

The logo of The University of Texas at Dallas, featuring a circular seal with the letters 'UTD' in the center, the text 'THE UNIVERSITY OF TEXAS AT DALLAS' around the top, and 'EST. 1969' at the bottom. Two stars are positioned on either side of the 'EST. 1969' text.

# Visual Rendering: Rasterization, Lighting and Shading, Fragment Processing

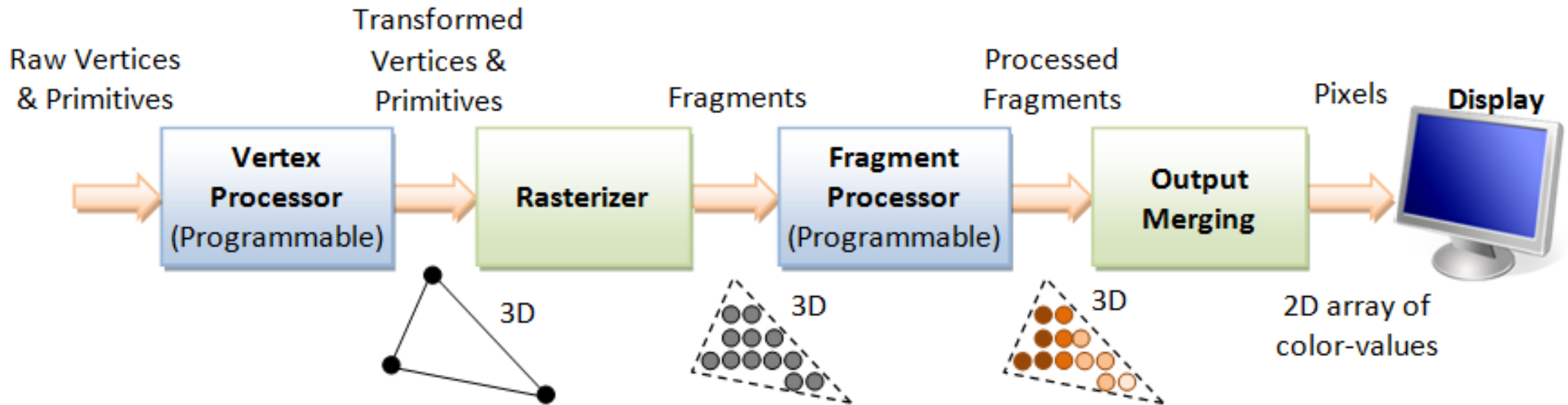
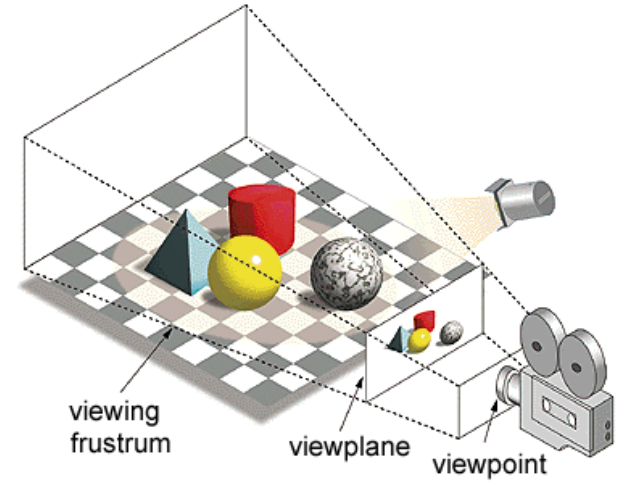
CS 6334 Virtual Reality

Professor Yu Xiang

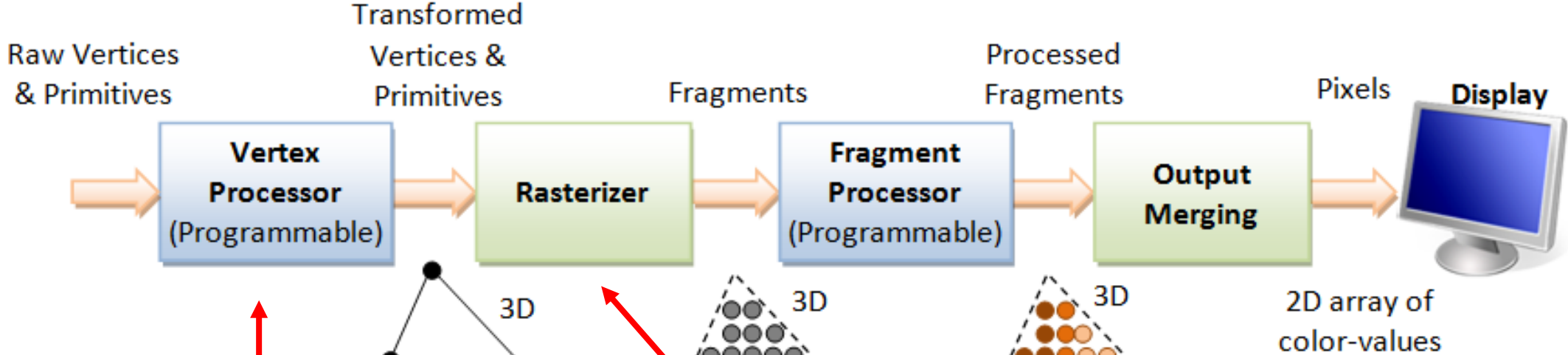
The University of Texas at Dallas

# Visual Rendering

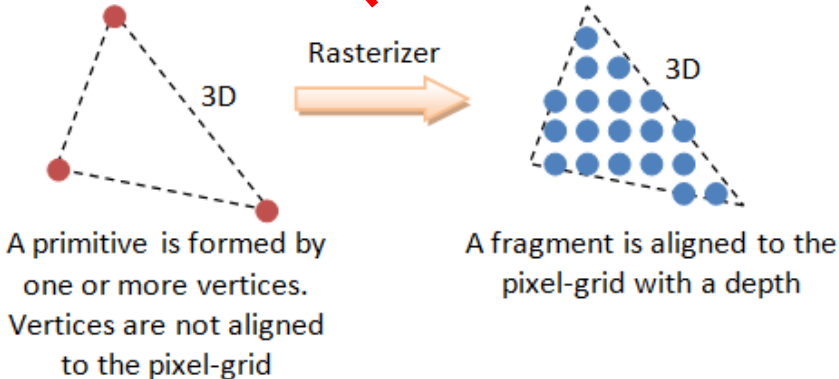
- Converting 3D scene descriptions into 2D images
- The graphics pipeline



# Rasterization



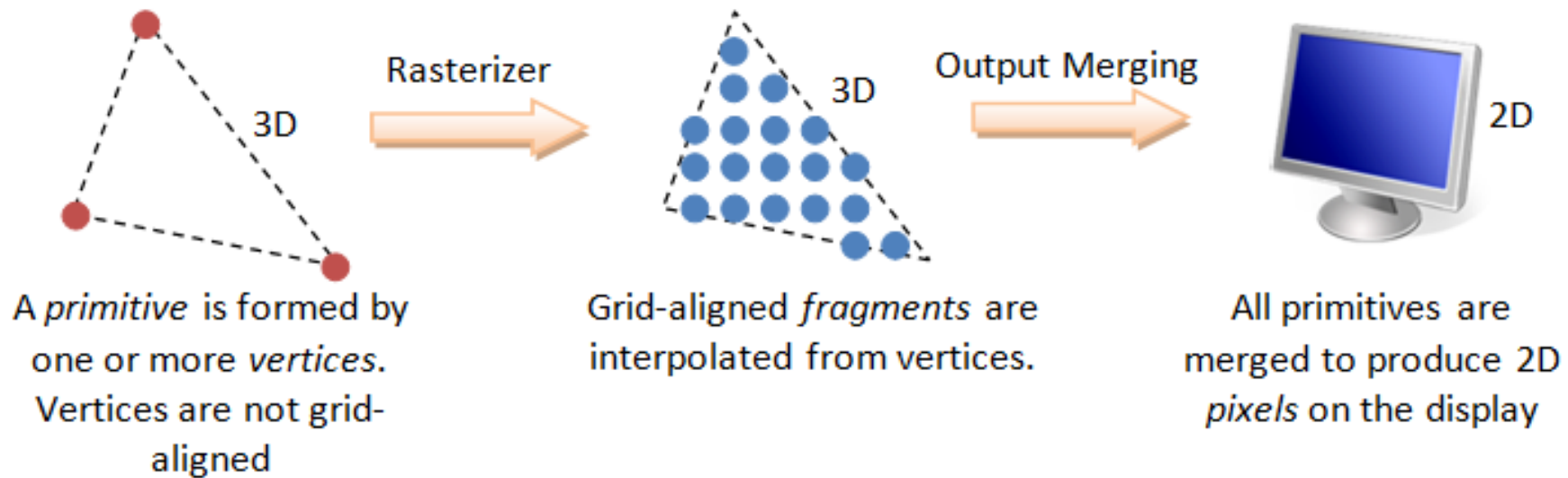
Vertex transforms



- Determine which pixels are inside the triangles
- Interpolate vertex attributes (e.g., color)

# Pixels vs. Fragments

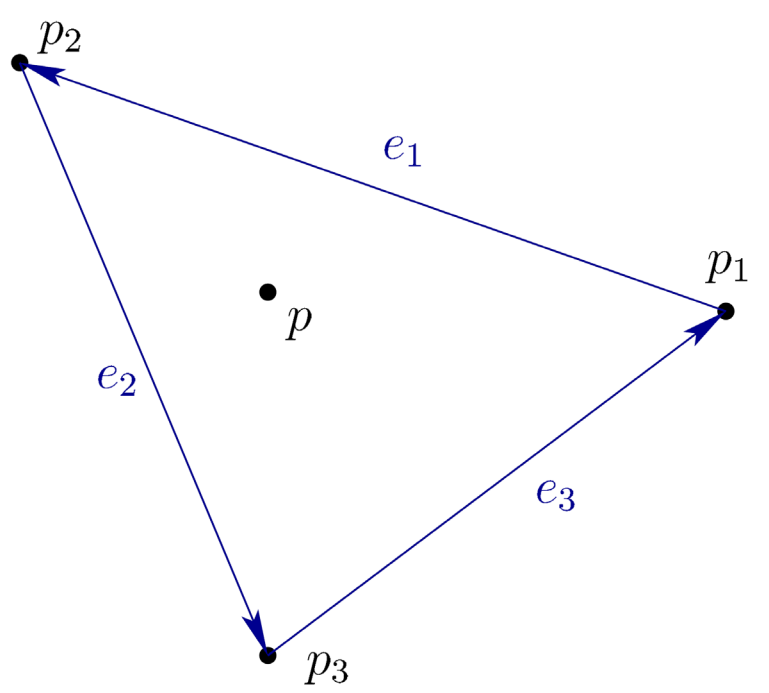
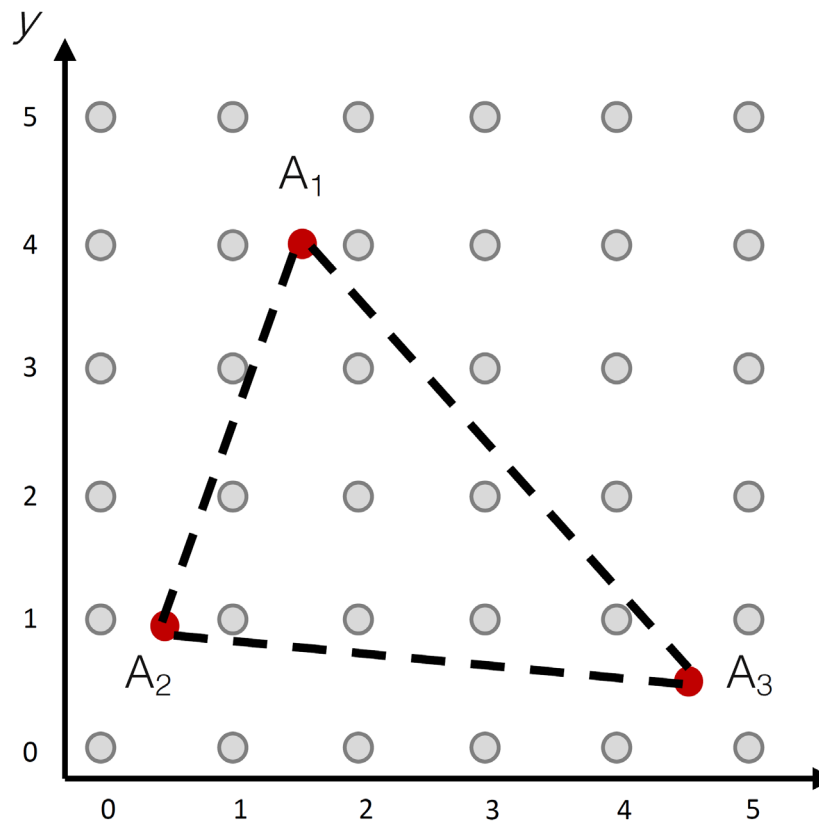
- Pixels are dots on the screen:  $(x, y)$  and RGB color
- Fragments:  $(x, y, z)$ ,  $z$  is the depth and other attributes (color, normal, texture coordinates, alpha value, etc.)



**Vertex, Primitives, Fragment and Pixel**

# Rasterization

- Determine which fragments are inside the triangle



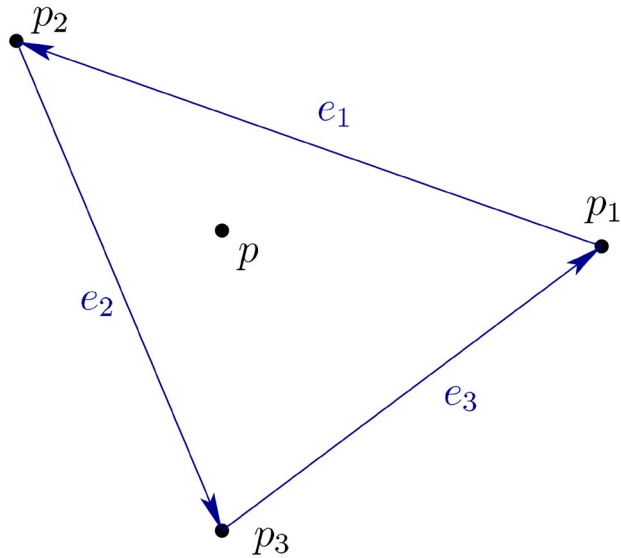
$$e_1 = p_2 - p_1$$
$$e_2 = p_3 - p_2$$
$$e_3 = p_1 - p_3$$

$p$  is inside if and only if

$$(p - p_1) \times e_1 < 0$$
$$(p - p_2) \times e_2 < 0$$
$$(p - p_3) \times e_3 < 0$$

magnitude of the cross products

# Barycentric Coordinates



Interpolate attributes of the vertices

$$p = \alpha_1 p_1 + \alpha_2 p_2 + \alpha_3 p_3$$

$$0 \leq \alpha_1, \alpha_2, \alpha_3 \leq 1$$

$$\alpha_1 + \alpha_2 + \alpha_3 = 1$$

Cramer's rule

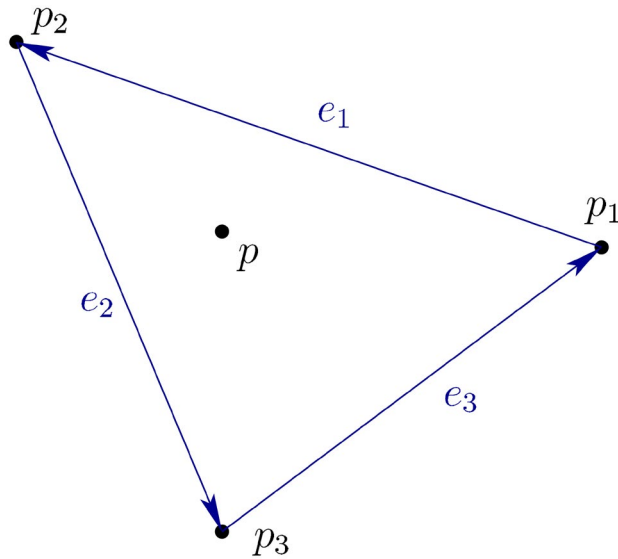
$$d_{ij} = e_i \cdot e_j \quad s = 1 / (d_{11}d_{22} - d_{12}d_{21})$$

$$\alpha_1 = s(d_{22}d_{31} - d_{12}d_{32})$$

$$\alpha_2 = s(d_{11}d_{32} - d_{12}d_{31})$$

$$\alpha_3 = 1 - \alpha_1 - \alpha_2.$$

# Barycentric Coordinates



$$p = \alpha_1 p_1 + \alpha_2 p_2 + \alpha_3 p_3$$

Color

$$R = \alpha_1 R_1 + \alpha_2 R_2 + \alpha_3 R_3$$

$$G = \alpha_1 G_1 + \alpha_2 G_2 + \alpha_3 G_3$$

$$B = \alpha_1 B_1 + \alpha_2 B_2 + \alpha_3 B_3.$$

Apply to other attributes, e.g., depth, texture coordinates, alpha value, etc.

# Depth Buffer for Visibility Testing

- When drawing multiple triangles, determine which one to draw and which one to discard
- If depth of fragment is smaller than the current value in the depth buffer, overwrite color and depth value using the current fragment



color buffer

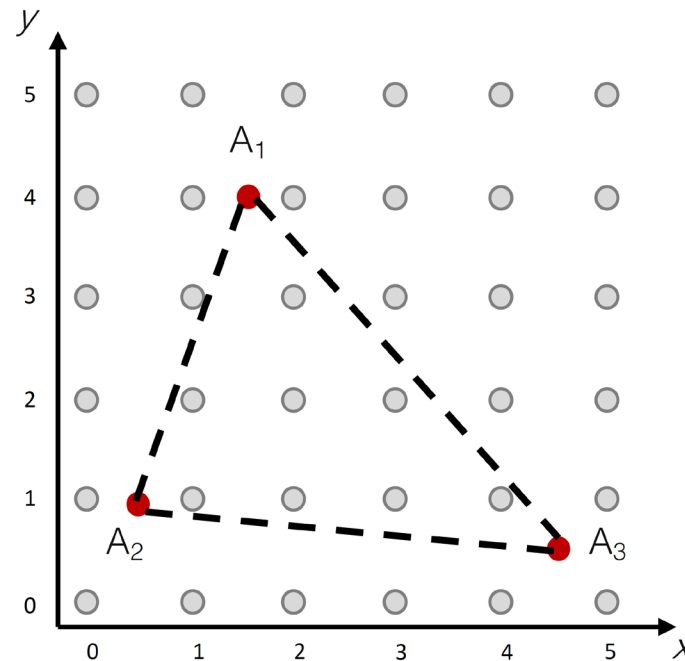


depth buffer



# Lighting and Shading

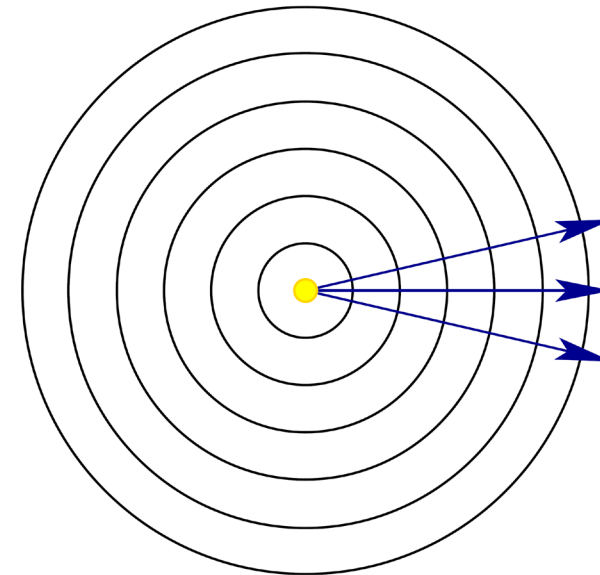
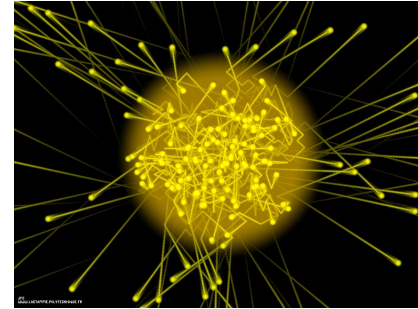
- How to determine color and what attributes to interpolate after rasterization



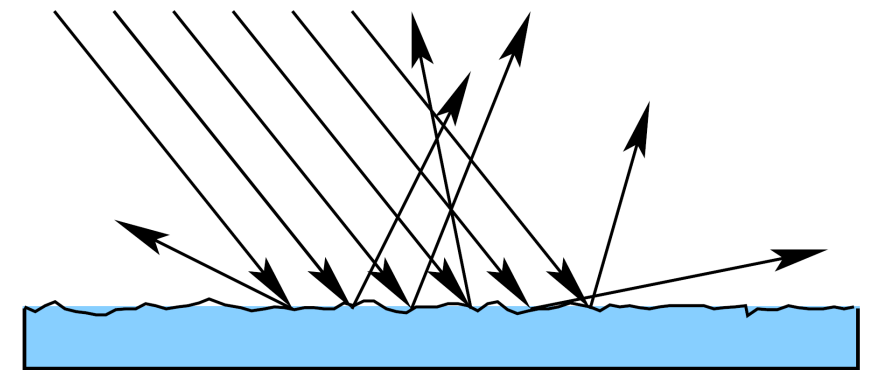
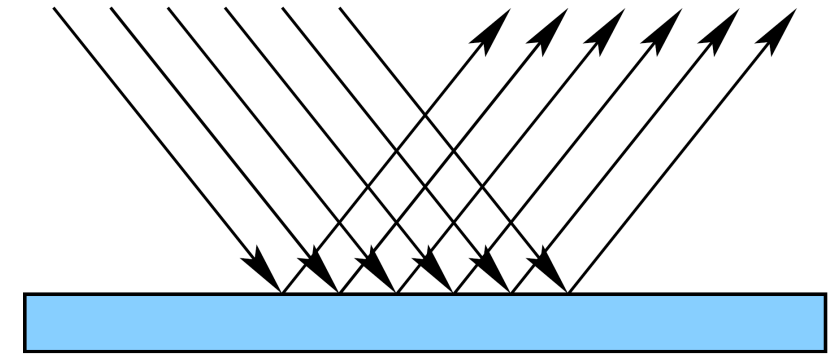
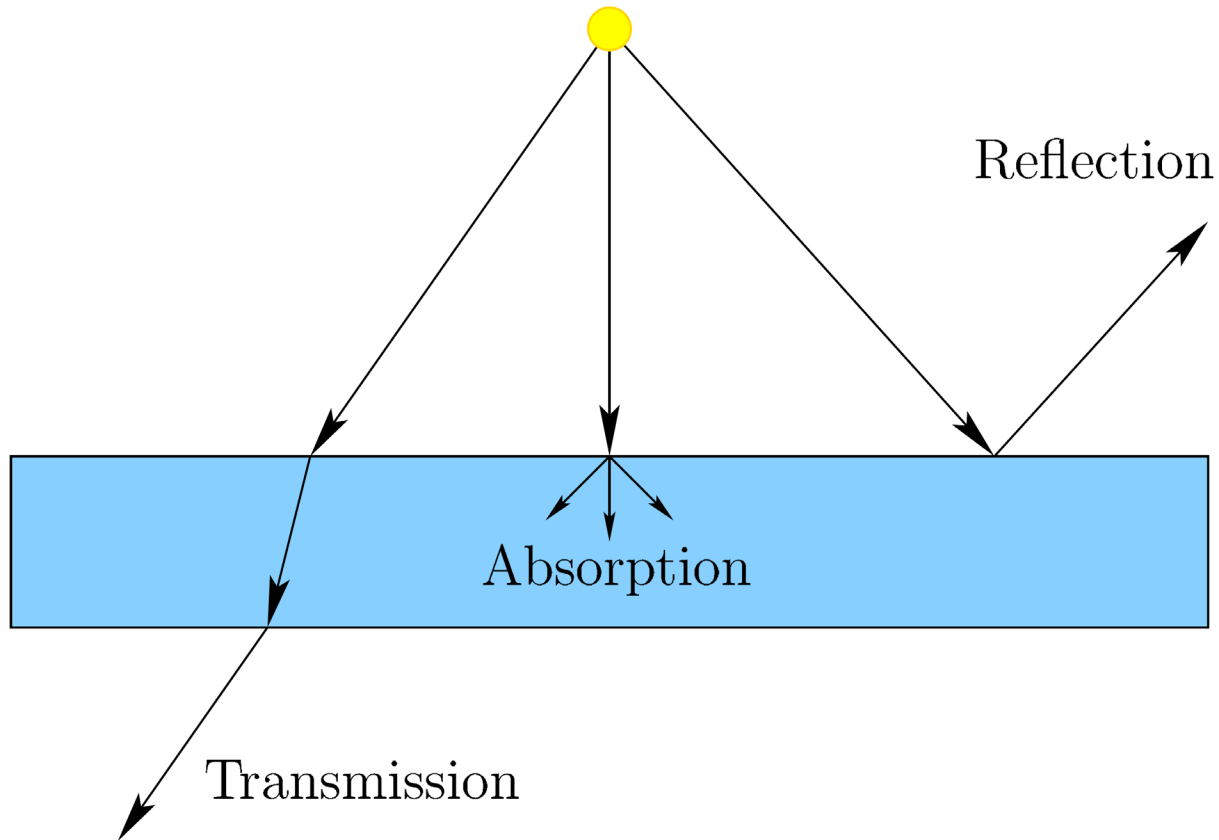
Rasterization: determine which fragments are inside the triangles

# Basic Behavior of Light

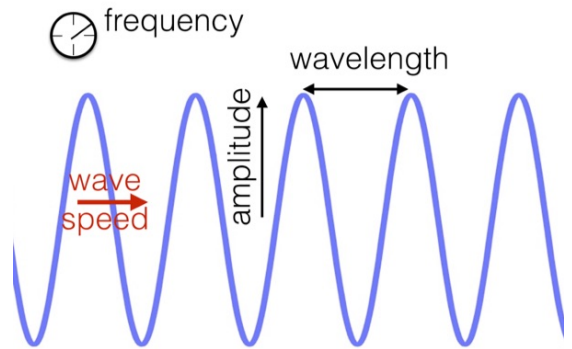
- Light can be described in three ways
  - Photons: tiny particles of energy moving through space at high speed
  - Waves: ripples through space
  - Rays: a ray traces the motion of a single hypothetical photon



# Interactions with Materials

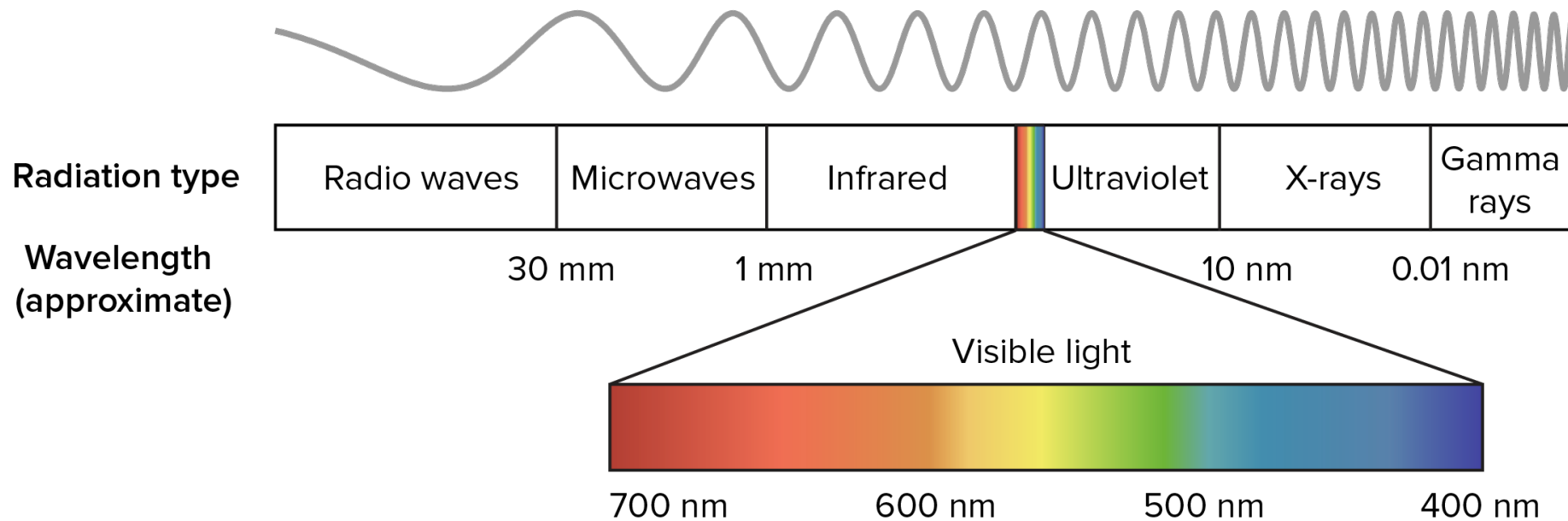


# Wavelengths and Colors



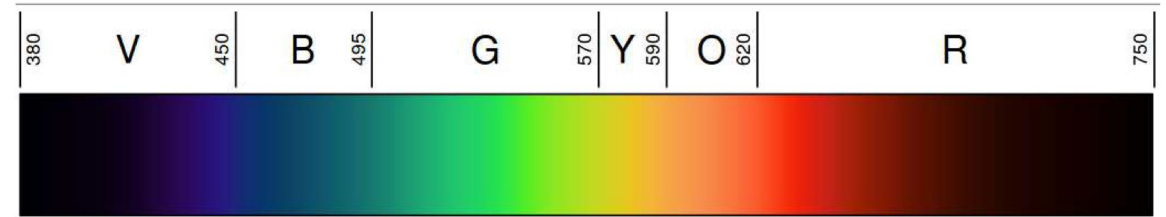
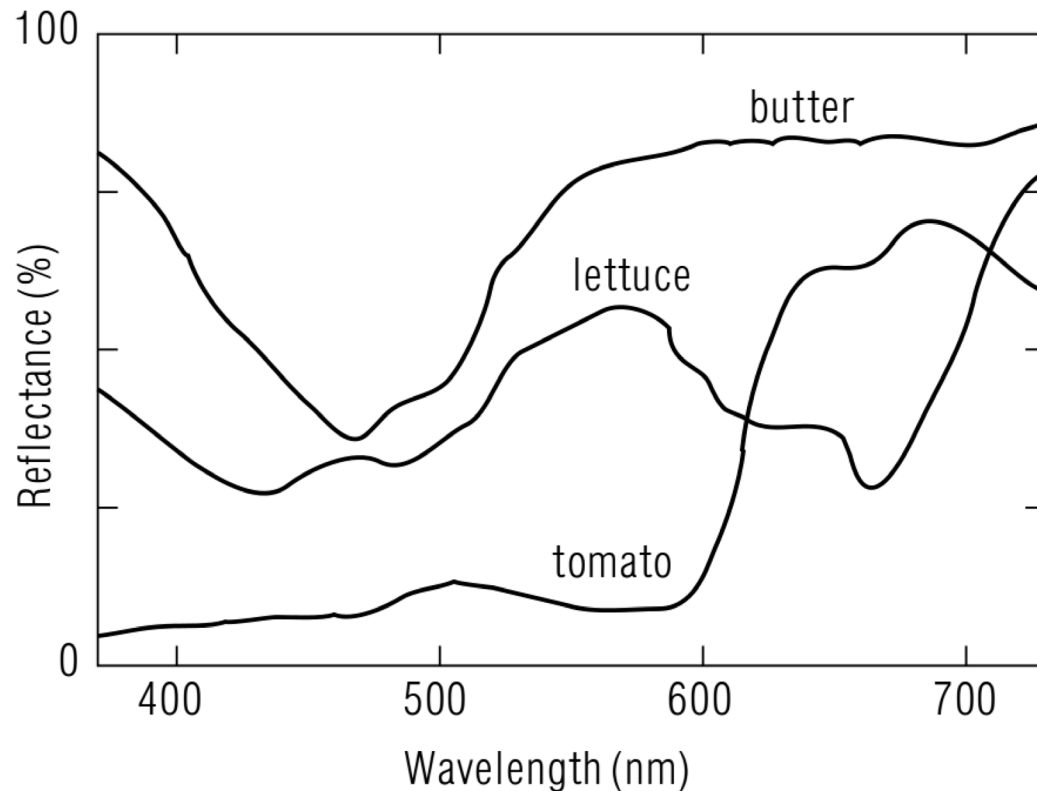
Wavelength  $\lambda = \frac{v}{f}$  Speed  
Frequency

Electromagnetic spectrum



# Reflection of Materials

- We see objects with different colors because the materials reflect specific colors differently



# Lambertian Lighting

Diffuse reflection

$$R = d_R I_R \max(0, n \cdot \ell)$$

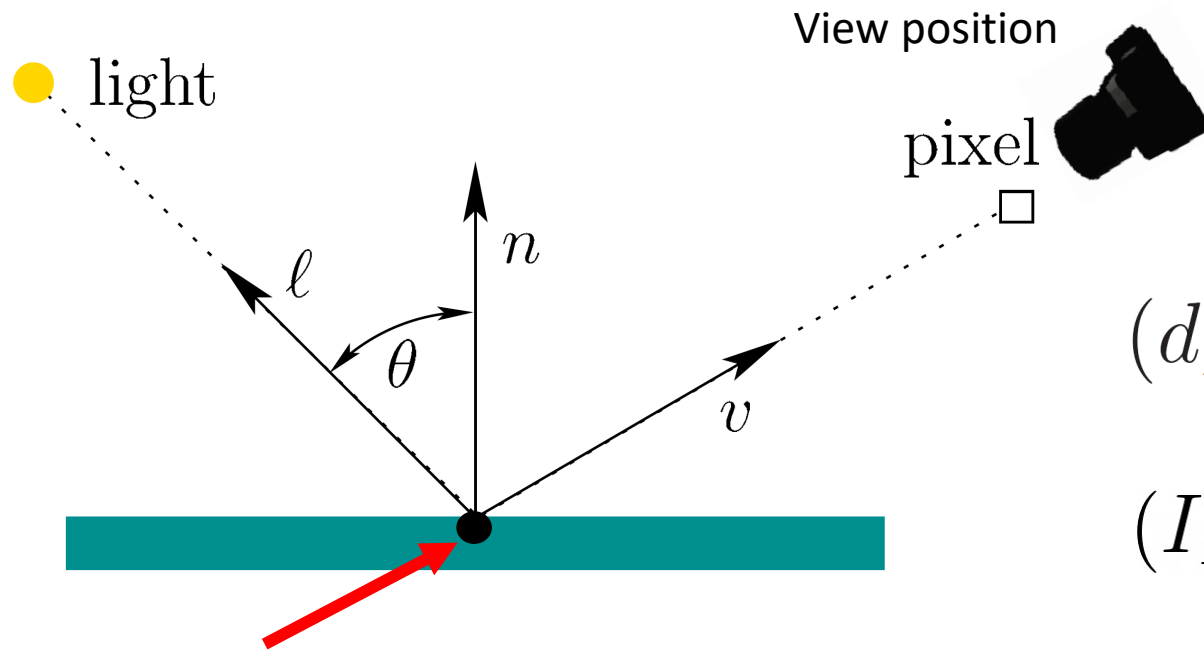
$$G = d_G I_G \max(0, n \cdot \ell)$$

$$B = d_B I_B \max(0, n \cdot \ell)$$

$$n \cdot \ell = \cos \theta$$

$(d_R, d_G, d_B)$  Reflectance property of the material (triangle)

$(I_R, I_G, I_B)$  Spectral power distribution of the light source

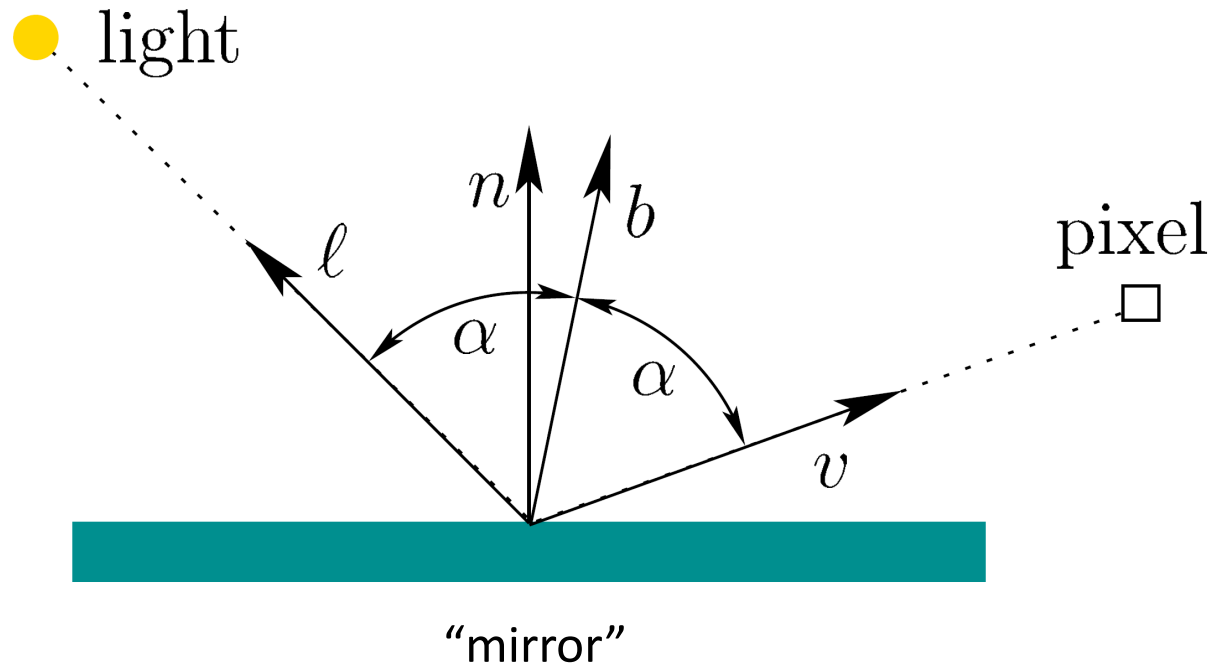


Think about this point as a vertex of a 3D mesh. We want to compute its color on the image

$$L = dI \max(0, n \cdot \ell) \quad n \cdot \ell < 0$$

Light behind triangle

# Blinn-Phong Lighting



Related to specular reflection

$$b = \frac{l + v}{\|l + v\|}$$

$x$  Material property that expresses the amount of surface shininess  
 $x=100$ , mild amount of shininess  
 $x=10000$ , almost like a mirror

$s$  Specular reflectance property of the material

$$L = dI \max(0, n \cdot l) + sI \max(0, n \cdot b)^x$$

# Ambient Lighting

- Independent of light/surface position, viewer, normal
- Adding some background color

$$L = dI \max(0, n \cdot \ell) + sI \max(0, n \cdot b)^x + L_a$$

Ambient light





# Multiple Light Sources and Attenuation

- N light sources

$$L = L_a + \sum_{i=1}^N dI_i \max(0, n \cdot l_i) + sI_i \max(0, n \cdot b_i)^x$$

- Attenuation: the greater the distance, the low the intensity

$$L = L_a + \sum_{i=1}^N \frac{1}{k_c + k_l c + k_q c^2} \left( dI_i \max(0, n \cdot l_i) + sI_i \max(0, n \cdot b_i)^x \right)$$

constant    linear    quadratic attenuation

$c$  Light source distance to surface

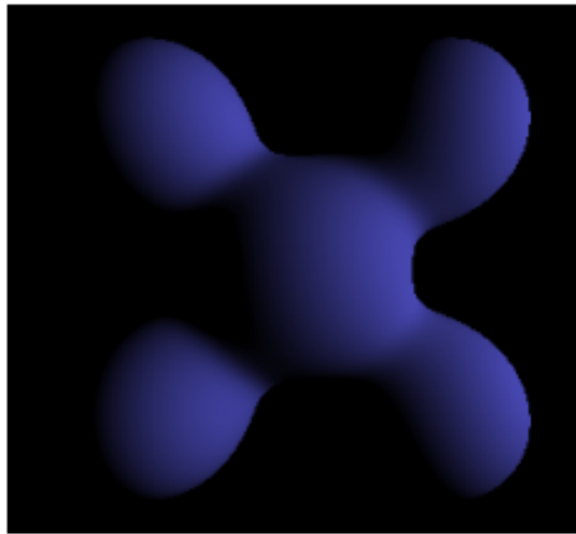
Used by OpenGL for ~25 years

# Phong Reflection Model



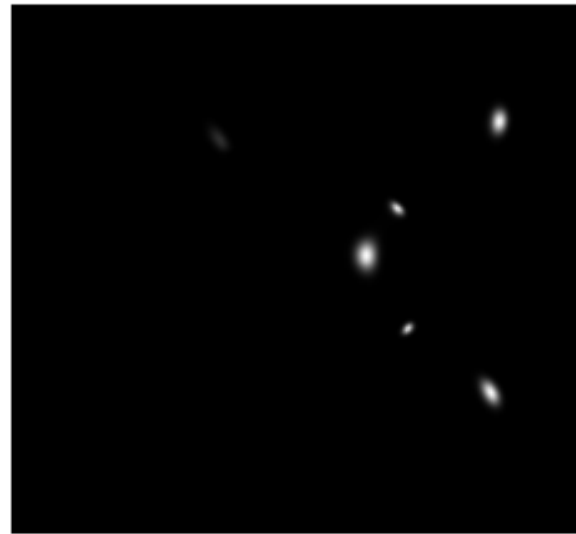
**Ambient**

+



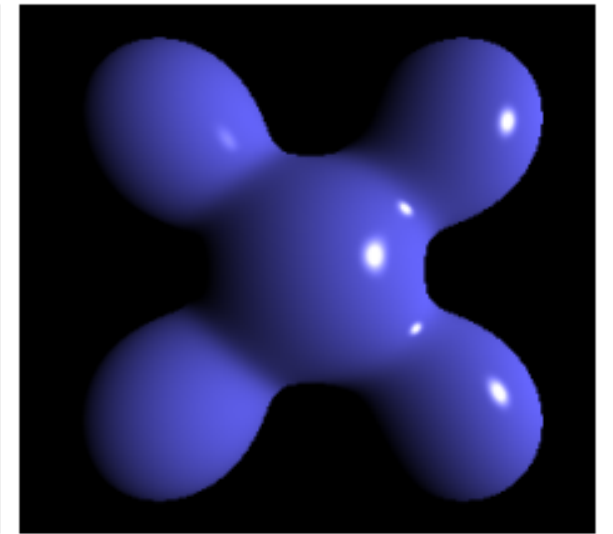
**Diffuse**

+



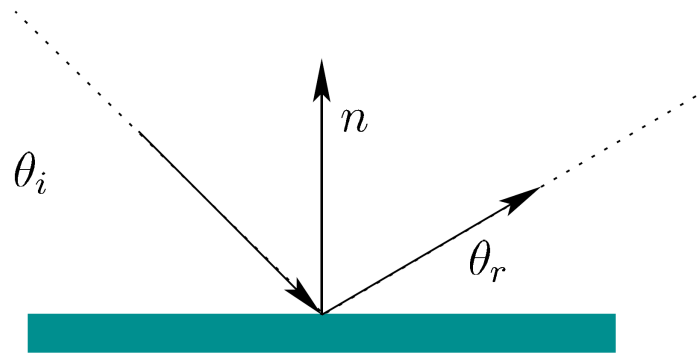
**Specular**

=

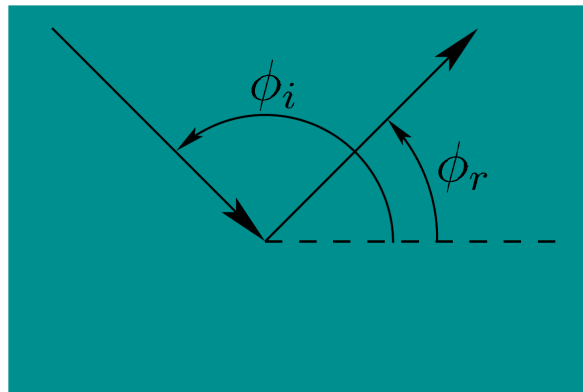


**Phong Reflection**

# Bidirectional Reflectance Distribution Function (BRDF)



Side view



Top view

Shading in a more precise and general way

$$f(\theta_i, \phi_i, \theta_r, \phi_r) = \frac{\text{radiance}}{\text{irradiance}}$$

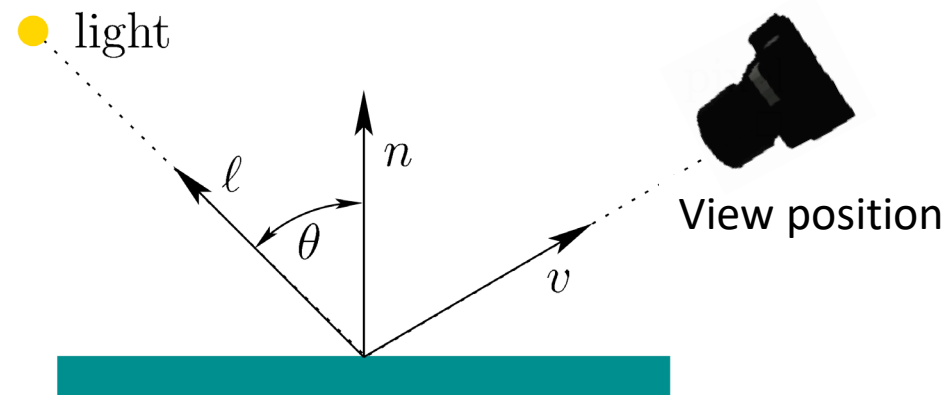
- Radiance: light energy reflected from the surface
- Irradiance: light energy arriving at the surface

For Lambertian shading, BRDF is a constant

- The surface reflects equally in all directions

# Lighting Calculations

- All lighting calculations can happen in world space
  - Transform vertices and normal into world space
- Calculate lighting, i.e., compute vertex color given material properties, light source color and position, vertex position, normal position, view position

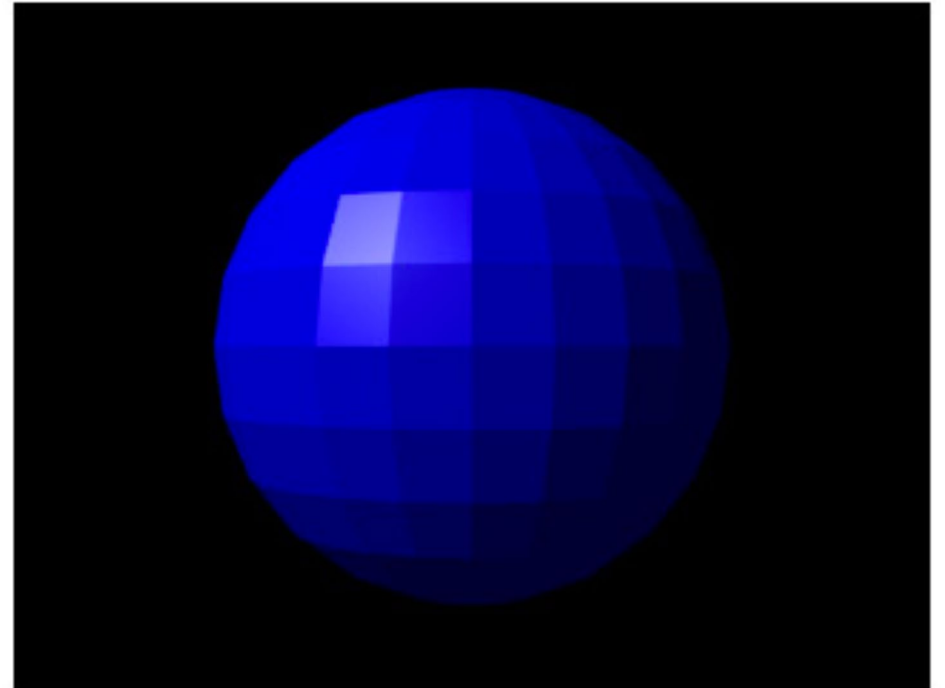


# Lighting vs. Shading

- Lighting: interaction between light and surface
  - Different mathematic models exist, e.g., Phong lighting model
  - What formula is being used to calculate intensity/color
- Shading: how to compute color for each fragment
  - What attributes to interpolate
  - Where to do lighting calculation

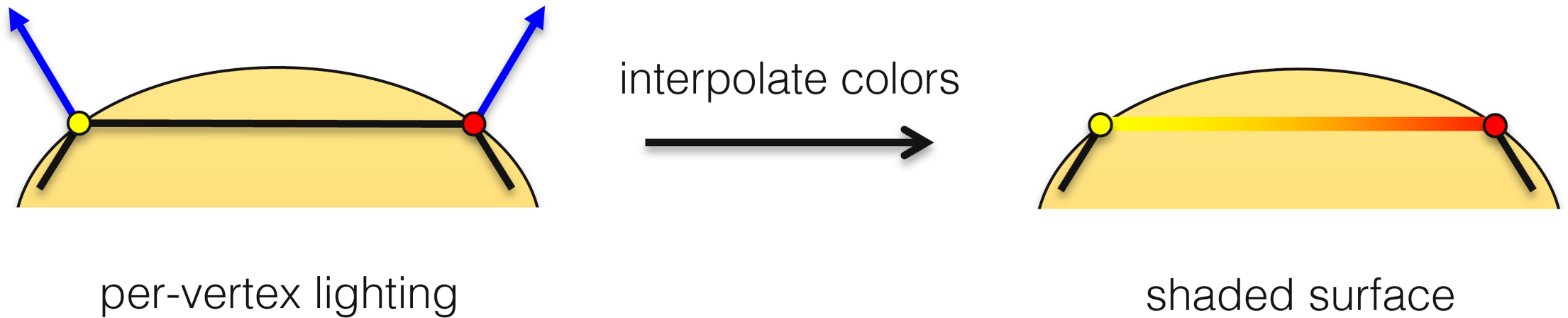
# Flat Shading

- Compute color only once per triangle (i.e., with Phong lighting)
  - Compute color for the first vertex or the centroid
- Pro: fast to compute
- Con: create a flat, unrealistic appearance

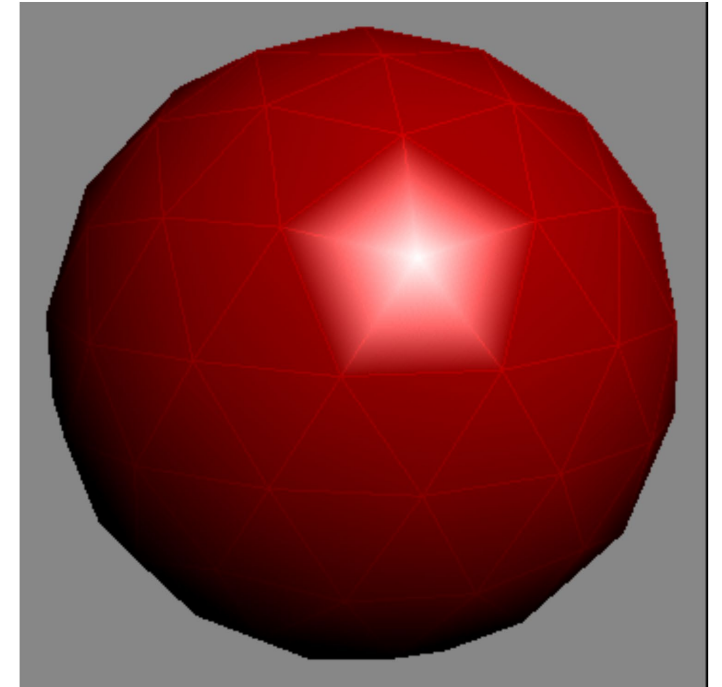
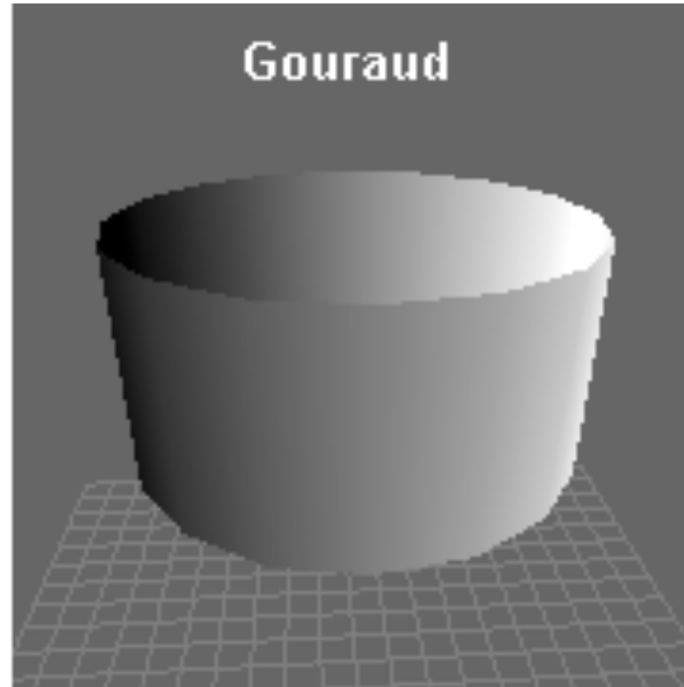
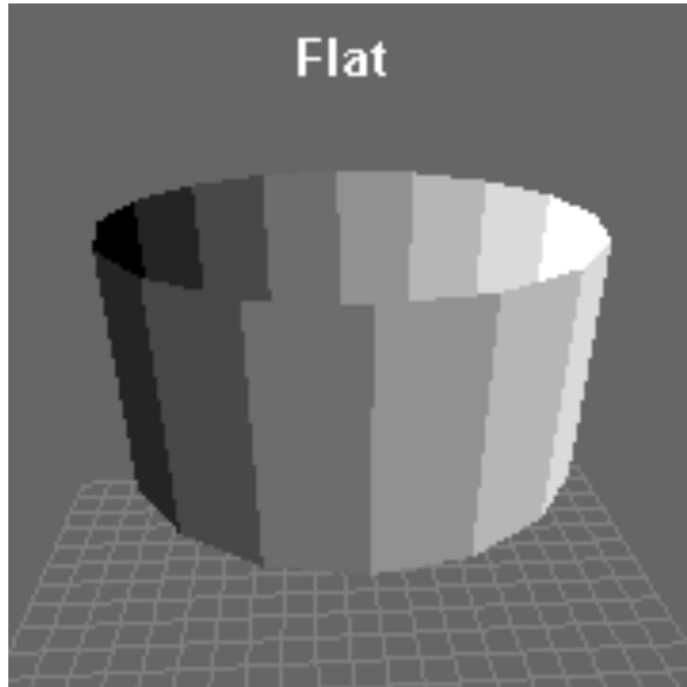


# Gouraud or Per-vertex Shading

- Compute color only once per vertex (i.e., with Phong lighting)
- Interpolate per-vertex color to all fragments within the triangle
- Pro: fast to compute
- Con: flat, unrealistic specular highlights



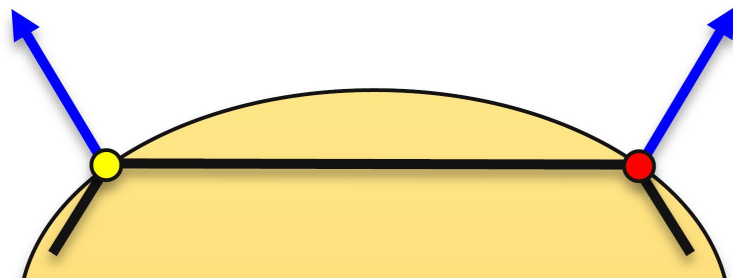
# Gouraud or Per-vertex Shading



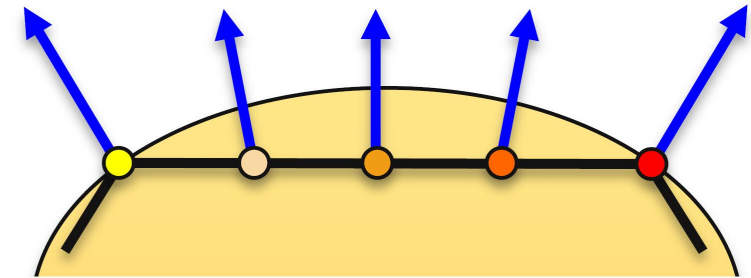


# Phong Shading or Per-fragment Shading

- Compute color only once per fragment (i.e., with Phong lighting)
- Need to interpolate per-vertex normal to all fragments to do the lighting calculation
- Pro: better appearance of specular highlights
- Con: slower to compute



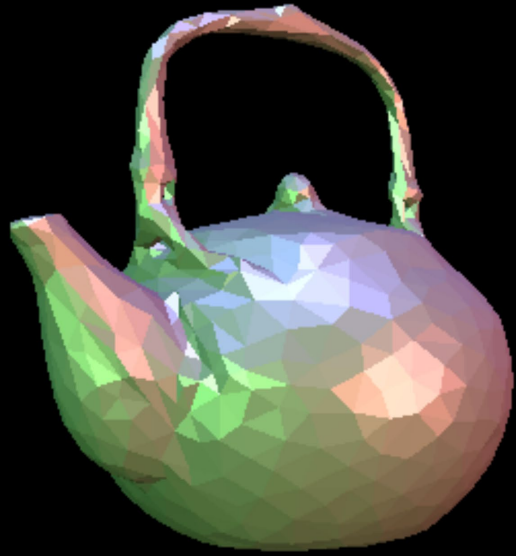
interpolate normals



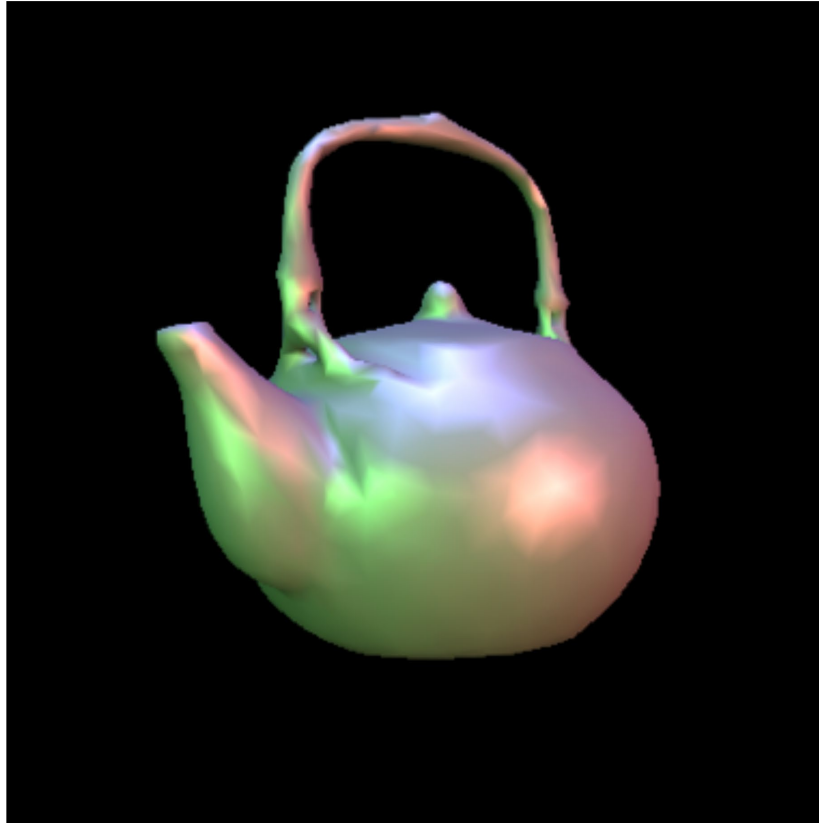
per-fragment lighting

# Shading

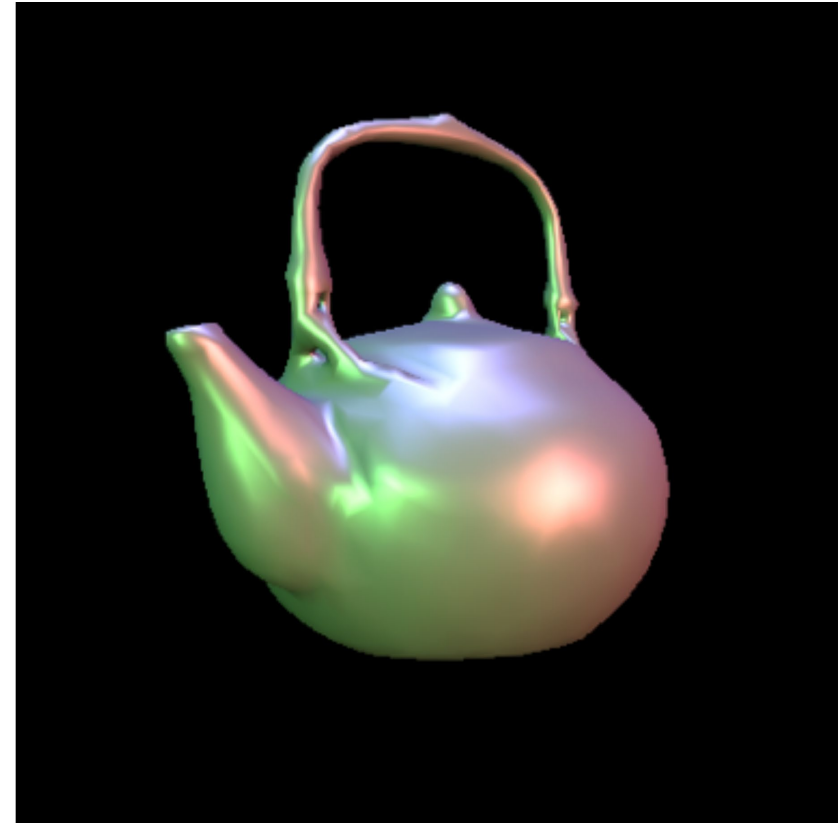
Flat Shading



Gouraud Shading

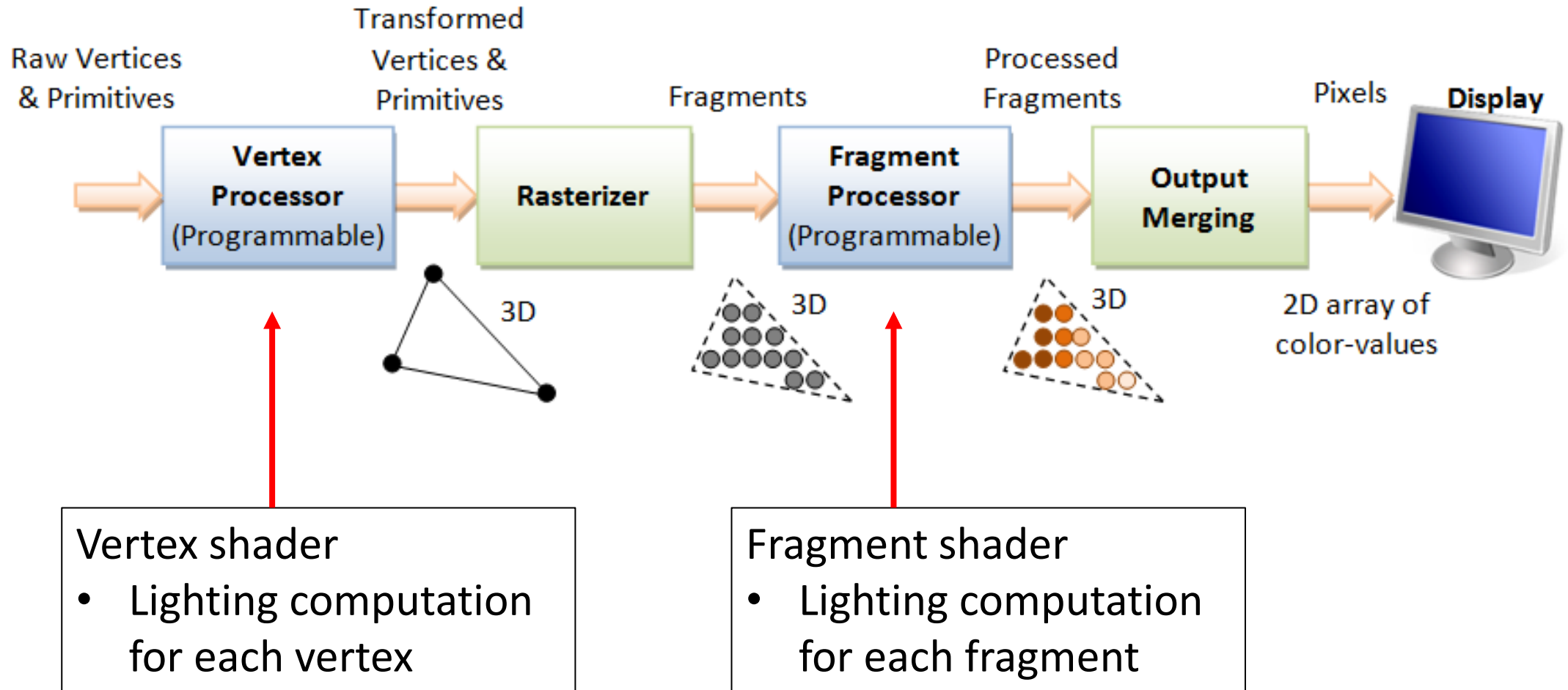


Phong Shading



<http://www.decew.net/OSS/timeline.php>

# Shader

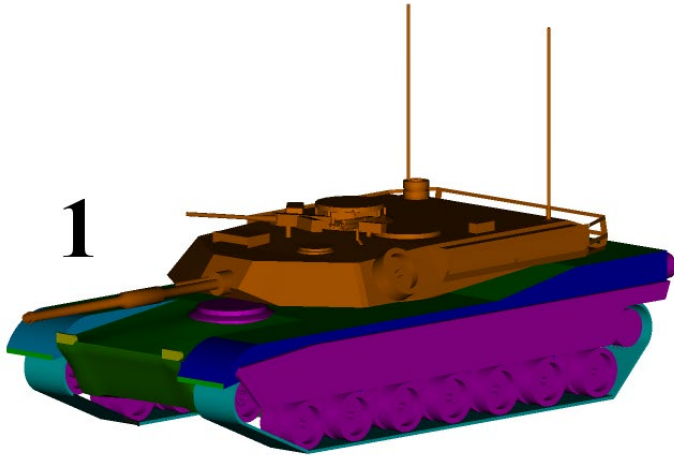


# Shader

- Shaders are small programs that are executed in parallel on GPUs for each vertex (vertex shader) or each fragment (fragment shader)
- Vertex shader (before rasterization)
  - Modelview projection transform of vertex and normal
  - If per-vertex lighting, compute lighting for each vertex
- Fragment shader (after rasterization)
  - If per-vertex lighting, assign color to each fragment
  - If per-fragment lighting, compute lighting for each fragment

# Texture Mapping

- Map textures (2D images) to 3D models



Without texture

- Need to specify vertex colors

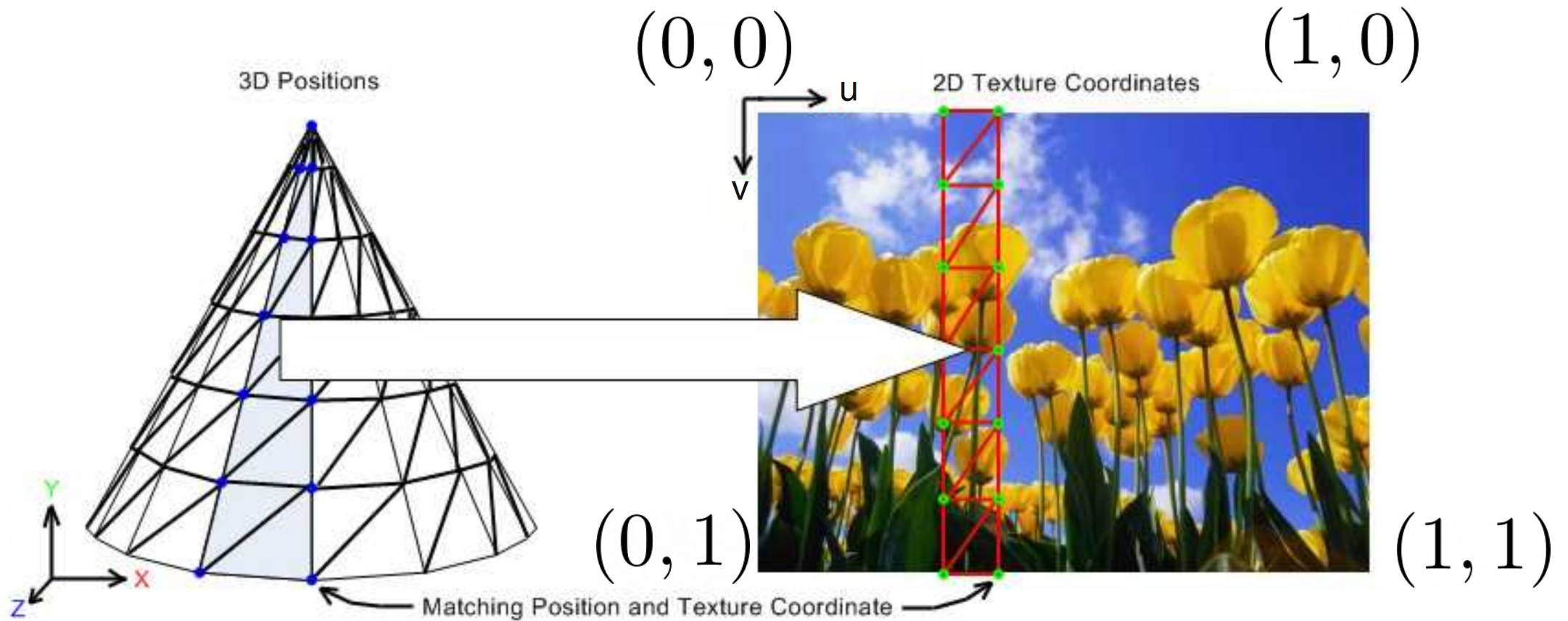


With texture

- Vertex colors from texture

# Texture Mapping

- UV coordinates (normalized)

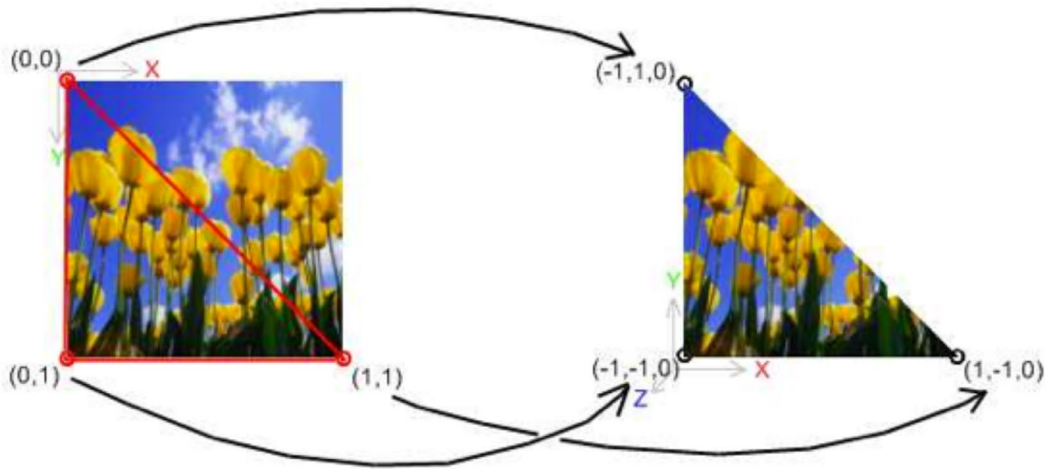


# Texture Mapping

- Same texture, different UV coordinates for mapping

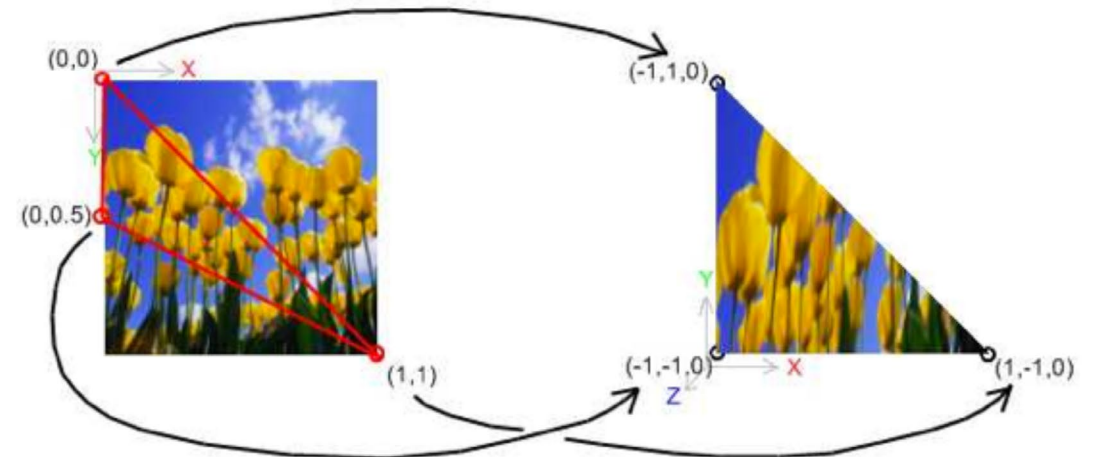
Texture Coordinates

Rendered Triangle



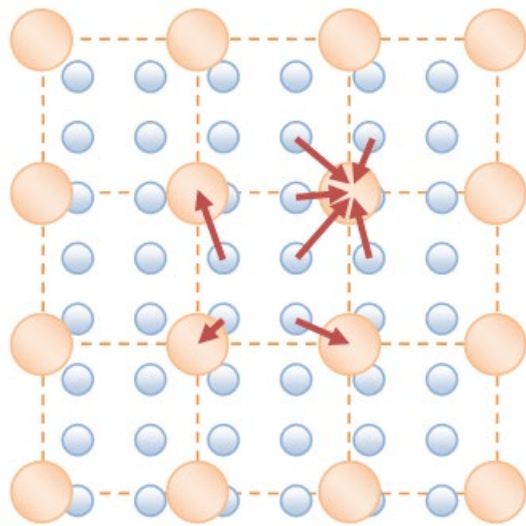
Texture Coordinates

Rendered Triangle

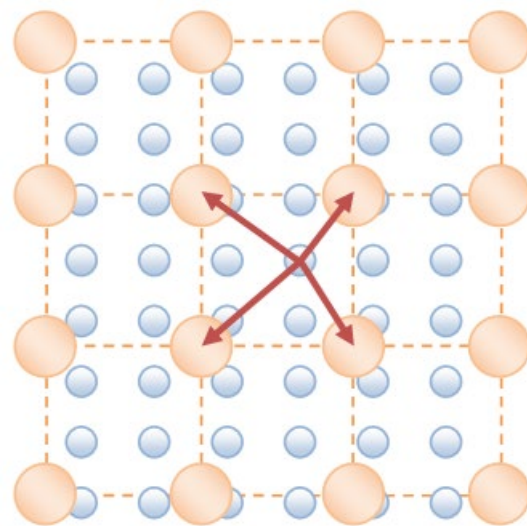


# Texture Mapping

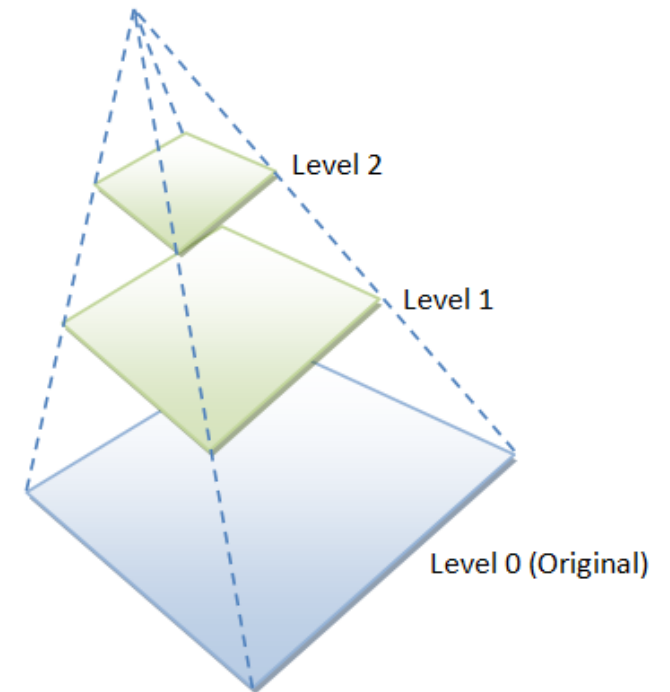
- Texture filtering: the resolution of the texture image is different from the displayed fragment
  - Magnification
  - Minification



Magnification - Nearest Point Sampling



Magnification - Bilinear Interpolation



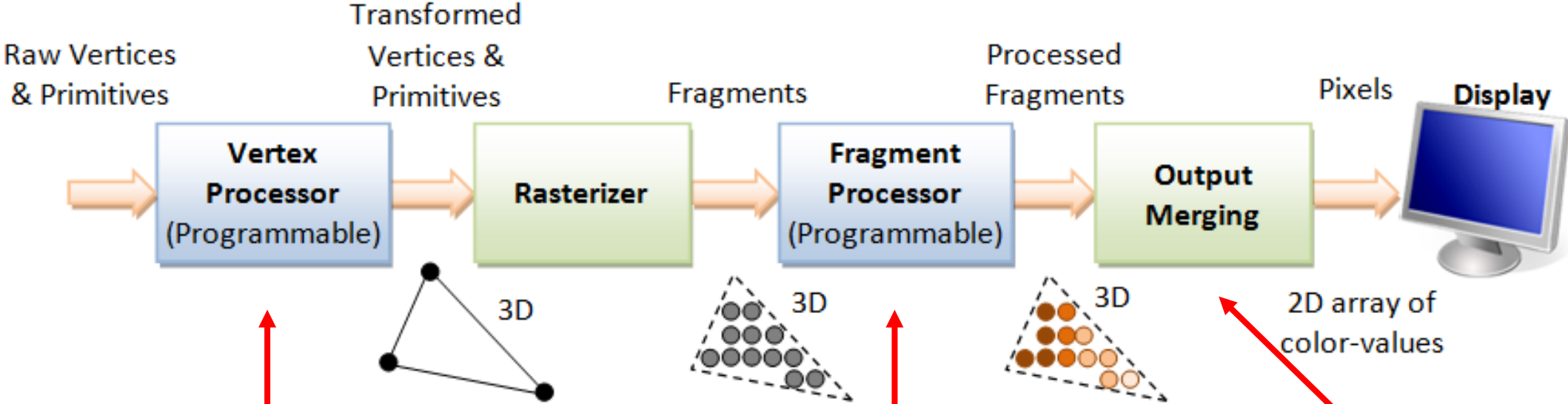
Minmapping



# Texture Mapping



# Review of the Graphics Pipeline



Vertex shader

- Vertex transforms
- Per-vertex lighting

Fragment shader

- Texturing
- Per-fragment lighting

Combine the fragments of all primitives into 2D color-pixel for display

# Further Reading

- 3D graphics with OpenGL, Basic Theory  
[https://www3.ntu.edu.sg/home/ehchua/programming/opengl/CG\\_BasicsTheory.html](https://www3.ntu.edu.sg/home/ehchua/programming/opengl/CG_BasicsTheory.html)
- Textbook: Shirley and Marschner “Fundamentals of Computer Graphics”, AK Peters, 2009
- Stanford EE267, Virtual Reality, Lecture 3  
<https://stanford.edu/class/ee267/syllabus.html>