CS 6334.001 Virtual Reality Homework 1

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Problem 1

(2 points)

Derivation of Rodrigues' rotation formula.

(1.1) Let $\mathbf{v} \in \mathbb{R}^3$ be a vector in 3D, and $\mathbf{k} \in \mathbb{R}^3$, $\|\mathbf{k}\| = 1$ be a unit vector describing an axis of rotation. If we rotate $\mathbf{v}$ around $\mathbf{k}$ by an anlge θ, show that the rotated vector $\mathbf{v}_{\text{rot}}$ is

$$\mathbf{v}_{\text{rot}} = \mathbf{v} \cos\theta + (\mathbf{k} \times \mathbf{v}) \sin\theta + \mathbf{k}(\mathbf{k} \cdot \mathbf{v})(1 - \cos\theta). \tag{1.1}$$

(1.2) Show that if $\mathbf{v}_{\text{rot}} = \mathbf{R}\mathbf{v}$, then

$$\mathbf{R} = \mathbf{I} + (\sin\theta)\mathbf{K} + (1 - \cos\theta)\mathbf{K}^2, \tag{1.2}$$

where

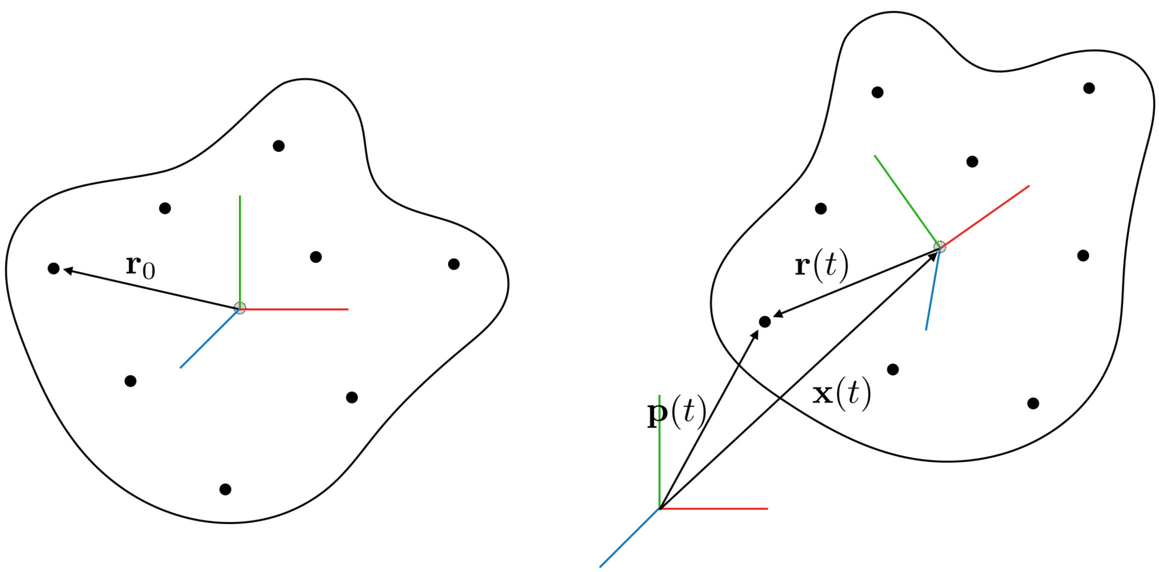
$$\mathbf{K} = \begin{bmatrix} 0 & -k_z & k_y \\ k_z & 0 & -k_z \\ -k_y & k_x & 0 \end{bmatrix} \tag{1.3}$$

is the cross-product matrix of $\mathbf{k} = (k_x, k_y, k_z)^T$.

(Hint) Read the derivation in Wikipedia, understand it and write down your answer based on your understanding.

Problem 2

(2 points)



(a) *Object space.* **(b)** *World space.*

Figure 1: (a) A rigid body in its object space. (b) The rotated rigid body at time t in the world space.

Figure 1 shows a rigid body in its object space and in the world space, respectively.

(2.1) For a rigid body with N particles, let the particle positions in the world space be $\mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_N$, and the masses of the particles be $m_1, m_2, \ldots, m_N$. Compute the center of mass $\mathbf{x}$ of the rigid body in the world space.

(2.2) Assume the origin of the object space is the center of mass of the rigid body. Let $\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_N$ be the vectors from the center of mass to each particle in the world space as shown in Figure 1(b). Show that

$$\sum_{i=1}^{N} m_i \mathbf{r}_i = 0. \tag{2.1}$$

Problem 3

(3 points)

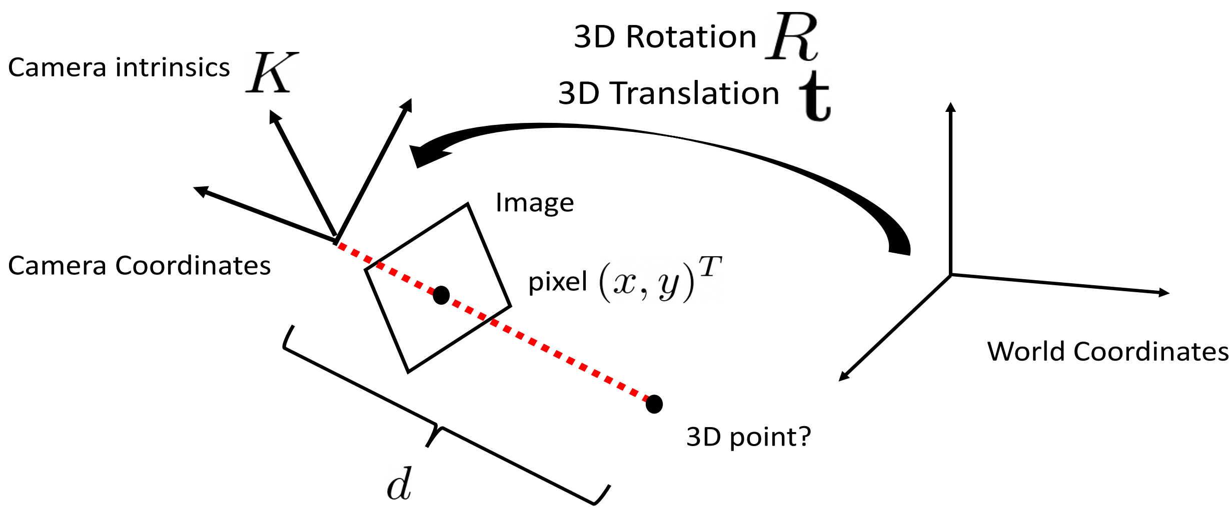


Figure 2: Backprojection of a pixel

Suppose a pinhole camera has a camera intrinsic matrix K. Let the camera extrinsics be a 3D rotation R and a 3D translation $\mathbf{t}$. Given a pixel $(x, y)^T$ in an image, assume the depth of the pixel is d, where depth is the distance between the 3D point of pixel and the camera center. Compute the coordinates of the 3D point in the world coordinate system.

Problem 4

(3 points)

Download the homework1_programming.zip file from eLearning, Assignments, Homework 1. Implement the randomly_place_objects() function in table_scene.py.

Run the table_scene.py in Python. Make sure the mug drops onto the table in the beginning, then random forces can be applied to the mug. Figure 3 shows an example of running the script. Submit your script to eLearning, and TA will run your script to verify it.

Here are some useful resources:

- Python basics `https://pythonbasics.org/`
- PyBullet `https://pybullet.org/wordpress/`

Figure 3: Example of running of the table_scene.py script