



# Head Tracking and IMUs

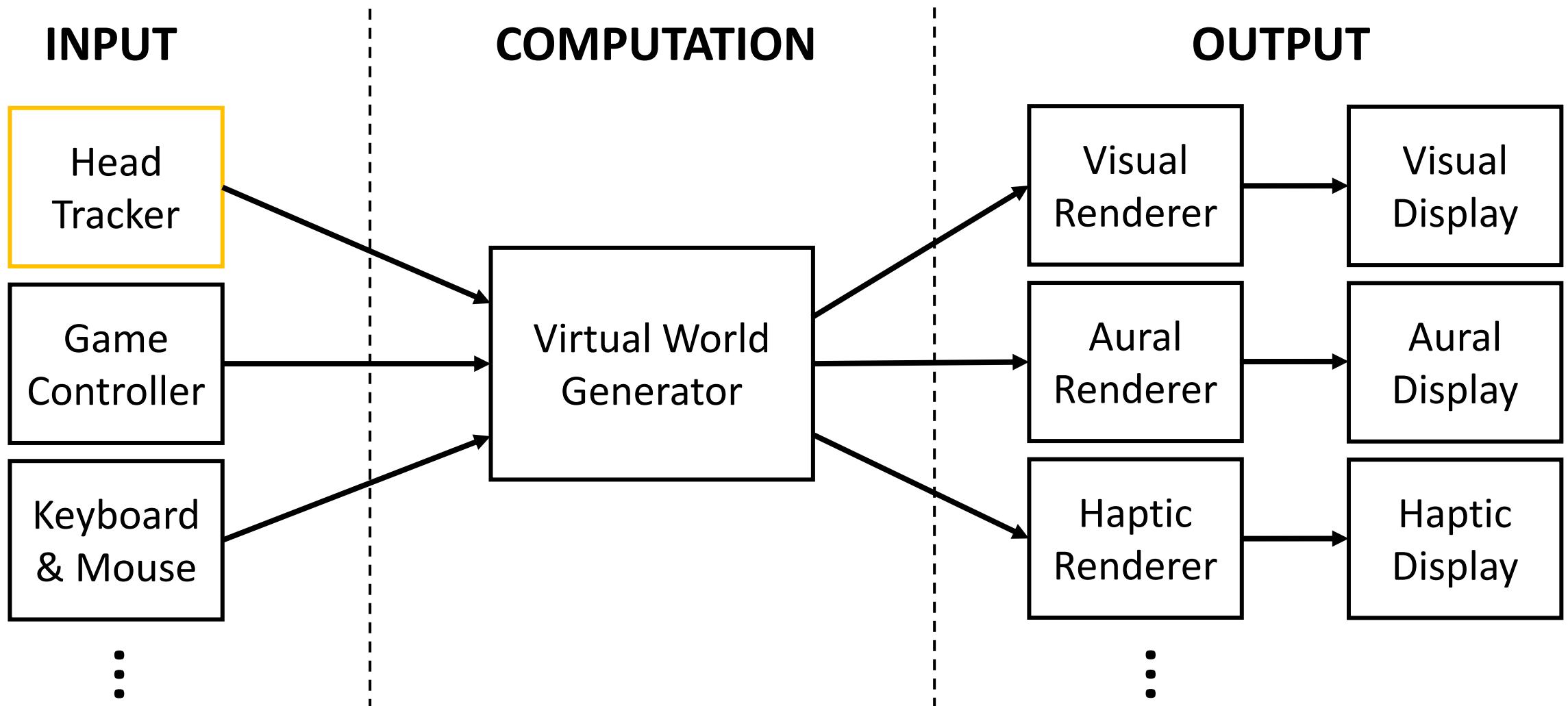
CS 6334 Virtual Reality

Professor Yu Xiang

The University of Texas at Dallas

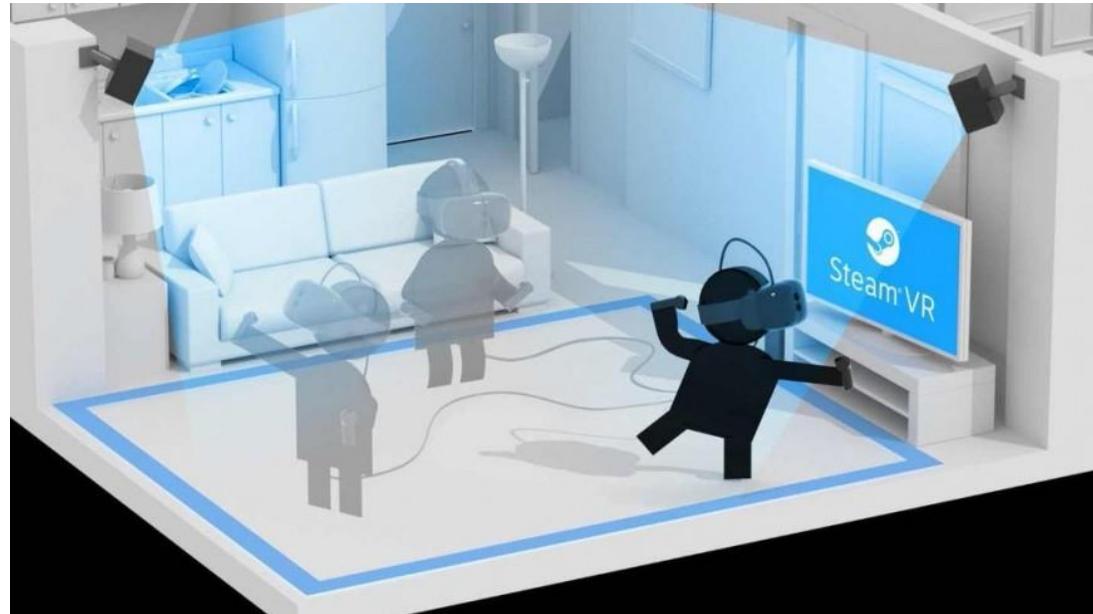
Some slides of this lecture are courtesy Gordon Wetzstein

# Review of VR Systems

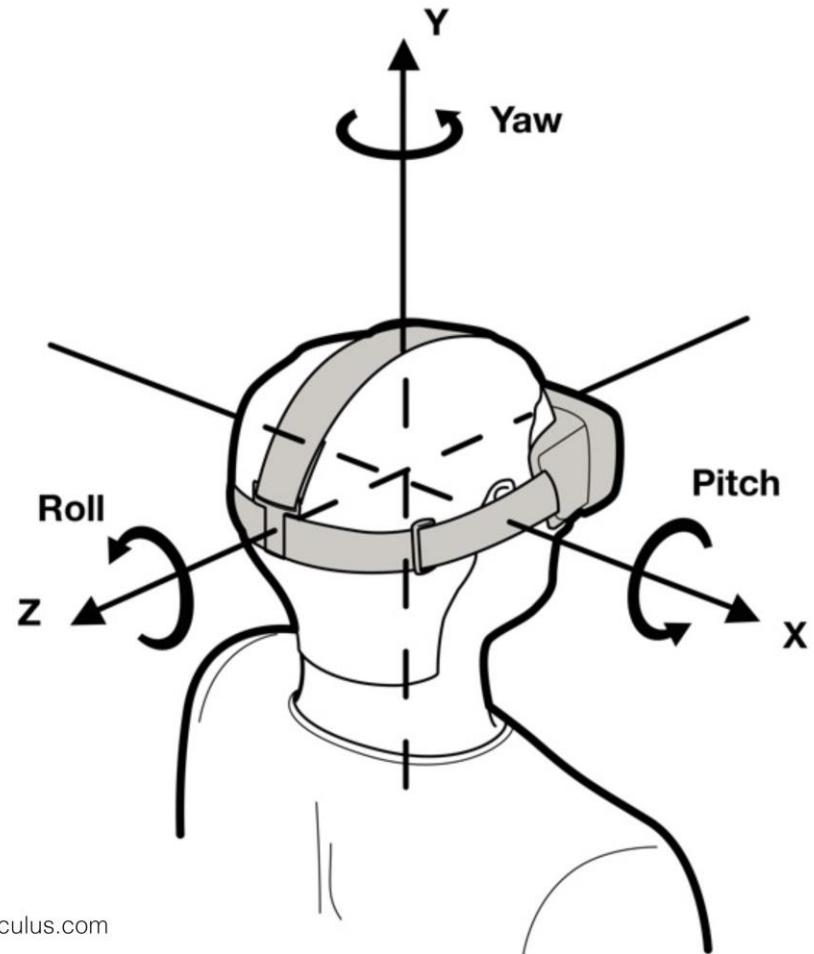


# Tracking in VR

- Tracking the user's sense organs
  - E.g., Head and eye
  - Render stimulus accordingly
- Tracking user's other body parts
  - E.g., human body and hands
  - Locomotion and manipulation
- Tracking the rest of the environment
  - Augmented reality
  - Obstacle avoidance in the real world

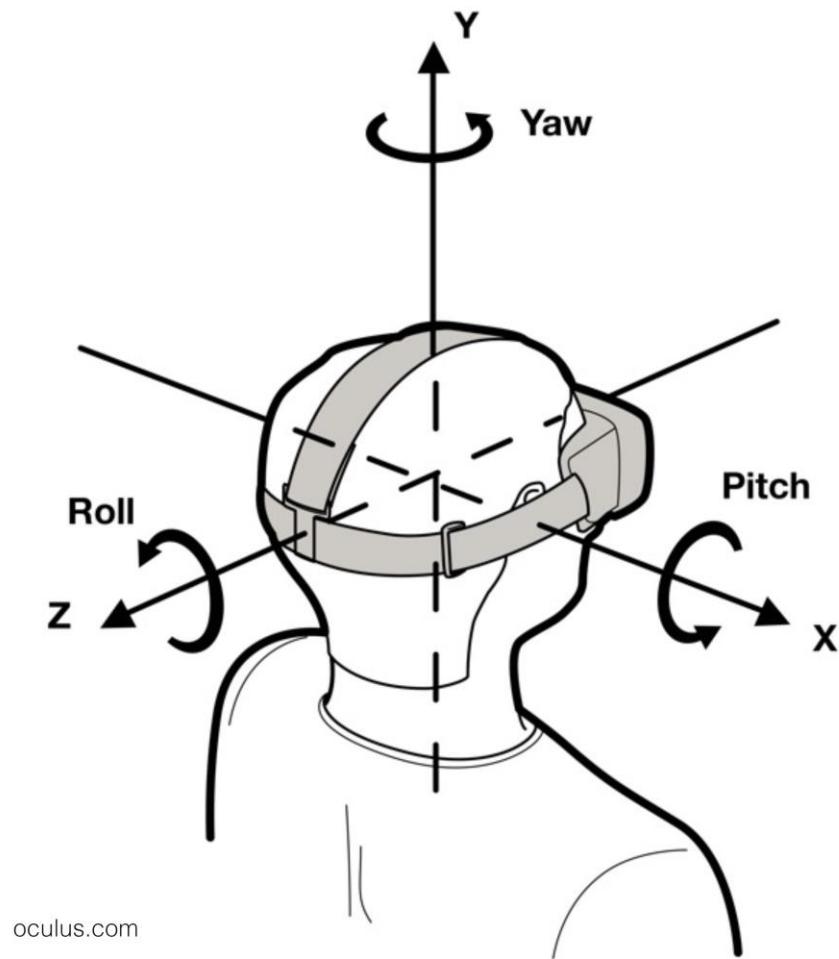


# Head Tracking



- Track orientation of the head
- Orientation is the rotation of the device w.r.t. world or inertial frame
- Euler angle representation: yaw, pitch, roll

# Head Tracking



- Determine the viewpoint of the user
- In visual rendering

vertex in clip space

$$v_{clip} = M_{proj} \cdot M_{view} \cdot M_{model} \cdot v$$

↓  
projection matrix    view matrix    model matrix  
↑                 ↑                 ↑

rotation    translation

$$M_{view} = R \cdot T(-eye)$$

$$R = R_z(-\theta_z) \cdot R_x(-\theta_x) \cdot R_y(-\theta_y)$$

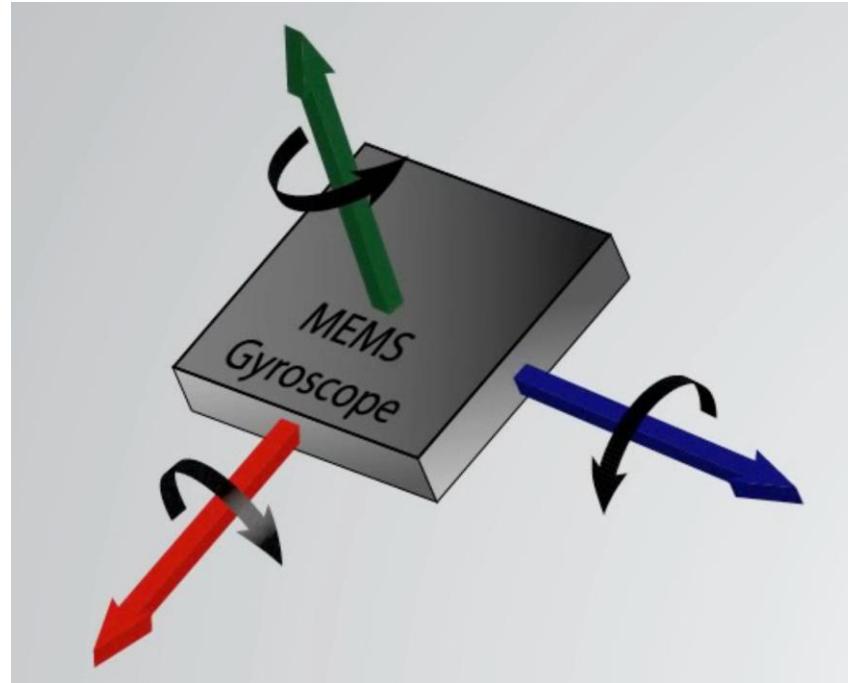
# Inertial Measurement Unit (IMU)

- Gyroscope measures angular velocity  $\tilde{\omega}$  in degrees/second
- Accelerometer measures linear acceleration  $\tilde{a}$  in m/s<sup>2</sup>
- Magnetometer measures magnetic field strength  $\tilde{m}$  in uT (micro Tesla) or Gauss, 1 Gauss = 100 uT

**All measurements taken in sensory/body coordinates**

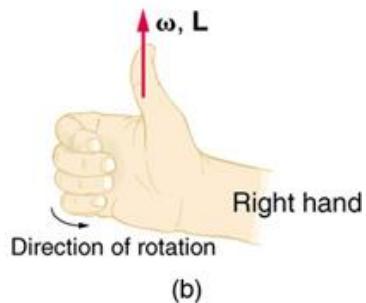
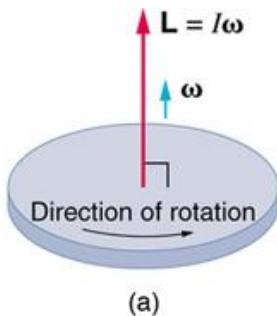
# Gyroscopes

- Measure angular velocity

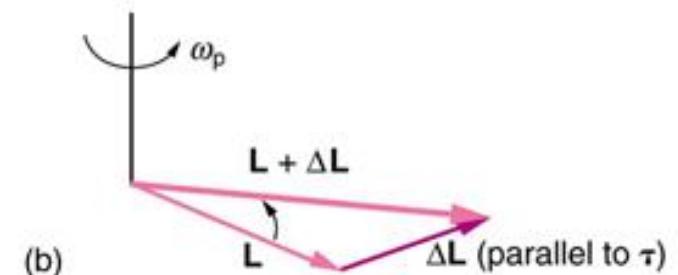
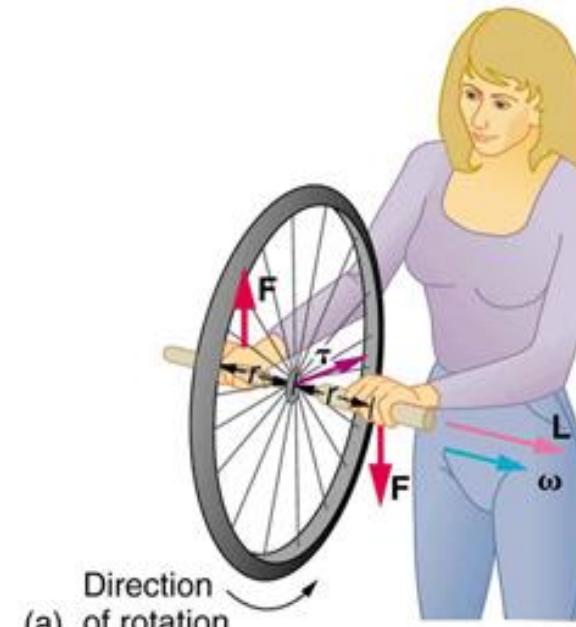
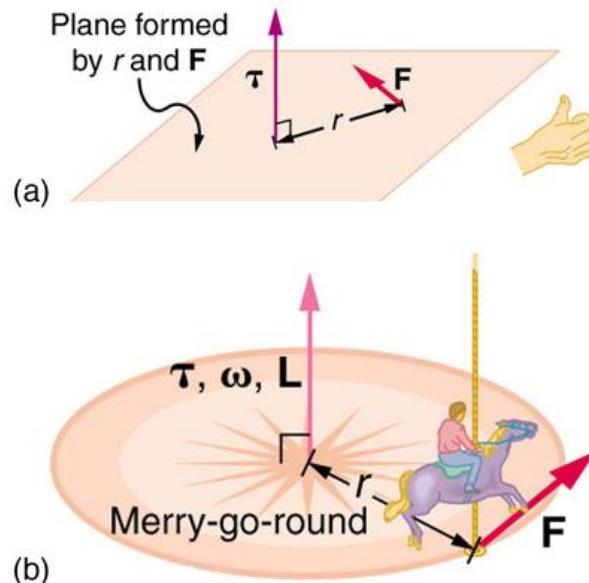


<https://robotacademy.net.au/lesson/how-gyroscopes-work/>

# Gyroscopic Effects: Vector Aspects of Angular Momentum



Angular velocity  
Angular momentum



<https://courses.lumenlearning.com/physics/chapter/10-7-gyroscopic-effects-vector-aspects-of-angular-momentum/>  
<https://www.youtube.com/watch?v=8H98BgRzpOM>

# Gyroscopes

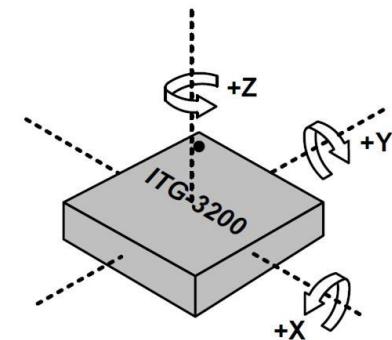
- Gyro model

$$\tilde{\omega} = \omega + b + \eta$$

measured angular velocity      true angular velocity      bias      additive, zero-mean Gaussian noise

$$\eta \sim N(0, \sigma_{gyro}^2)$$

3DOF: 3-axis gyro that measures 3 orthogonal axes



# Gyroscopes

- From gyro measurement to orientation
  - Taylor expansion

$$\theta(t + \Delta t) \approx \theta(t) + \frac{\partial}{\partial t} \theta(t) \Delta t + \varepsilon, \varepsilon \sim O(\Delta t^2)$$

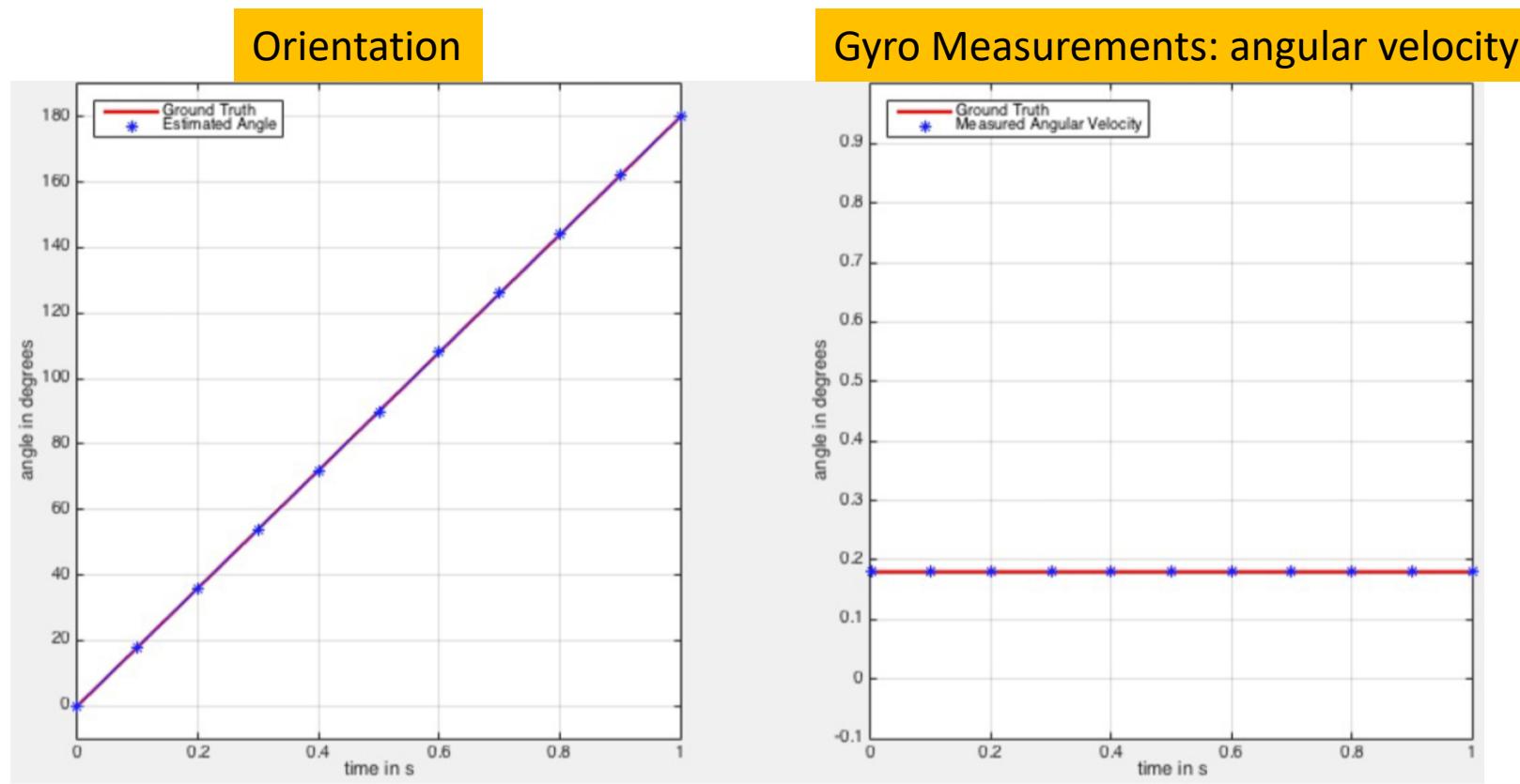
Diagram illustrating the Taylor expansion:

- Angle at current time step:  $\theta(t + \Delta t)$
- Angle at previous time step:  $\theta(t)$
- Gyro measurement (angular velocity):  $\frac{\partial}{\partial t} \theta(t)$
- Time step:  $\Delta t$
- Approximation error:  $\varepsilon$

The diagram shows the Taylor expansion of the angle  $\theta(t + \Delta t)$  around the angle at the previous time step  $\theta(t)$ . The expansion is given by the equation  $\theta(t + \Delta t) \approx \theta(t) + \frac{\partial}{\partial t} \theta(t) \Delta t + \varepsilon$ , where  $\varepsilon \sim O(\Delta t^2)$  represents the approximation error. Red arrows point from the labels to the corresponding terms in the equation.

# Gyro Integration

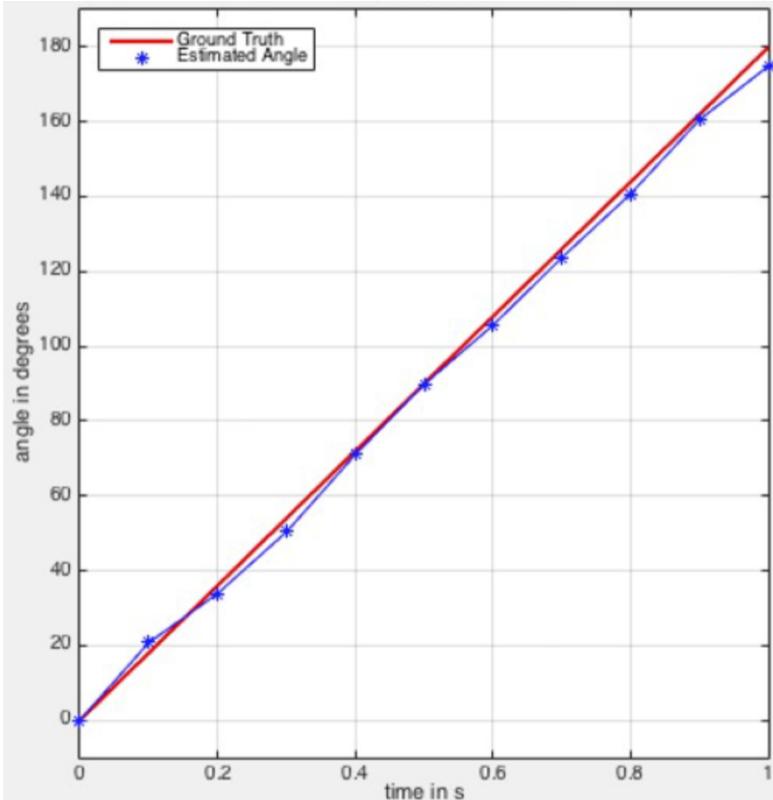
- Linear motion, no noise, no bias



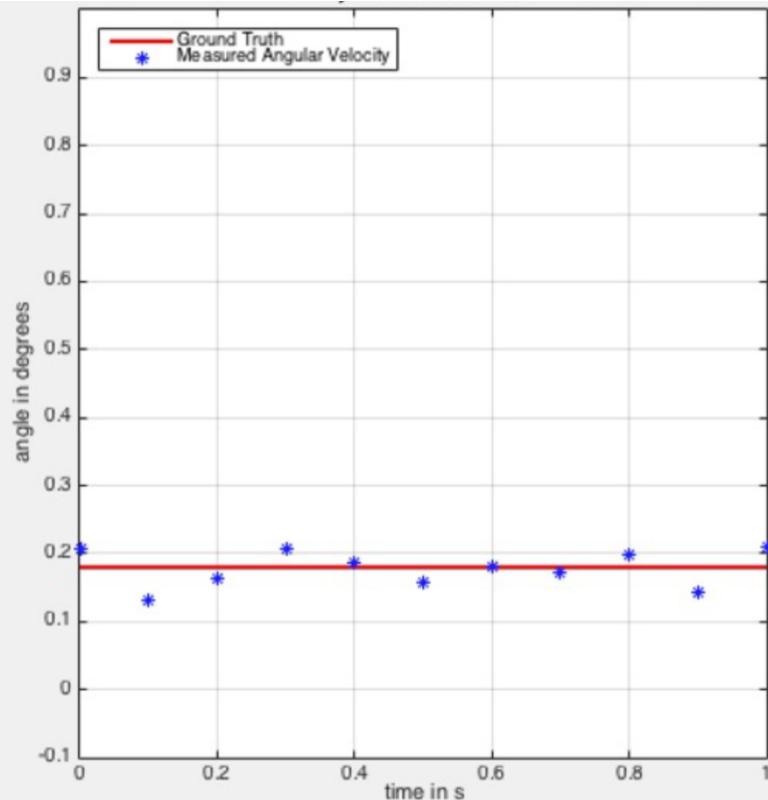
# Gyro Integration

- Linear motion, noise, no bias

Orientation



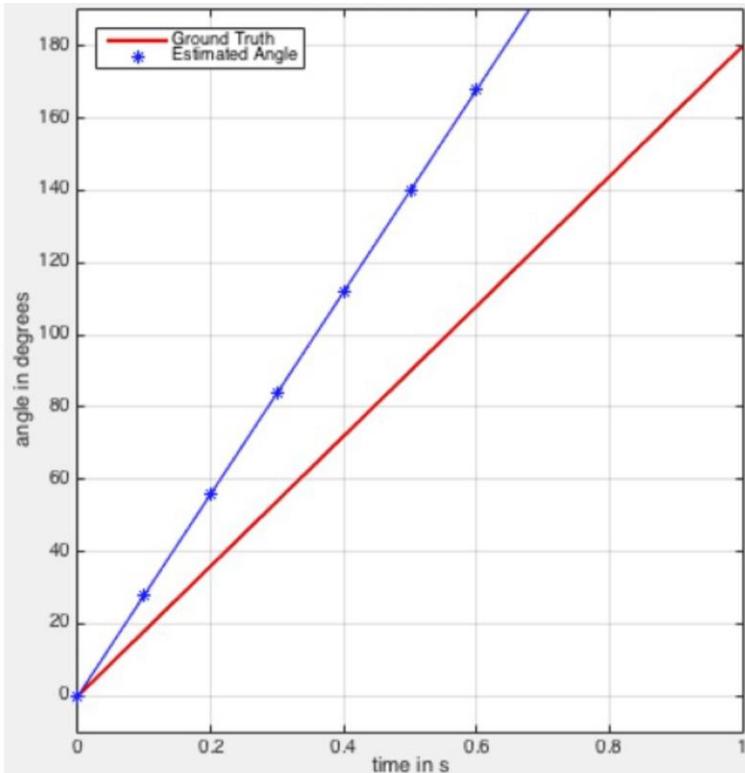
Gyro Measurements: angular velocity



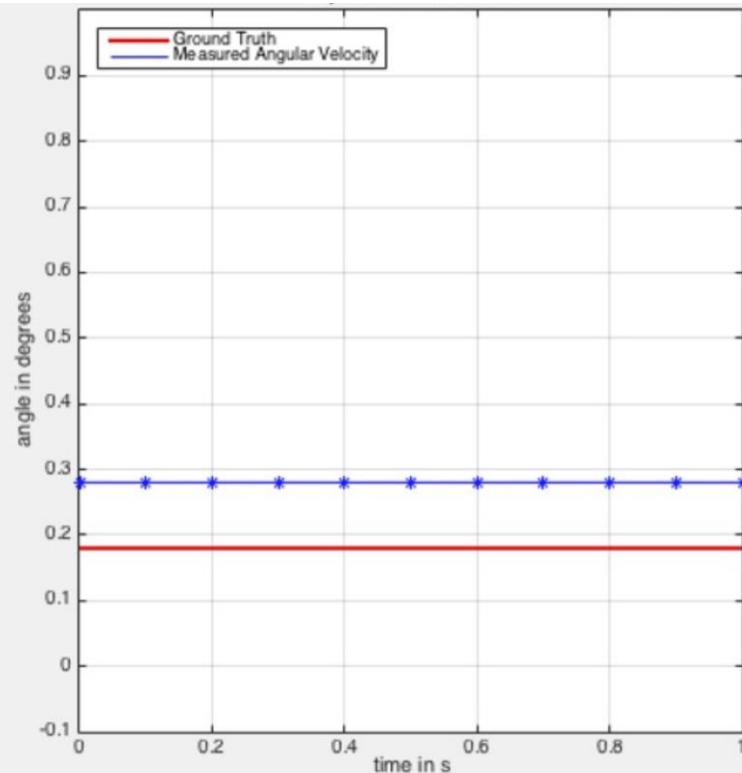
# Gyro Integration

- Linear motion, no noise, bias

Orientation



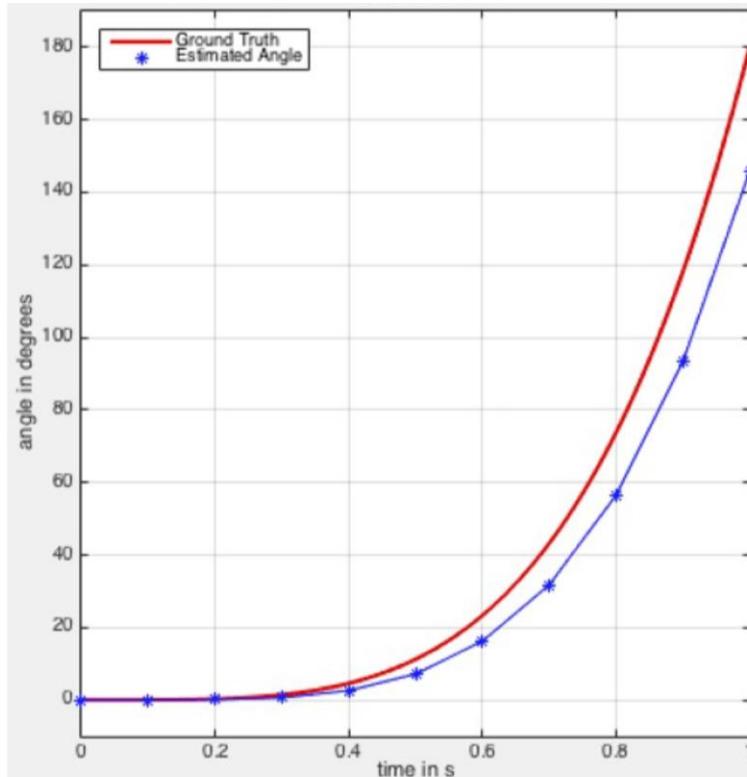
Gyro Measurements: angular velocity



# Gyro Integration

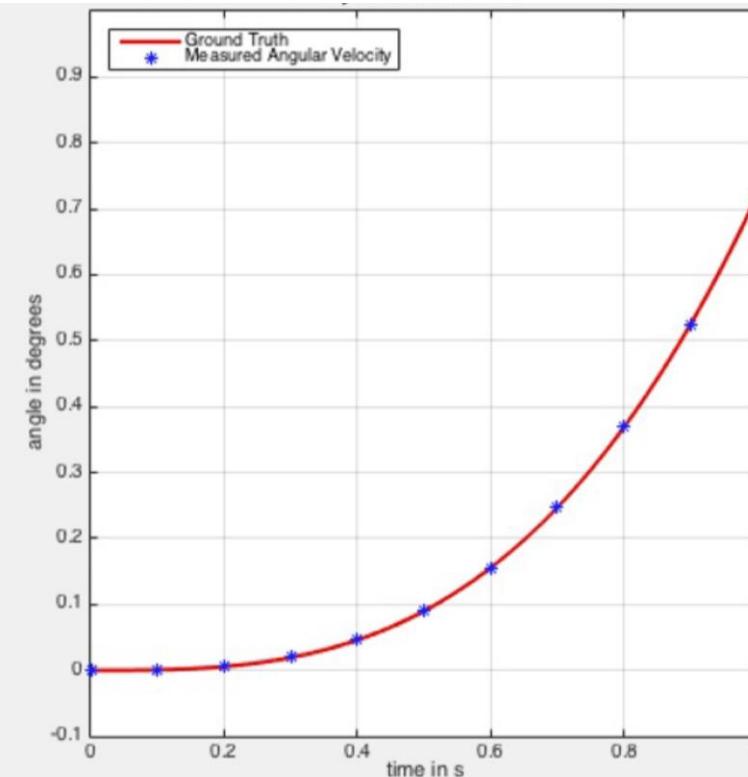
- Nonlinear motion, no noise, no bias

Orientation



Due to  
approximation  
error in Taylor  
expansion for  
nonlinear motion

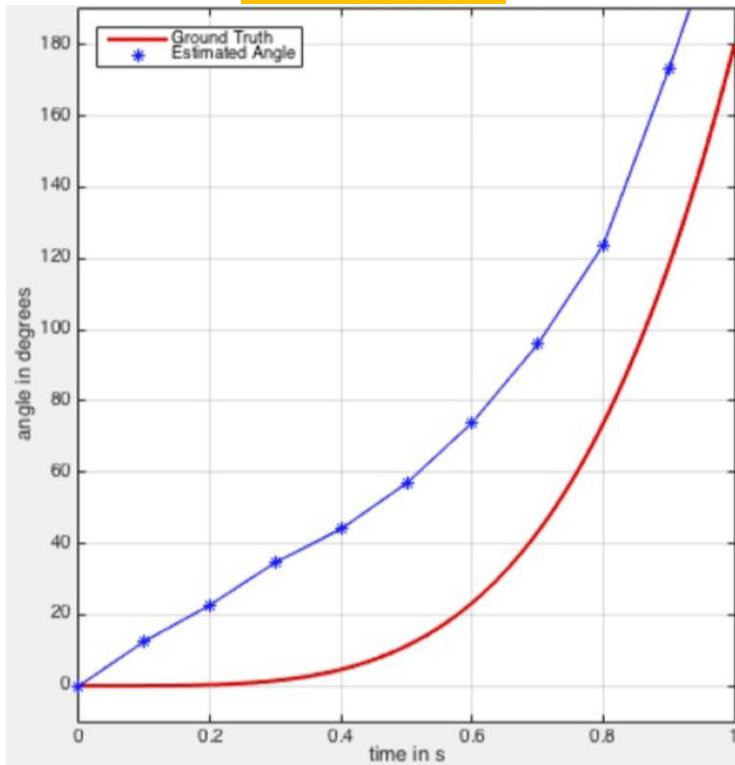
Gyro Measurements: angular velocity



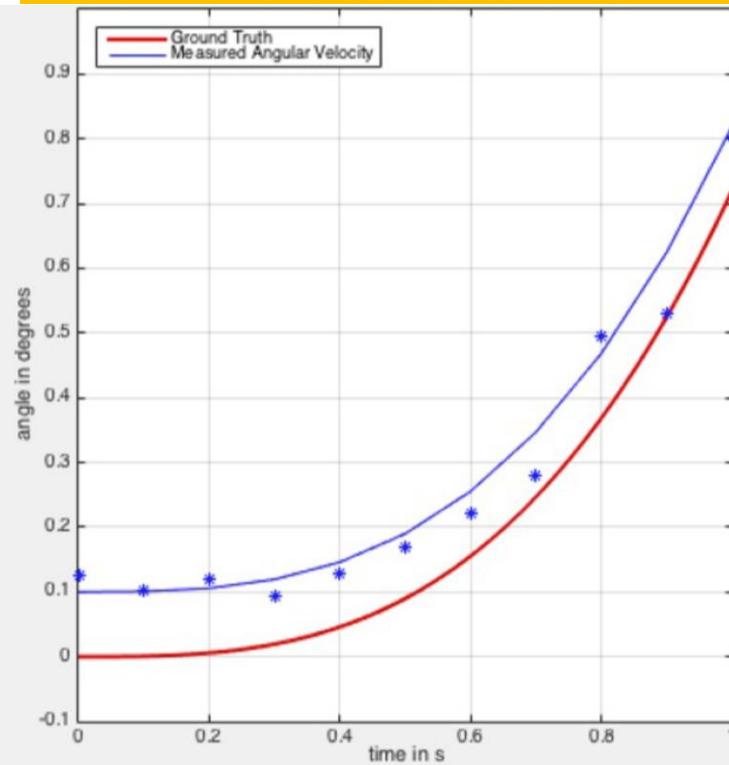
# Gyro Integration

- Nonlinear motion, noise, bias

Orientation



Gyro Measurements: angular velocity



# Gyro Integration

- Works well for linear motion, no noise, no bias (unrealistic)
- Integration drift
  - Errors in measured angular velocity result in errors in orientation
  - Errors accumulate in time
- Gyro integration is accurate in short time, but not reliable in long term due to drift
- Bias/noise variance can be estimated, other sensor measurements can be used to correct drift, e.g., vision, accelerometer

# Accelerometers

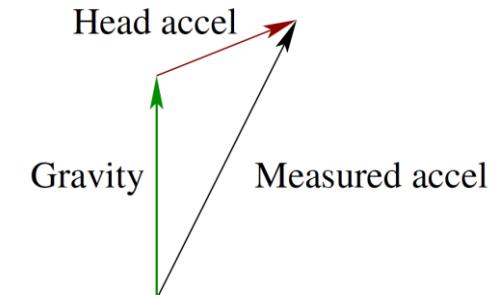
- Measure linear acceleration

$$\tilde{a} = a^{(g)} + a^{(l)} + \eta, \quad \eta \sim N(0, \sigma_{acc}^2)$$

Gravity acceleration (pointing up)      external acceleration      additive, zero-mean Gaussian noise

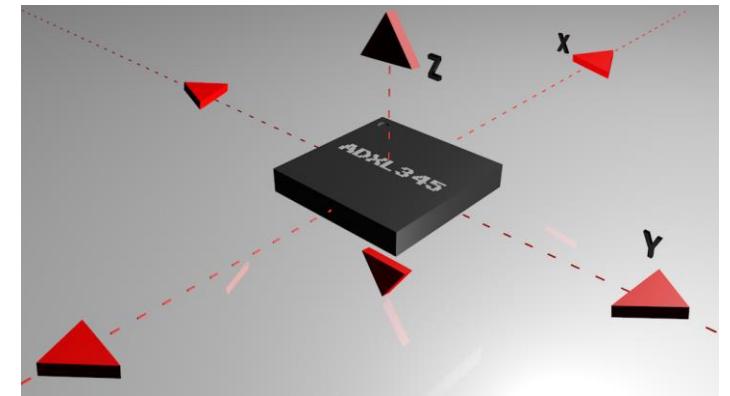
The diagram illustrates the mathematical equation for measured acceleration. It shows three vectors originating from the same point: a vertical red arrow pointing upwards labeled "Gravity acceleration (pointing up)", a horizontal red arrow pointing to the right labeled "external acceleration", and a diagonal red arrow pointing up and to the right labeled "additive, zero-mean Gaussian noise". These three vectors are summed to produce the total measured acceleration vector.

Think about the force of the table pushing the device upwards



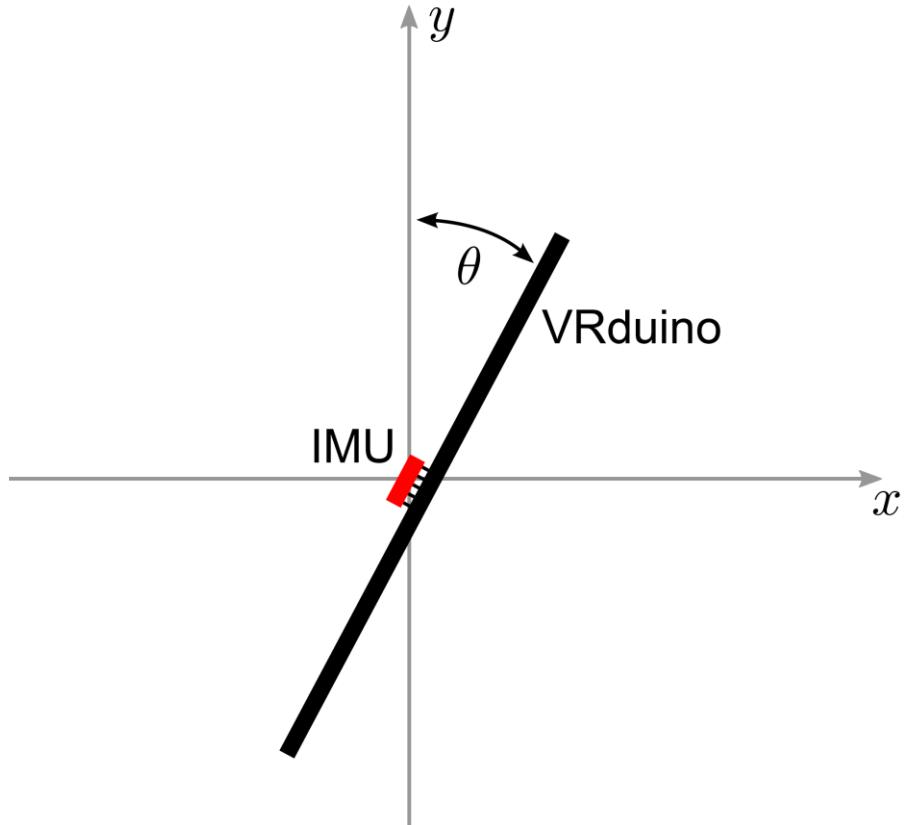
# Accelerometers

- Pros
  - Points up on average with magnitude of 1g
  - Accurate in long term because there is no drift
- Cons
  - Noisy measurements
  - Unreliable in short run due to motion and noise
- Complementary to gyro measurements
- Fusing gyro and accelerometer data: 6DOF sensor fusion



# Orientation Tracking Example

- Track angle  $\theta$  in 2D space
- Sensors
  - 1 gyro
  - 2 accelerometers
- Goal: understand sensor fusion



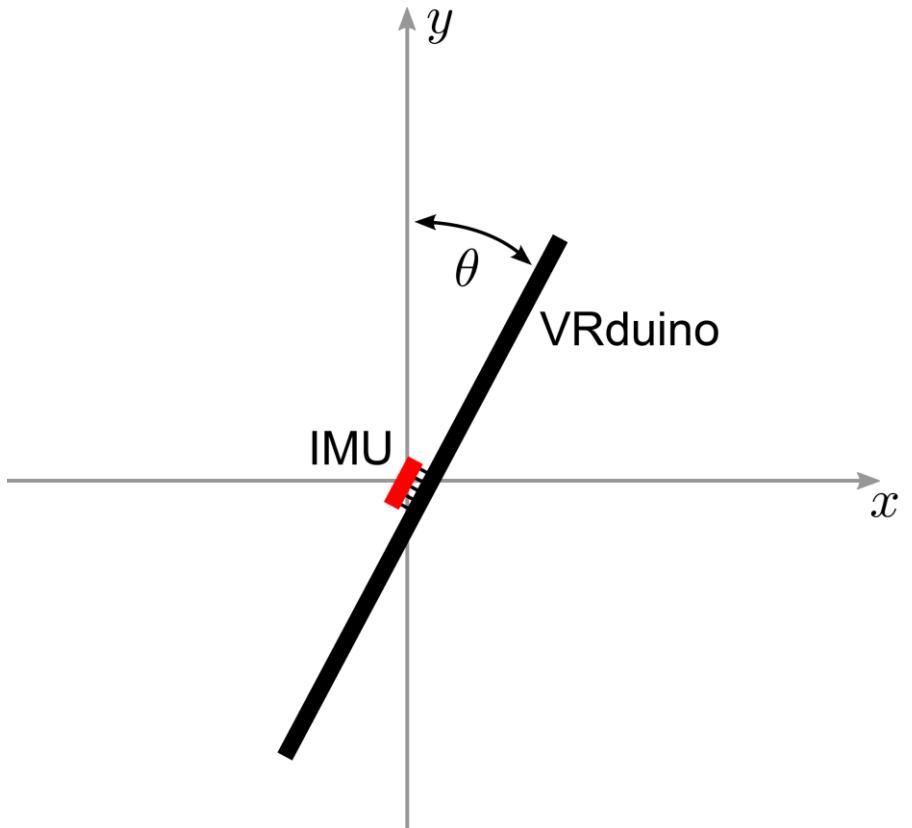
# Orientation Tracking Example

- Gyro integration

$$\theta_{gyro}^{(t)} = \theta_{gyro}^{(t-1)} + \tilde{\omega} \Delta t$$

$$\theta_{gyro}^{(0)} = 0$$

Problem: Drift

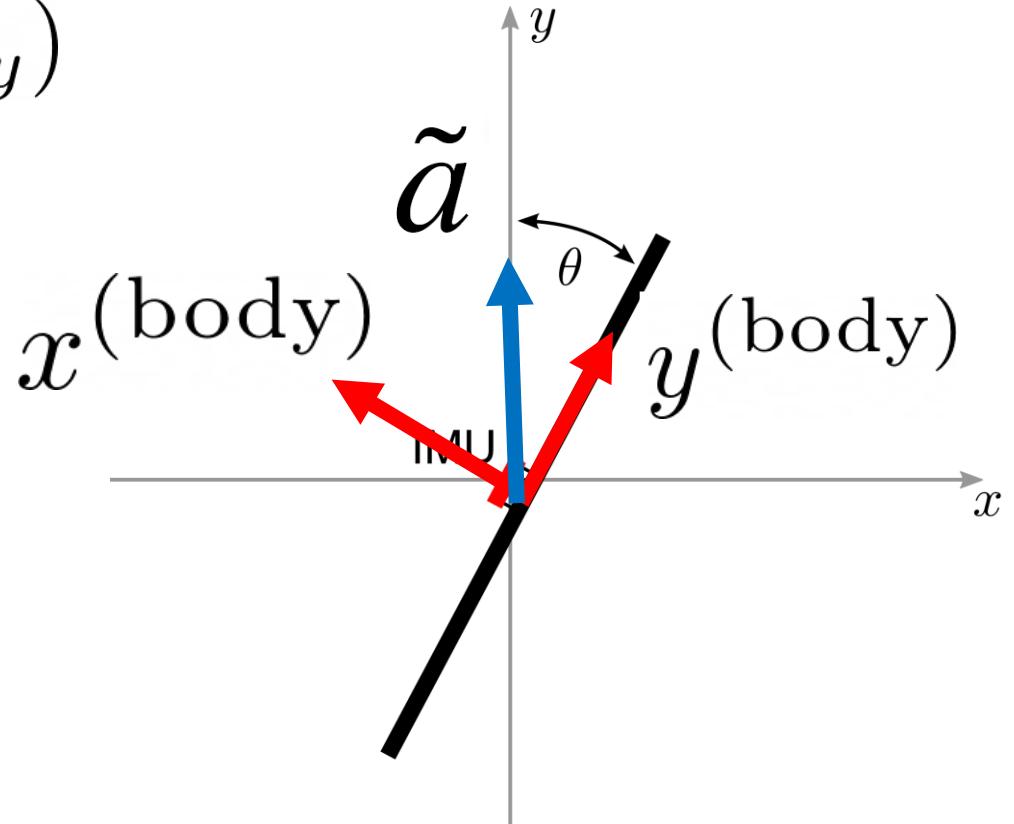


# Orientation Tracking Example

- Angle from accelerometers
  - Measurements in body  $\tilde{a} = (\tilde{a}_x, \tilde{a}_y)$
  - Corresponds to gravity in world

$$\theta_{acc} = \tan^{-1}\left(\frac{\tilde{a}_x}{\tilde{a}_y}\right)$$

Problem: noises

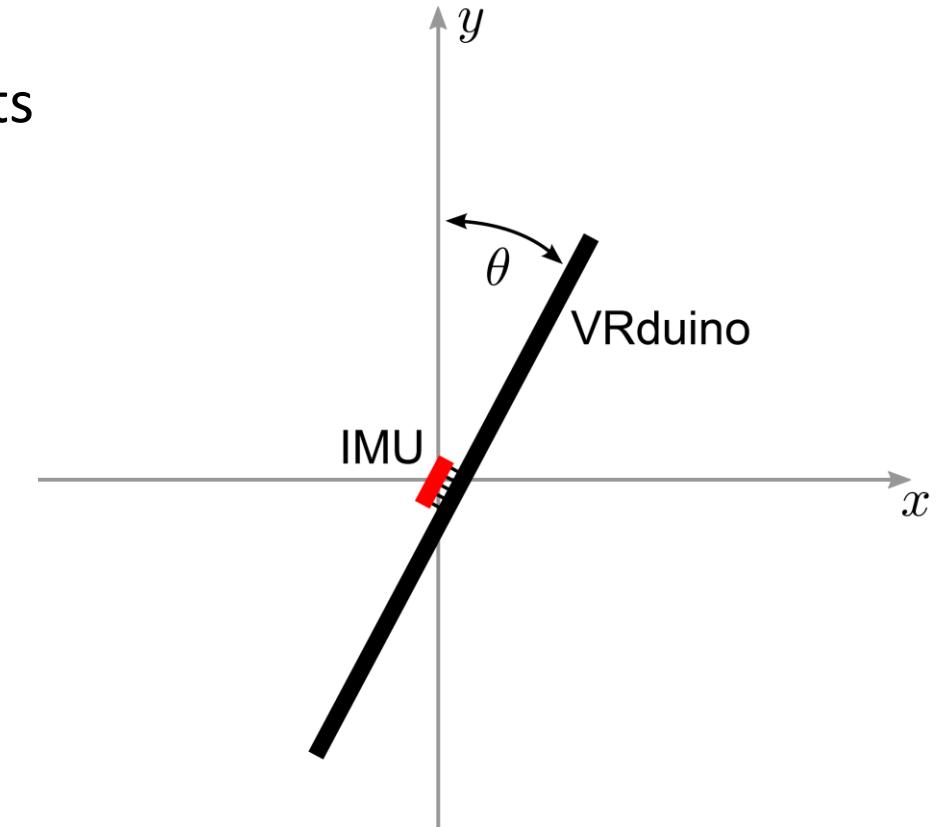


# Orientation Tracking Example

- Sensor fusion
  - Combine gyro and accelerometer measurements

$$\theta^{(t)} = \alpha(\theta^{(t-1)} + \tilde{\omega}\Delta t) + (1 - \alpha)\text{atan2}(\tilde{a}_x, \tilde{a}_y)$$

complementary filter

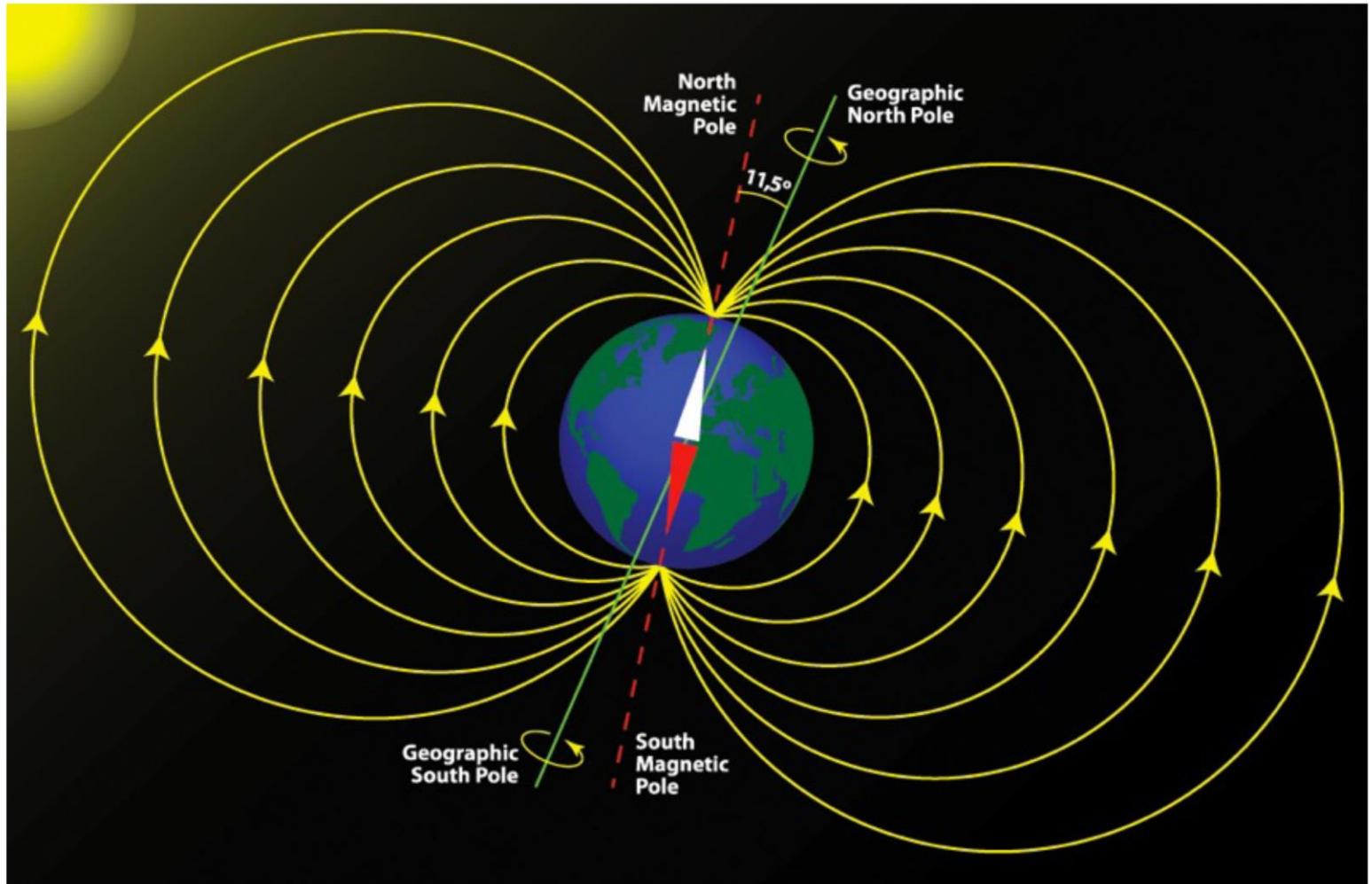


# Magnetometers

wikipedia



Compass



lifescience.com

# Magnetometers

- Measure earth's magnetic field in Gauss or uT (micro Tesla)
- 3 orthogonal axes
  - Vector pointing along the magnetic field
- Actual direction depends on latitude and longitude
- Distortions due to metal or electronics objects in room or in HMD

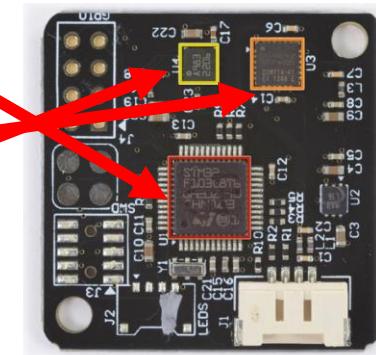
# Magnetometers

- Pros
  - Complementary to accelerometers
- Cons
  - Affected by metal, distortions of magnetic field
  - Need to know location even when calibrated (e.g., GPS)
- Together with gyros and accelerometers, 9 DOF sensor fusion

# Oculus Rift



- STMicroelectronics 32F103C8 ARM Cortex-M3 microcontroller



- Invensense MPU-6000 (gyroscope + accelerometer)

- Honeywell HMC5983 magnetometer

- 3-axis gyro, 3-axis accelerometer, 3-axis magnetometer all on one chip



S. LaValle, A. Yershova, M. Katsev, M. Antonov. Head Tracking for the Oculus Rift. ICRA'14

# Head Orientation Tracking in Oculus Rift

- Gyro integration
  - Problem: drift
- Accelerometers for tilt correction
  - All drift errors except for rotation about the vertical axis
- Magnetometers for yaw correction
  - Rotation about the vertical axis

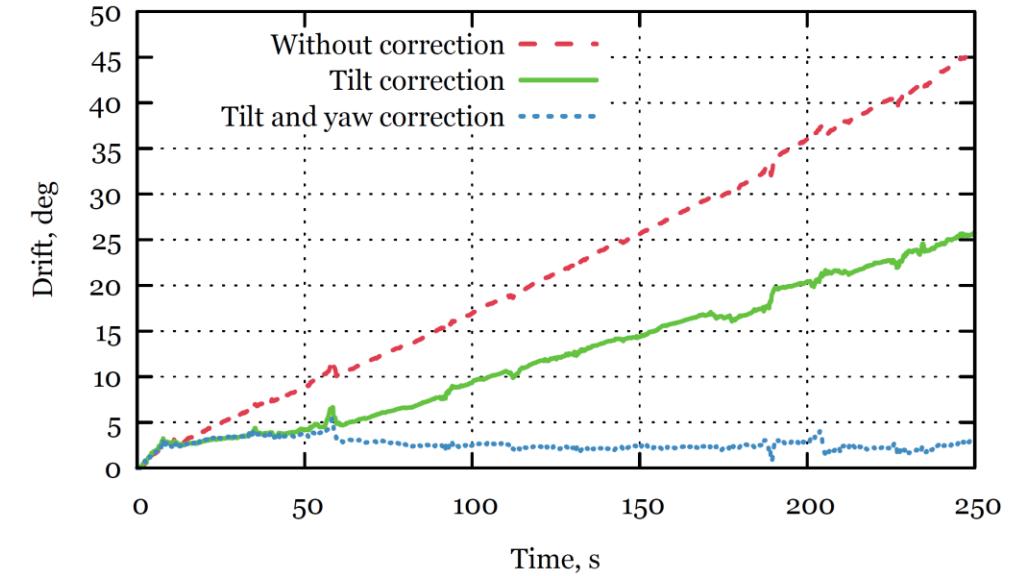
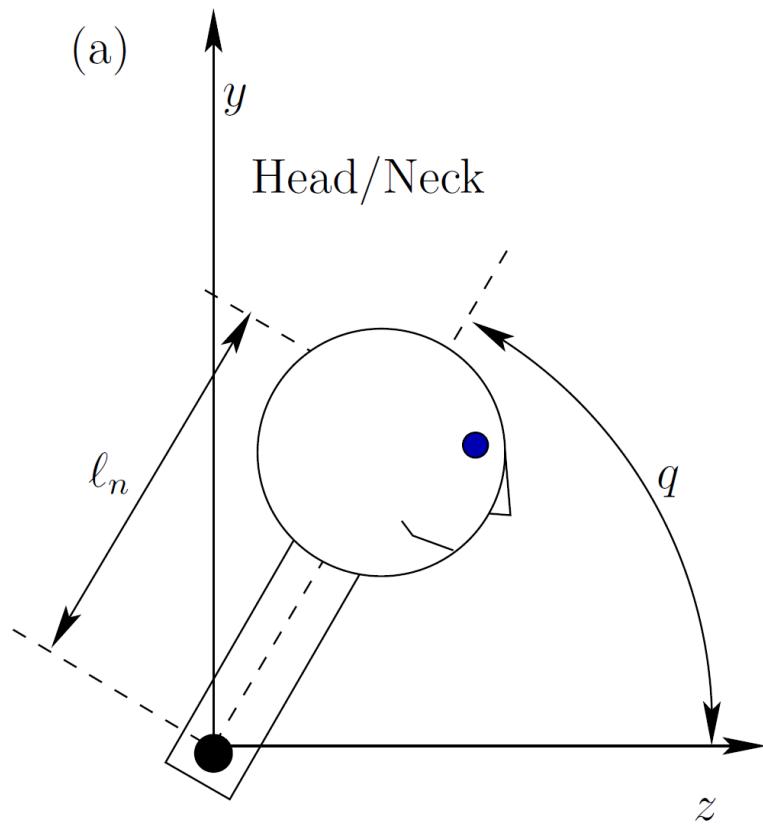


Fig. 7. Effect of correction methods on the overall drift.

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# Head Position Tracking in Oculus Rift



$$p = f(q)$$

Head position      Quaternion of head orientation

$$p = f(q) = q * (0, \ell_n, 0) * q^{-1}$$

Use quaternion to rotate a vector

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# Further Reading

- Sections 9.1, 9.2 in Virtual Reality, Steven LaValle
- Stanford EE 267 course note on 3DOF orientation tracking and IMUs
  - [https://stanford.edu/class/ee267/notes/ee267\\_notes\\_imu.pdf](https://stanford.edu/class/ee267/notes/ee267_notes_imu.pdf)
- Head Tracking for the Oculus Rift
  - <http://msl.cs.illinois.edu/~lavalle/papers/LavYerKatAnt14.pdf>