CS 6334.001 Virtual Reality Homework 2

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Problem 1

(2 points) Convex Lens.

In Figure 1, an object with height h_1 and distance s_1 from a thin convex lens is imaged to the other side of the lens. The focal length of the lens is f. The image of the object is with height h_2 and distance s_2 from the lens. Show that

$$\frac{1}{s_1} + \frac{1}{s_s} = \frac{1}{f}. \tag{1.1}$$

(Hint) Apply triangle similarity theorems. Consider the relationship between height and distance first.

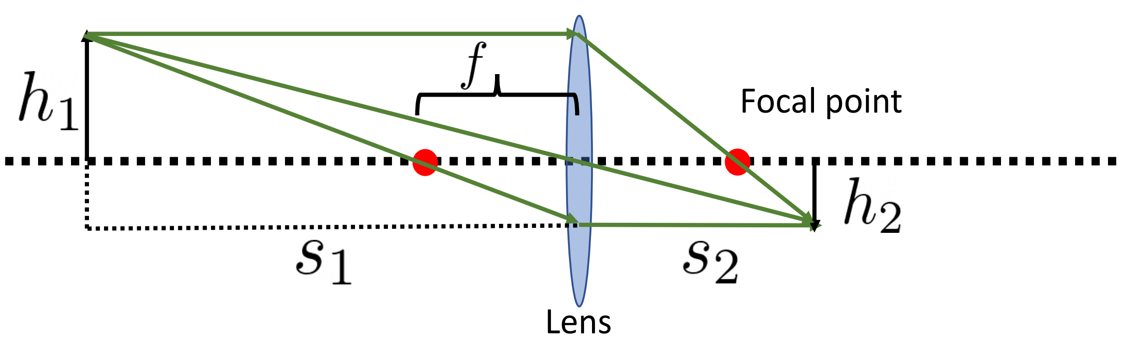

Figure 1: Imaging process of a thin convex lens

Problem 2

(2 points) Barycentric Coordinates.

Let $\mathbf{p}_1 = (x_1, y_1, z_1)^T$, $\mathbf{p}_2 = (x_2, y_2, z_2)^T$ and $\mathbf{p}_3 = (x_3, y_3, z_3)^T$ be three vertices of a triangle as shown in Figure 2. Let $\mathbf{p} = (x, y, z)^T$ be a 3D point inside the triangle. Then $\mathbf{p}$ can be expressed using its barycentric coordinates

$$\mathbf{p} = \alpha_1\mathbf{p}_1 + \alpha_2\mathbf{p}_2 + \alpha_3\mathbf{p}_3, \tag{2.1}$$

where $0 \le \alpha_1, \alpha_2, \alpha_3 \le 1$ and $\alpha_1 + \alpha_2 + \alpha_3 = 1$. Show that

$$\begin{aligned} \alpha_1 &= \frac{(y_2 - y_3)(x - x_3) + (x_3 - x_2)(y - y_3)}{(y_2 - y_3)(x_1 - x_3) + (x_3 - x_2)(y_1 - y_3)}, \\ \alpha_2 &= \frac{(y_3 - y_1)(x - x_3) + (x_1 - x_3)(y - y_3)}{(y_2 - y_3)(x_1 - x_3) + (x_3 - x_2)(y_1 - y_3)}, \\ \alpha_3 &= 1 - \alpha_1 - \alpha_2. \end{aligned} \tag{2.2}$$

(Hint) Build a linear system of α_1, α_2 and then apply Cramer's rule to solve it. Do not follow slide 6 in lecture 7. I think these equations from the VR textbook may not be correct.

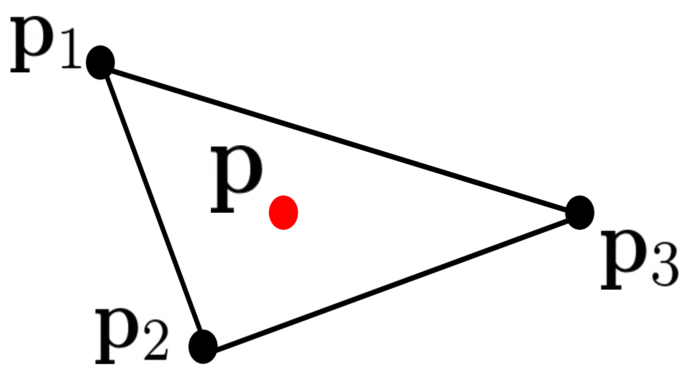

Figure 2: Illustration of barycentric coordinates

Problem 3

(2 points) Fundamental Matrix.

Let's consider a stereo camera rig with the world origin at the first camera. The second camera is translated by t_x along the x-axis from the first camera as shown in Figure 3. The projection matrices of the two cameras are

$$P = K[I|\mathbf{0}] \tag{3.1}$$

$$P' = K[I|\mathbf{t}], \tag{3.2}$$

where I is a 3×3 identity matrix and $\mathbf{t} = (t_x, 0, 0)^T$ is the translation of the right camera, and K is the camera intrinsic matrix:

$$K = \begin{bmatrix} f_x & 0 & p_x \\ 0 & f_y & p_y \\ 0 & 0 & 1 \end{bmatrix}. \tag{3.3}$$

Show that the fundamental matrix of this stereo rig is

$$F = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -f_x t_x \\ 0 & f_x t_x & 0 \end{bmatrix}. \tag{3.4}$$

(Hint) The pseudoinverse of P is

$$P^+ = \begin{bmatrix} K^{-1} \\ \mathbf{0}^T \end{bmatrix}. \tag{3.5}$$

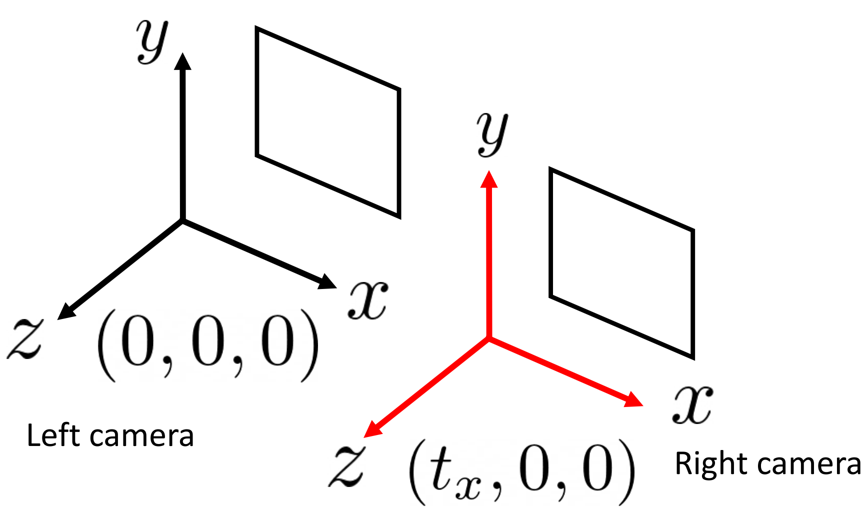

Figure 3: A stereo camera rig

Problem 4

(4 points)

Download the homework2_programming.zip file from eLearning, Assignments, Homework 2. Implement backprojection of pixels in the get_observation() function in table_scene_rendering.py.

After your implementation, run the table_scene_rendering.py in Python. Figure 4 shows an example of running the script. Submit your script to eLearning, and TA will run your script to verify it.

Here are some useful resources:

- Python basics `https://pythonbasics.org/`
- PyBullet `https://pybullet.org/wordpress/`
- Numpy `https://numpy.org/doc/stable/user/basics.html`

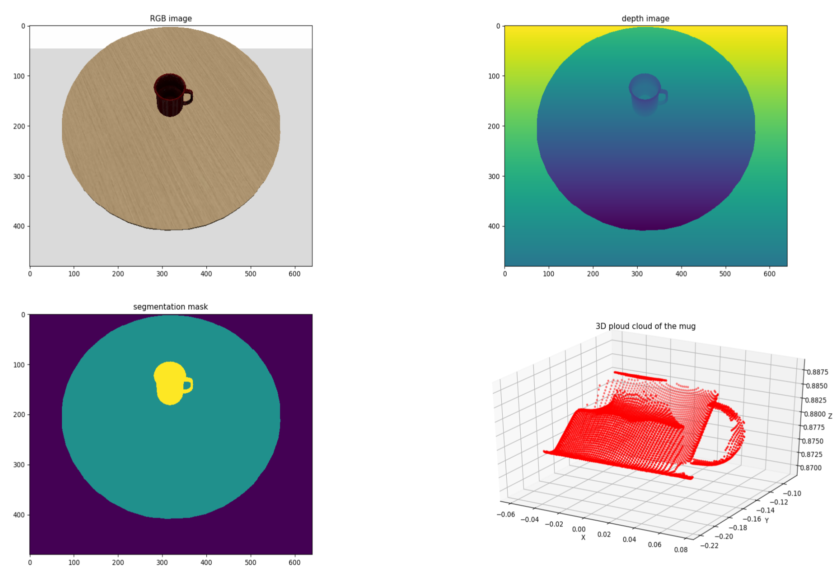

Figure 4: Example of running of the table_scene_rendering.py script