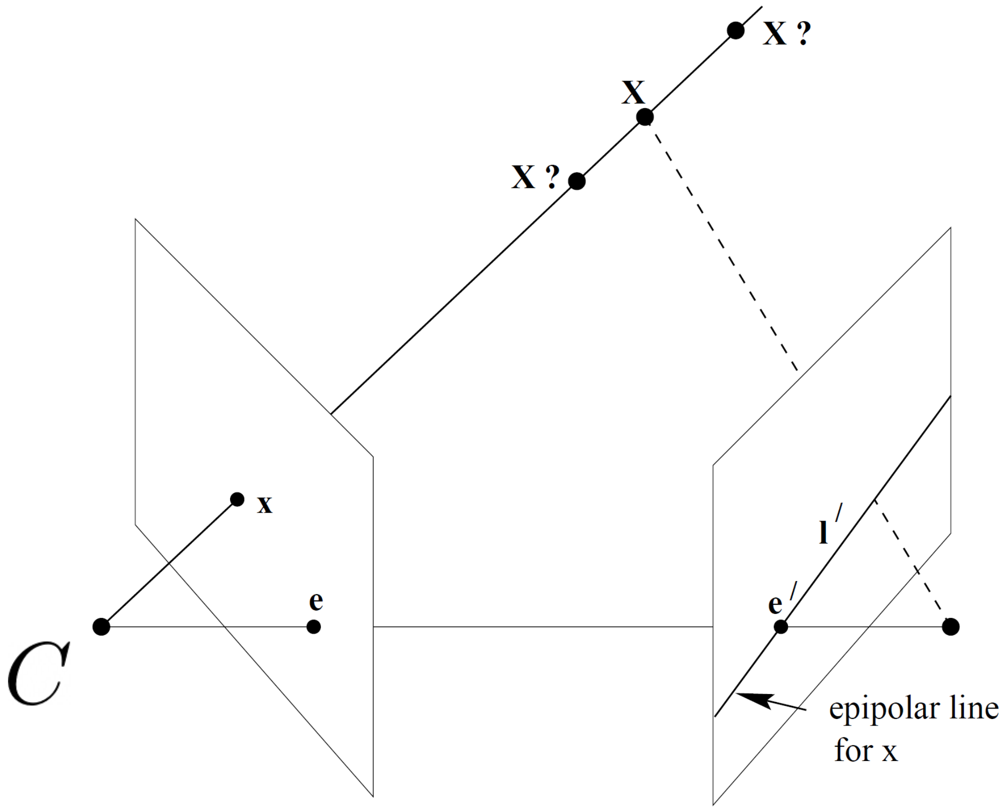
$$\mathbf{l}' = (P'C) \times (P'P^{+}\mathbf{x})$$

Epipole $\mathbf{e}' = (P'C)$

$$\mathbf{l}' = [\mathbf{e}']_{\times}(P'P^{+}\mathbf{x})$$

Cross product matrix

# Fundamental Matrix




- Epipolar line

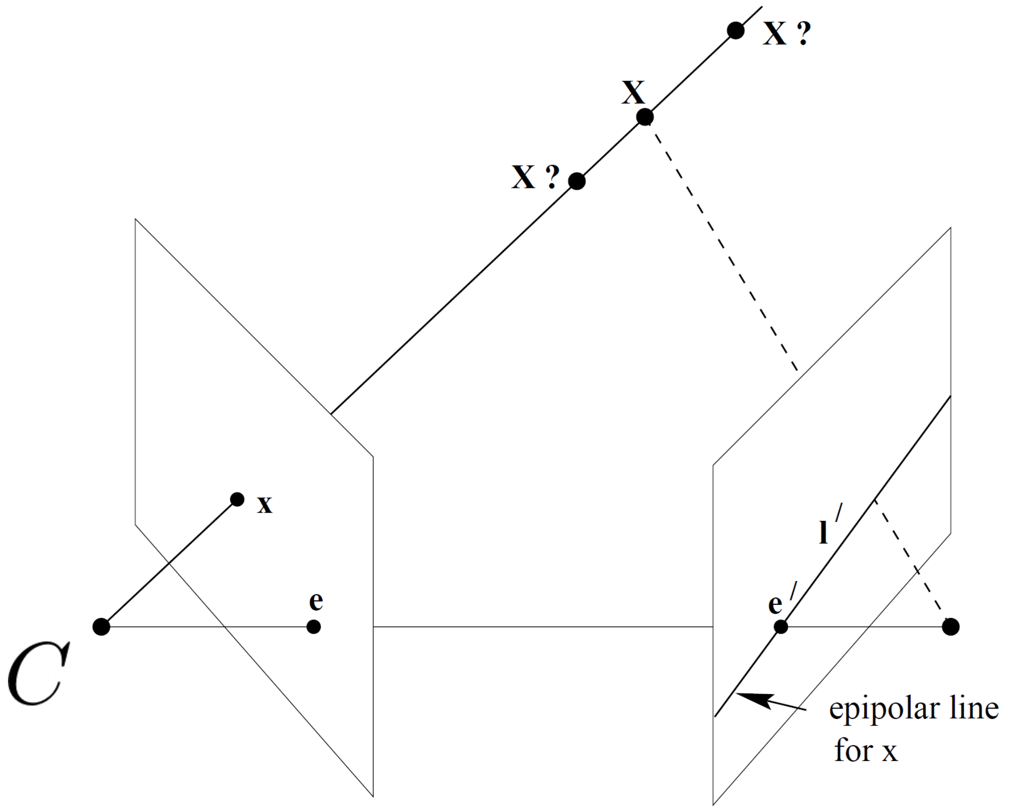
$$\mathbf{l}' = [\mathbf{e}']_\times (P' P^+ \mathbf{x}) = F\mathbf{x}$$

- Fundamental matrix

$$F = [\mathbf{e}']_\times P' P^+$$

3x3

# Properties of Fundamental Matrix




$\mathbf{x}'$ is on the epiploar line $\mathbf{l}' = F\mathbf{x}$

$$\mathbf{x}'^T F \mathbf{x} = 0$$

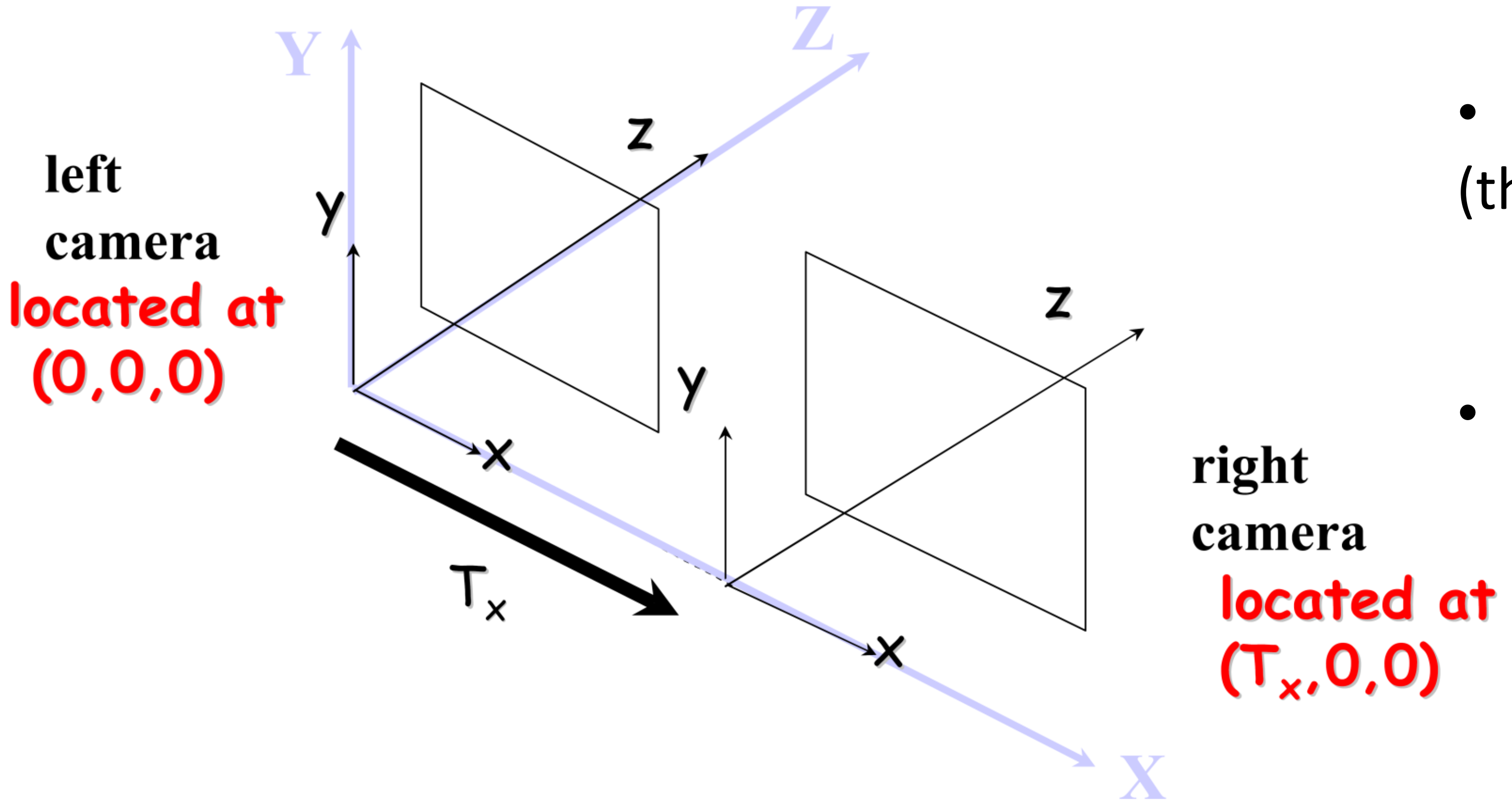
- Transpose: if F is the fundamental matrix of (P, P'), then $F^T$ is the fundamental matrix of (P', P)
- Epipolar line: $\mathbf{l}' = F\mathbf{x}$ $\quad \mathbf{l} = F^T\mathbf{x}'$
- Epipole: $\mathrm{e}'^{\mathsf{T}}\mathrm{F} = \mathbf{0}$ $\quad \mathbf{Fe} = \mathbf{0}$

$$\mathrm{e}'^{\mathsf{T}}(\mathrm{F}\mathbf{x}) = (\mathrm{e}'^{\mathsf{T}}\mathrm{F})\mathbf{x} = 0 \text{ for all } \mathbf{x}$$

- 7 degrees of freedom

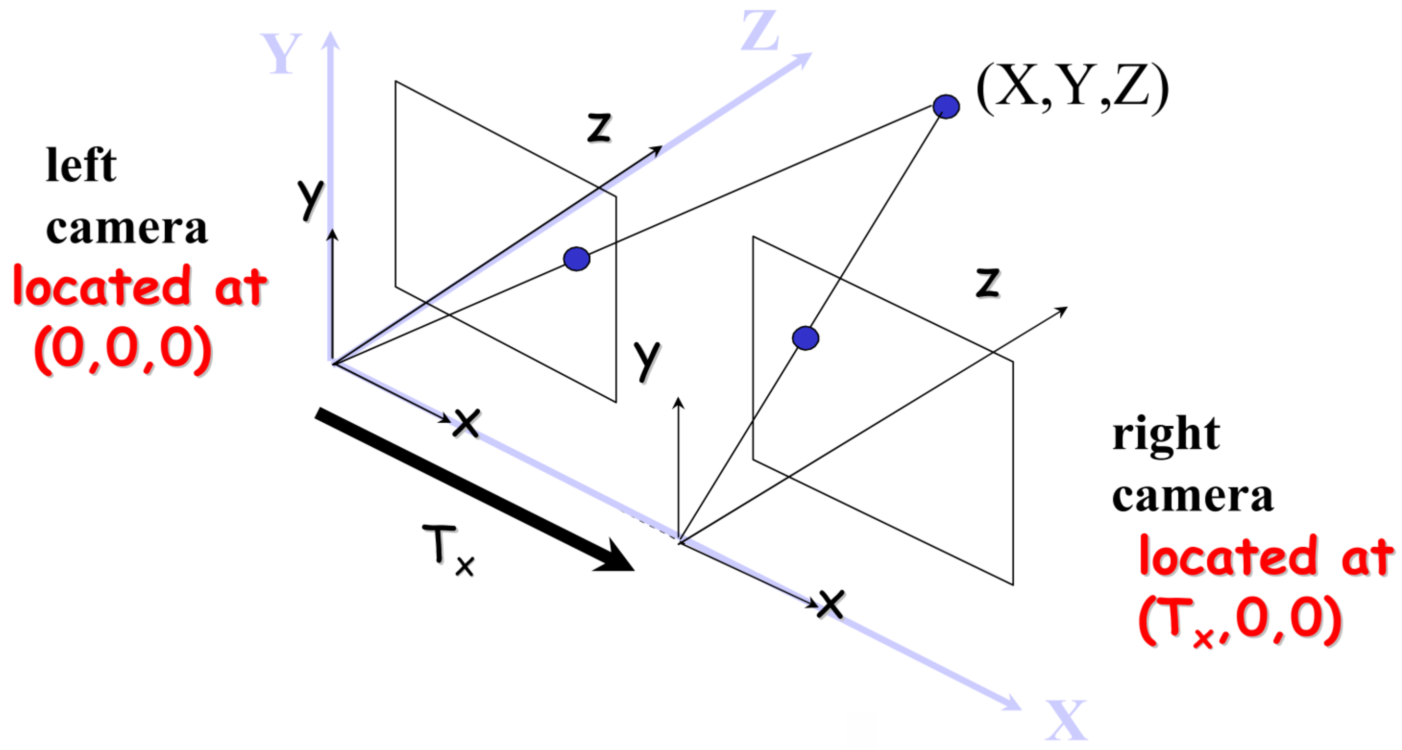
$$\det \mathrm{F} = 0$$

# Special Case: A Stereo System




- The right camera is shifted by $T_x$ (the stereo baseline)
- The camera intrinsics are the same

# Special Case: A Stereo System




- Left camera

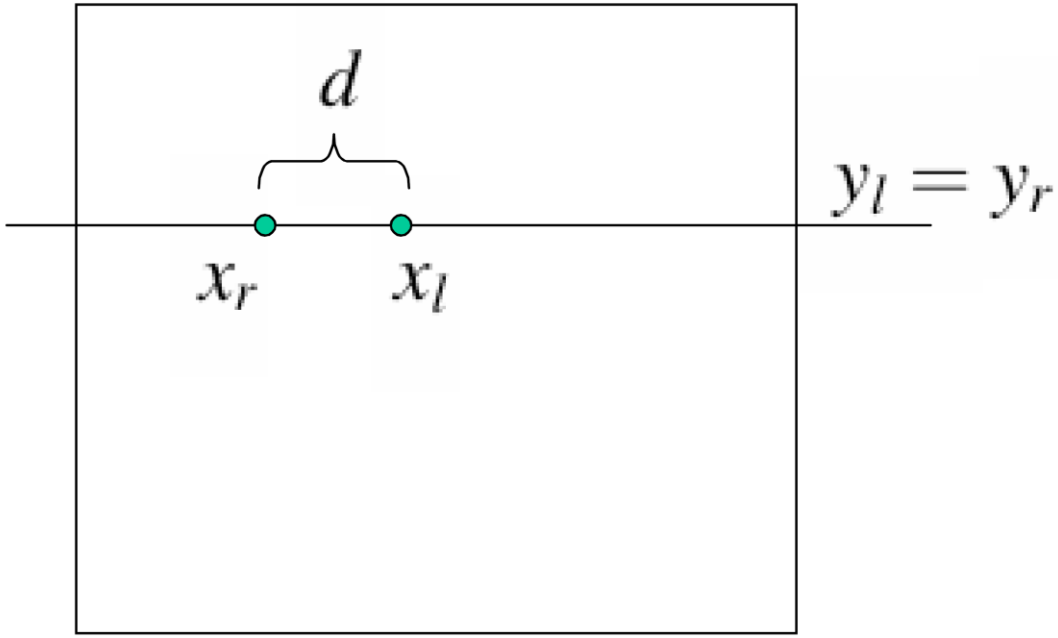
$$x_l = f\frac{X}{Z} + p_x \qquad y_l = f\frac{Y}{Z} + p_y$$

- Right camera

$$x_r = f\frac{X - T_x}{Z} + p_x$$

$$y_r = f\frac{Y}{Z} + p_y$$

# Stereo Disparity




- Disparity

$$
\begin{aligned}
d &= x_l - x_r \\
&= \left(f\frac{X}{Z} + p_x\right) - \left(f\frac{X - T_x}{Z} + p_x\right) \\
&= f\frac{T_x}{Z}
\end{aligned}
$$

- Depth

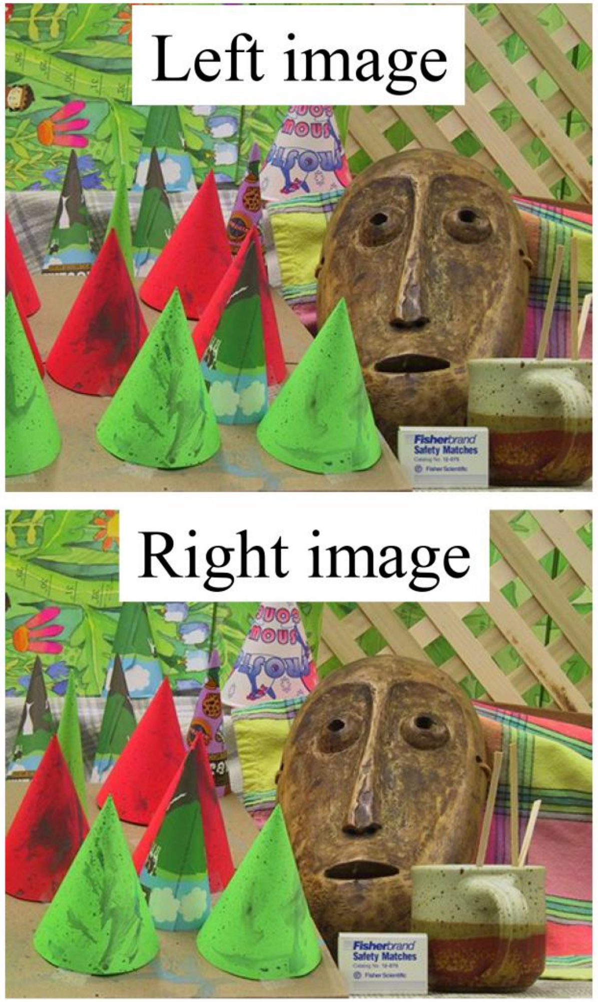
$$Z = f\frac{T_x}{d}$$

Baseline

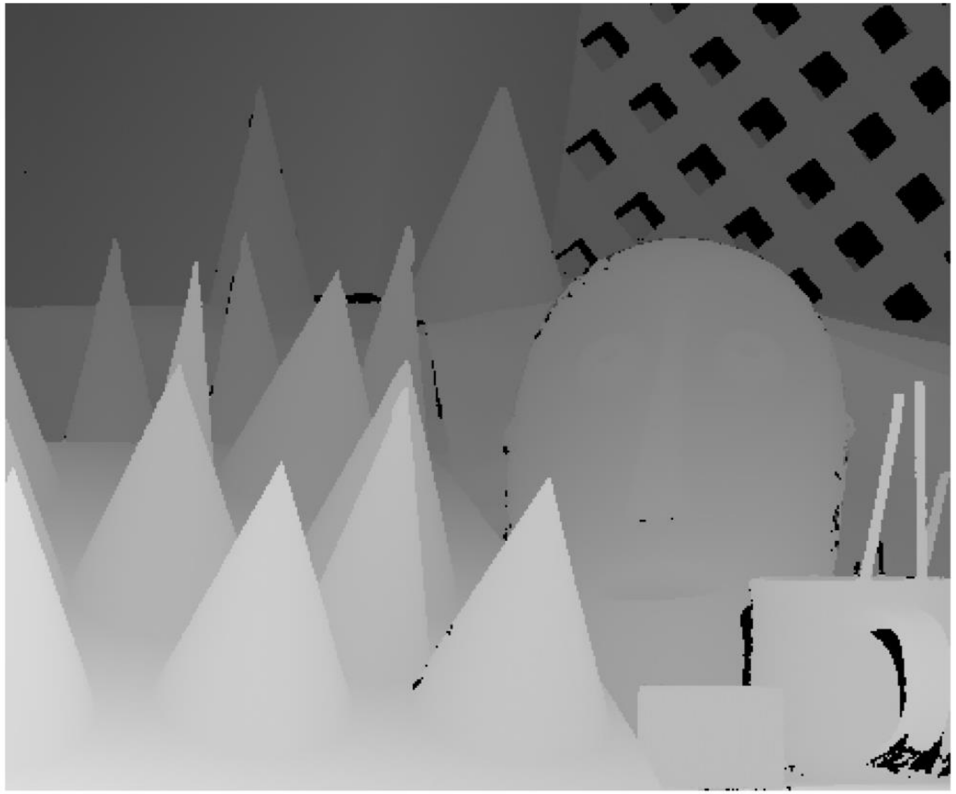
Disparity

Recall motion parallax: near objects move faster (large disparity)

# Stereo Example



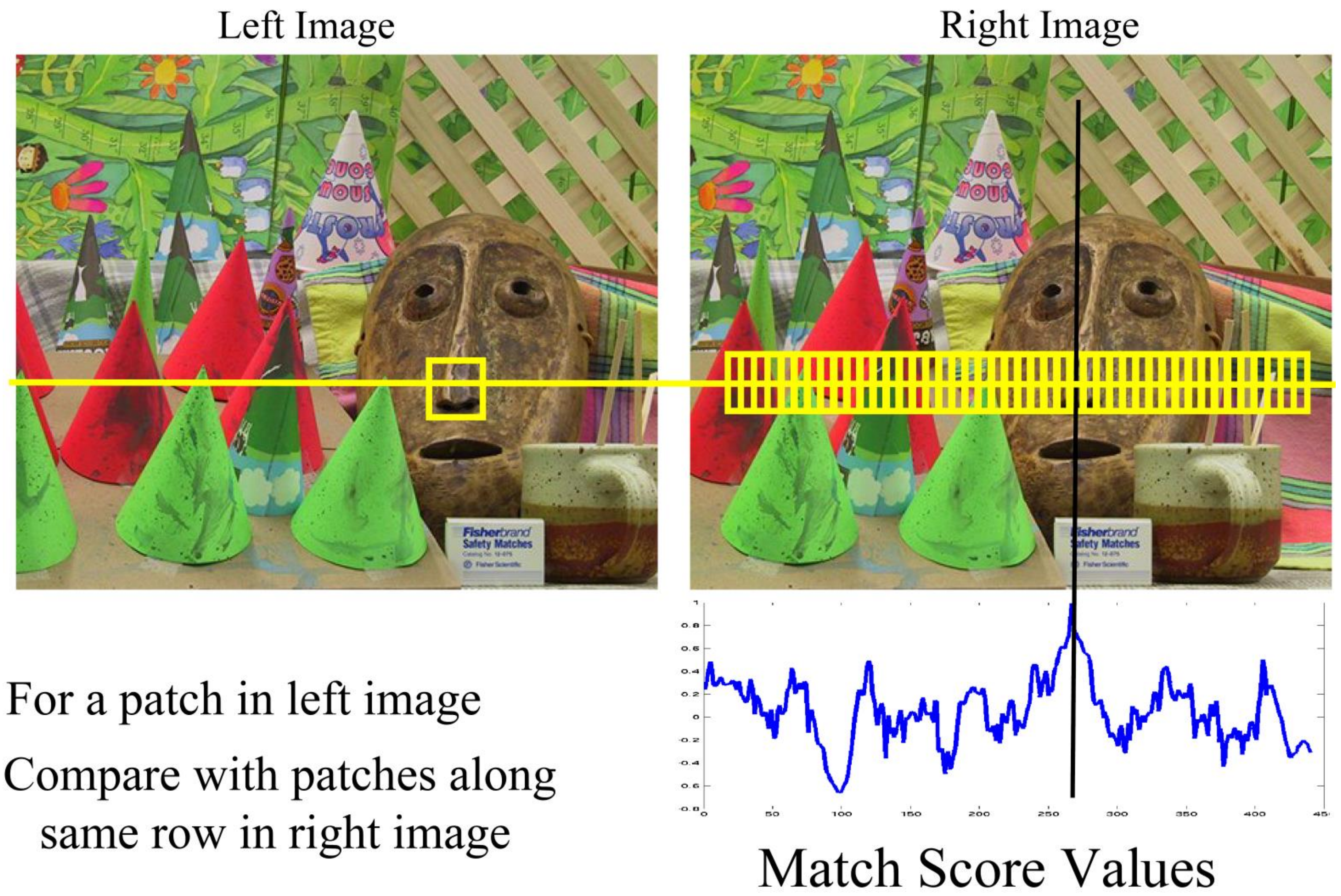

Disparity values (0-64)



$$d = f\frac{T_x}{Z}$$

Note how disparity is larger (brighter) for closer surfaces.

# Computing Disparity

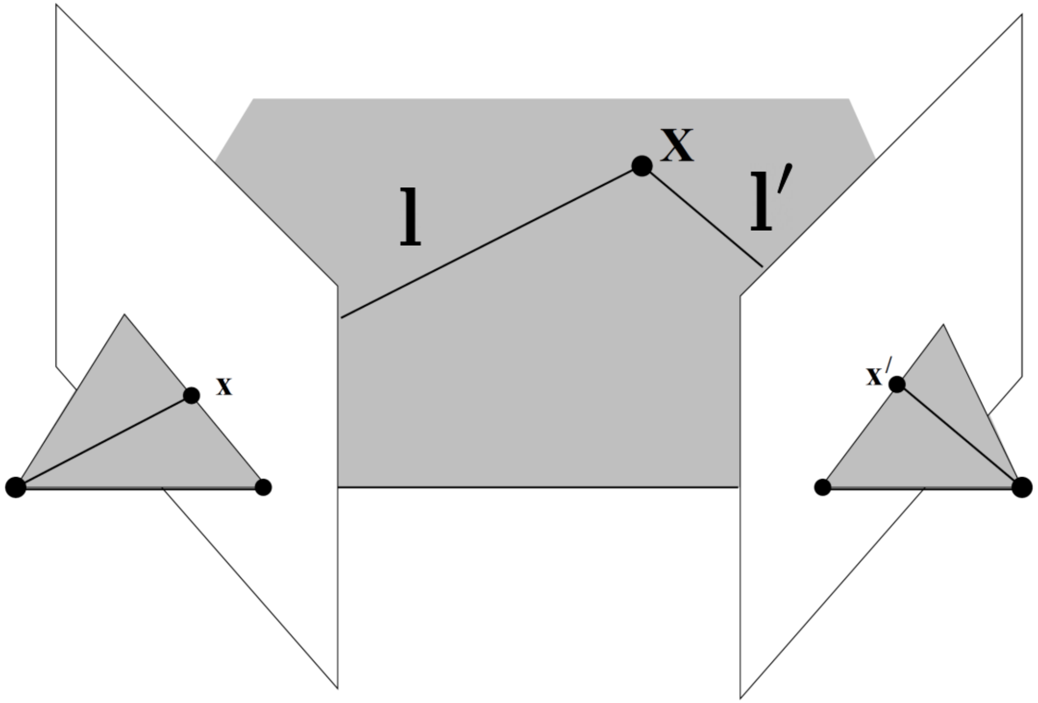



- Eipipolar lines are horizontal lines in stereo
- For general cases, we can find correspondences on eipipolar lines
- Depth from disparity

$$Z = f\frac{T_x}{d}$$

# Triangulation

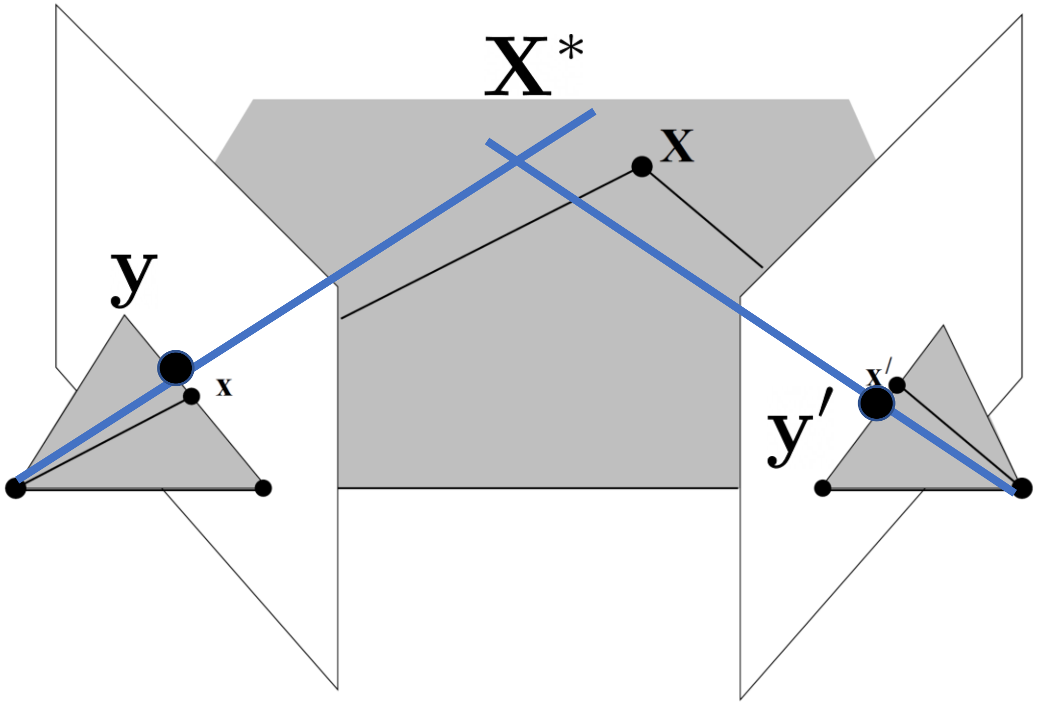
- Compute the 3D point given image correspondences




Intersection of two backprojected lines

$$\mathbf{X} = \mathbf{l} \times \mathbf{l}'$$

# Triangulation




- In practice, we find the correspondences
  $\mathbf{y} \quad \mathbf{y}'$
- The backprojected lines may not intersect
- Find X* that minimizes

$$d(\mathbf{y}, P\mathbf{X}^*) + d(\mathbf{y}', P'\mathbf{X}^*)$$

Projection matrix

# Summary

- Depth perception
  - Monocular cues
  - Stereo cues

- Computational models for stereo vision
  - Epipolar geometry
  - Stereo Systems
  - Triangulation

# Further Reading

- Section 6.1, Virtual Reality, Steven LaValle
- Multiview Geometry in Computer Vision, Richard Hartley and Andrew Zisserman, Chapter 9, Epipolar Geometry and Fundamental Matrix
- Stanford CS231A: Computer Vision, From 3D Reconstruction to Recognition, Lecture 5 https://web.stanford.edu/class/cs231a/syllabus.html