Robot Control: Motion Control with Velocities

CS 6301 Special Topics: Introduction to Robot Manipulation and Navigation

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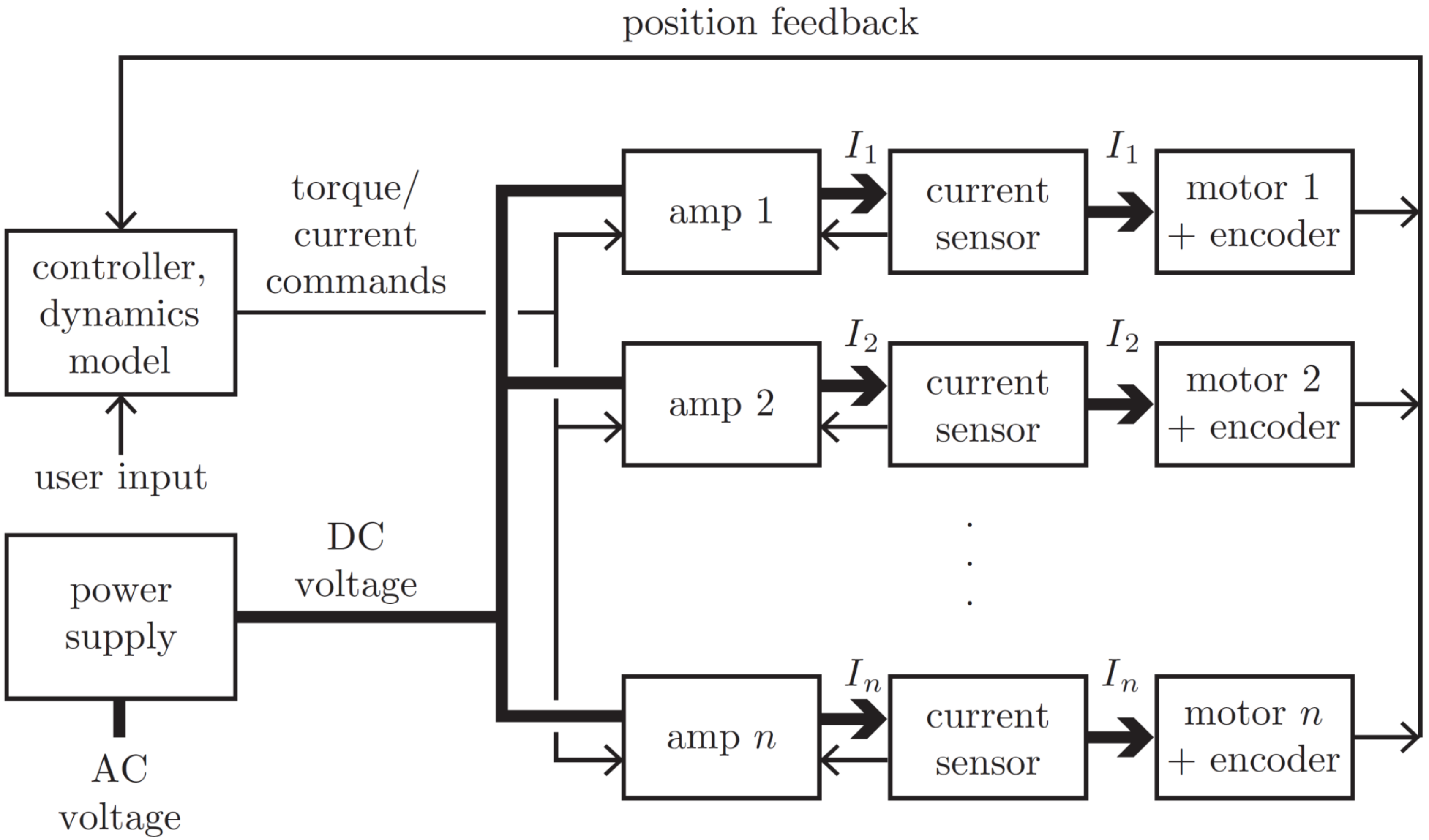
Robot Control

- Convert task specifications to forces and torques at the actuators

- Types
  - Motion control
  - Force control
  - Hybrid motion-force control
  - Impedance control

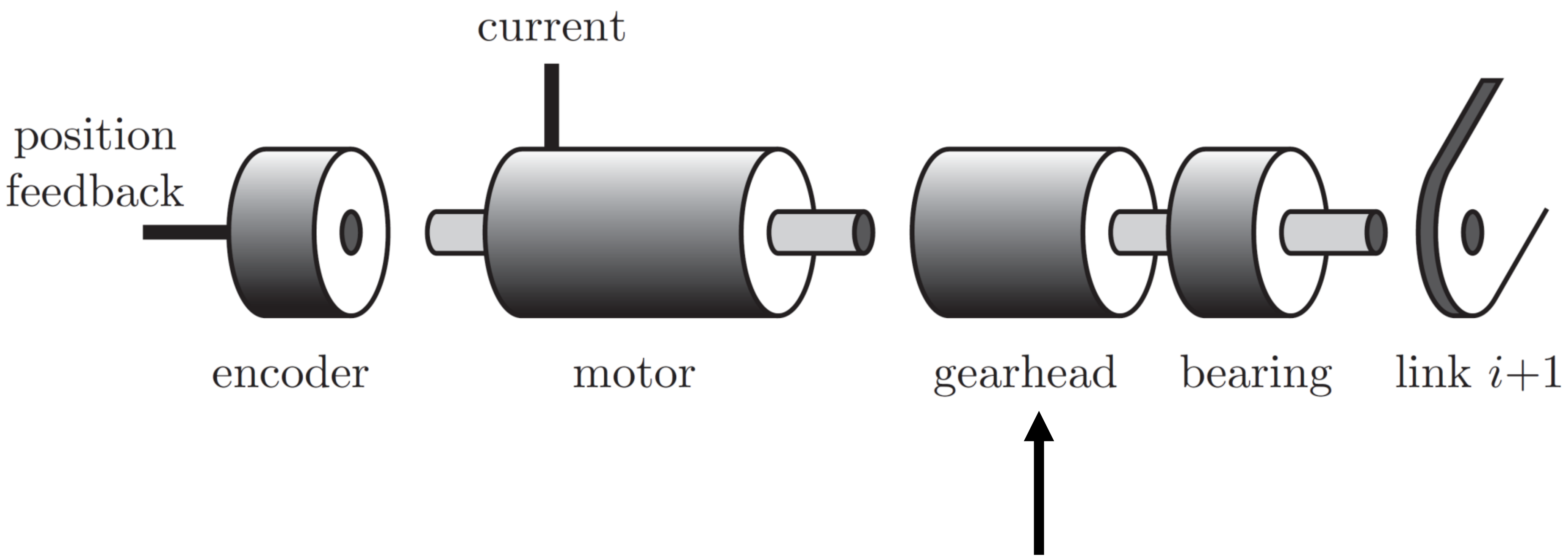
- Feedback control
  - Use sensors for position, velocity and force
  - Compare with the desired behavior to compute the control signals

Actuation with DC Electric Motors



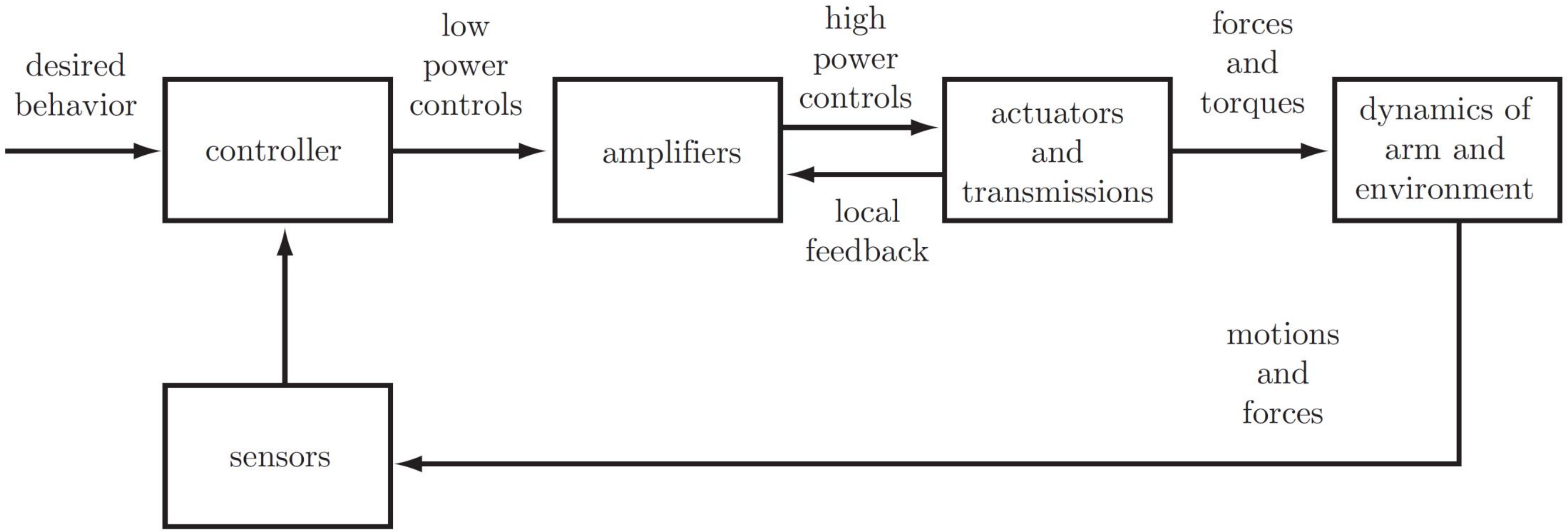
amplifier adjust the voltage sense the joint position

Actuation with DC Electric Motors



Increase torque
Reduce speed

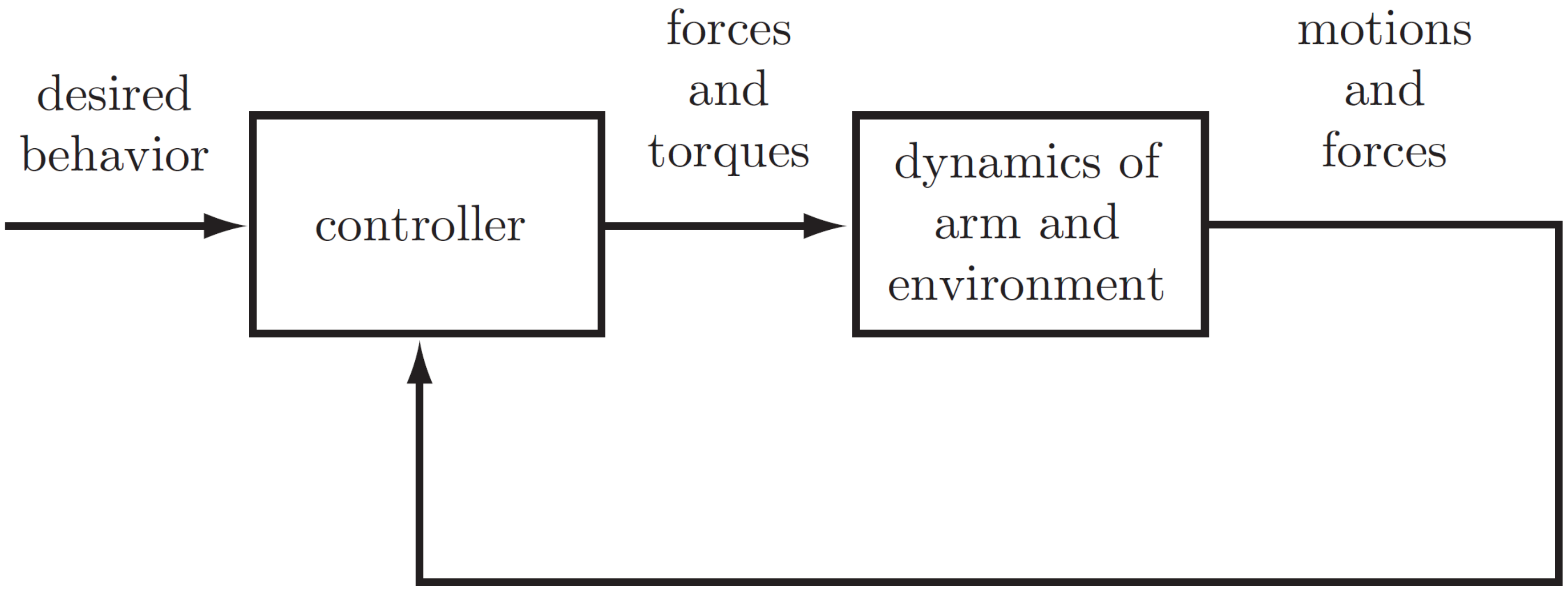
Control System Overview



- Potentiometers, encoders, or resolvers for joint position and angle sensing
- Tachometers for joint velocity sensing
- Joint force-torque sensors
- Multi-axis force-torque sensors at the “wrist" between the end of the arm and the end-effector

Control System Overview

- A simplified system



Controlled Dynamics of a Single Joint

- Desired joint position $\theta_d(t)$
- The current joint position $\theta(t)$
- Joint error $\theta_e(t) = \theta_d(t) - \theta(t)$

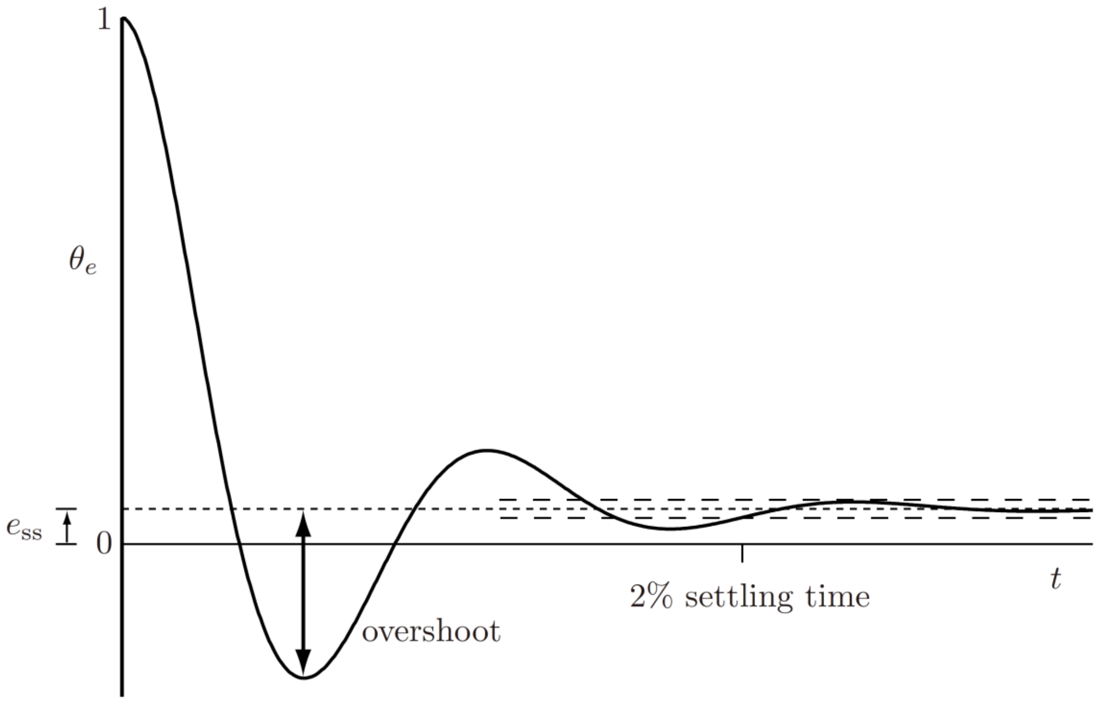
- Error dynamics: the differential equation governing the evolution of the joint error

- Feedback controller: create an error dynamics to make $\theta_e(t)$ become zero or a small value when t increases

Error Response

- How well a controller works?
  - Specify a nonzero initial error $\theta_e(0)$ and see how the controller reduces the error

- Error response $\theta_e(t), t > 0$
  - Initial conditions $\theta_e(0) = 1$
    $\dot{\theta}_e(0) = \ddot{\theta}_e(0) = \cdots = 0$
  - Steady-state error $\theta_e(t)$ as $t \to \infty$

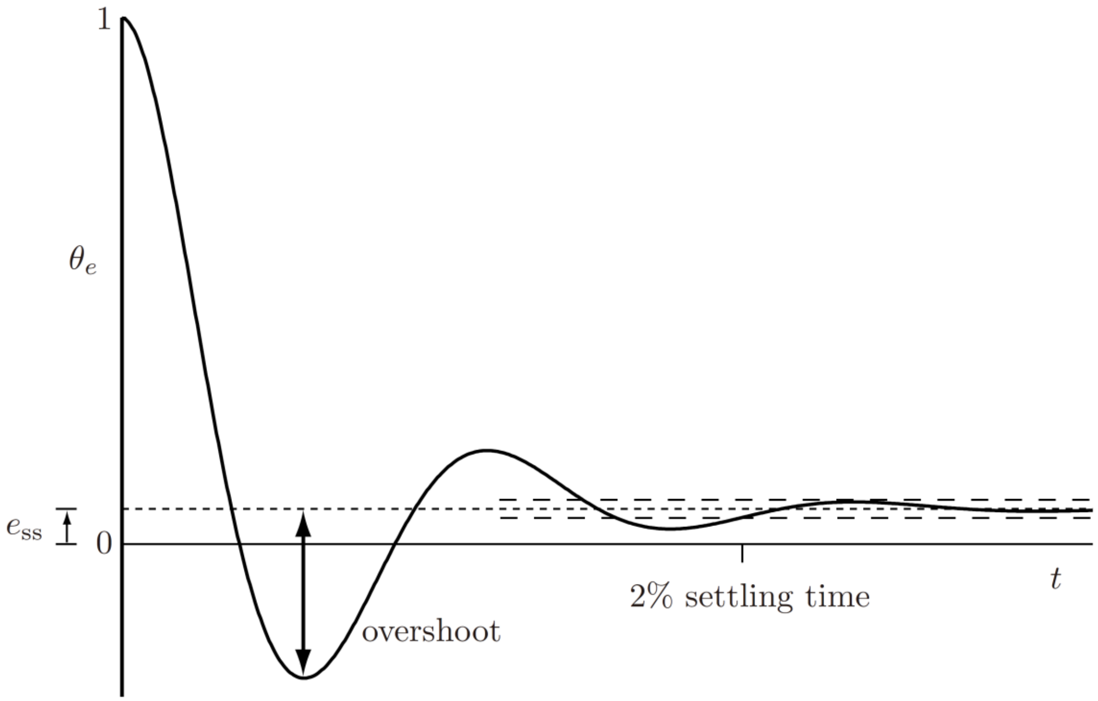


Error Response

- (2%) Settling time: first time T such that $|\theta_e(t) - e_{ss}| \leq 0.02(\theta_e(0) - e_{ss})$ for all $t \geq T$

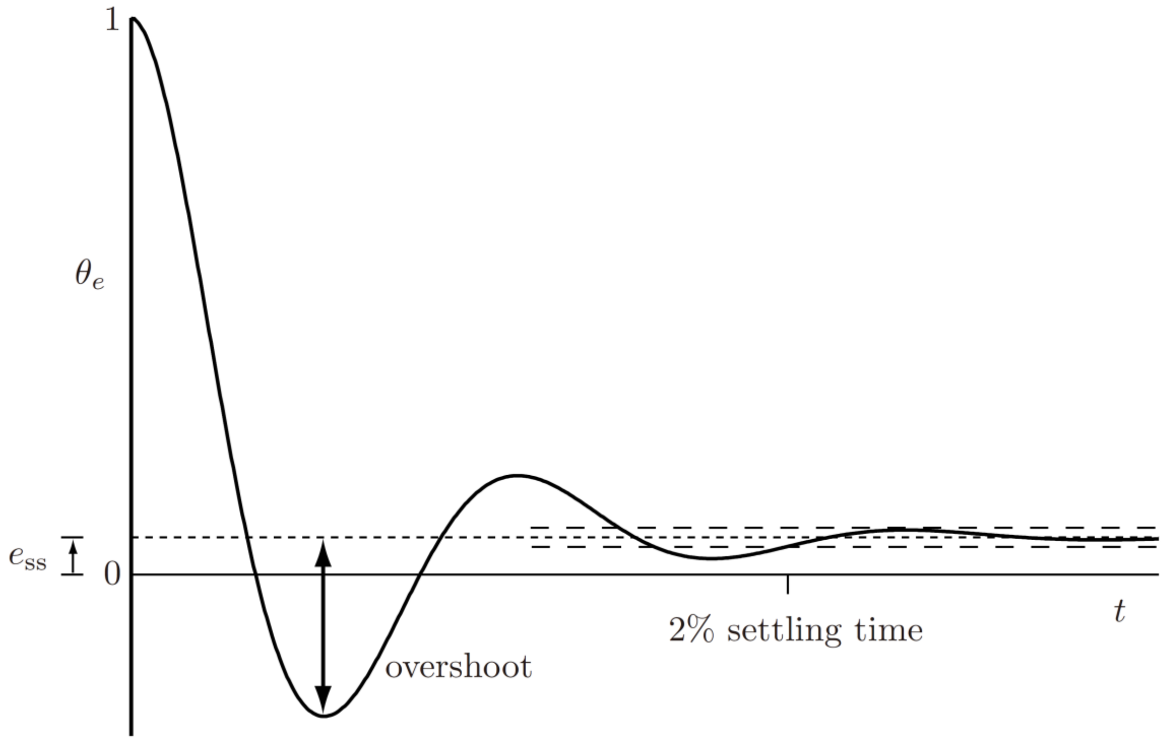
- Overshoot

$$\text{overshoot} = \left| \frac{\theta_{e,\min} - e_{ss}}{\theta_e(0) - e_{ss}} \right| \times 100\%$$



Error Response

- A good error response
  - Little or no steady-state error
  - Little or no overshoot
  - A short 2% settling time



Motion Control with Velocity Inputs

- Typically, we assume direct control of the forces or torques at robot joints
- In some cases, we can assume that there is direct control of the joint velocities
  - The velocity of a joint is determined directly by the frequency of the pulse train sent to the stepper motor https://en.wikipedia.org/wiki/Stepper_motor
- Motion control with velocity inputs
  - Given a desired trajectory of a robot in joint space or in task space

$$\theta_d(t) \qquad X_d(t)$$

Motion Control of a Single Joint

- Feedforward control or open-loop control
  - Given a desired joint trajectory $\theta_d(t)$
  - Choose the velocity command $\dot{\theta}(t) = \dot{\theta}_d(t)$
  - Cons: accumulating position errors

- Feedback control
  - Measure the joint position continuously for feedback

Motion Control of a Single Joint

- Proportional controller or P controller

$$\dot{\theta}(t) = K_p(\theta_d(t) - \theta(t)) = K_p\theta_e(t)$$

Control gain

$$K_p > 0$$

- When $\theta_d(t)$ is a constant $\dot{\theta}_d(t) = 0$

Setpoint control

  - Error dynamics

$$\dot{\theta}_e(t) = \cancelto{0}{\dot{\theta}_d(t)} - \dot{\theta}(t)$$

$$\dot{\theta}_e(t) = -K_p\theta_e(t) \quad \rightarrow \quad \dot{\theta}_e(t) + K_p\theta_e(t) = 0$$

First-Order Error Dynamics

$$\dot{\theta}_e(t) + \frac{1}{\tau}\theta_e(t) = 0$$

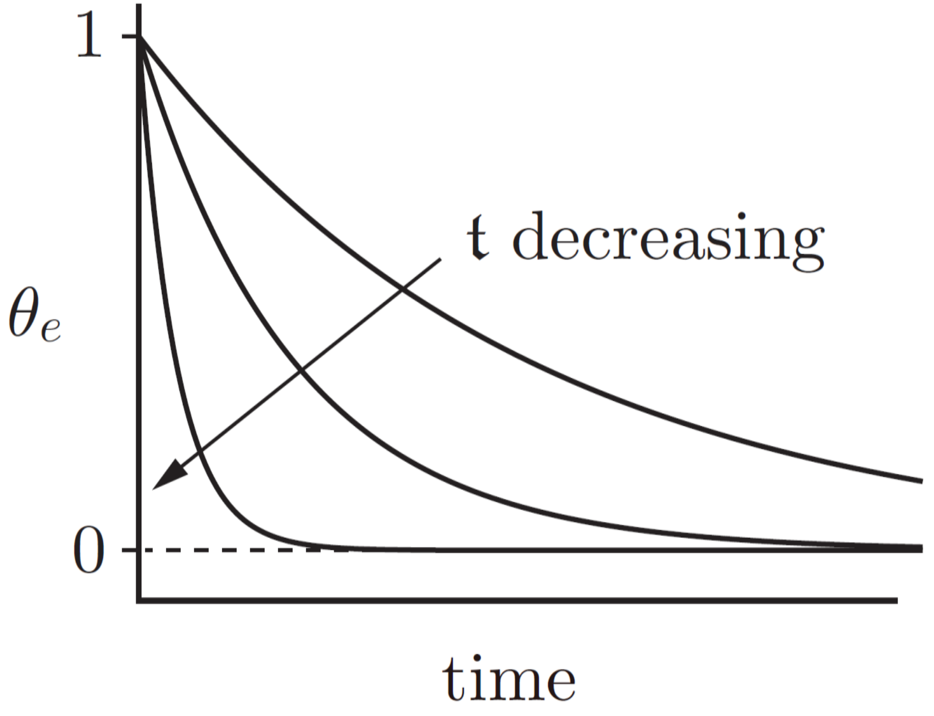
time constant τ

Solution $\theta_e(t) = e^{-t/\tau}\theta_e(0)$

Setpoint control

$$\dot{\theta}_e(t) + K_p\theta_e(t) = 0 \qquad \tau = 1/K_p$$

- 0 steady state error
- No overshoot
- 2% settling time $4/K_p$



P Controller

$$\dot{\theta}(t) = K_p(\theta_d(t) - \theta(t)) = K_p\theta_e(t)$$

- When $\theta_d(t)$ is not constant but $\dot{\theta}_d(t)$ is constant $\dot{\theta}_d(t) = c$
- Error dynamics

$$\dot{\theta}_e(t) = \dot{\theta}_d(t) - \dot{\theta}(t) = c - K_p\theta_e(t)$$

Solution

$$\theta_e(t) = \frac{c}{K_p} + \left(\theta_e(0) - \frac{c}{K_p}\right) e^{-K_p t} \longrightarrow c/K_p$$

steady-state error

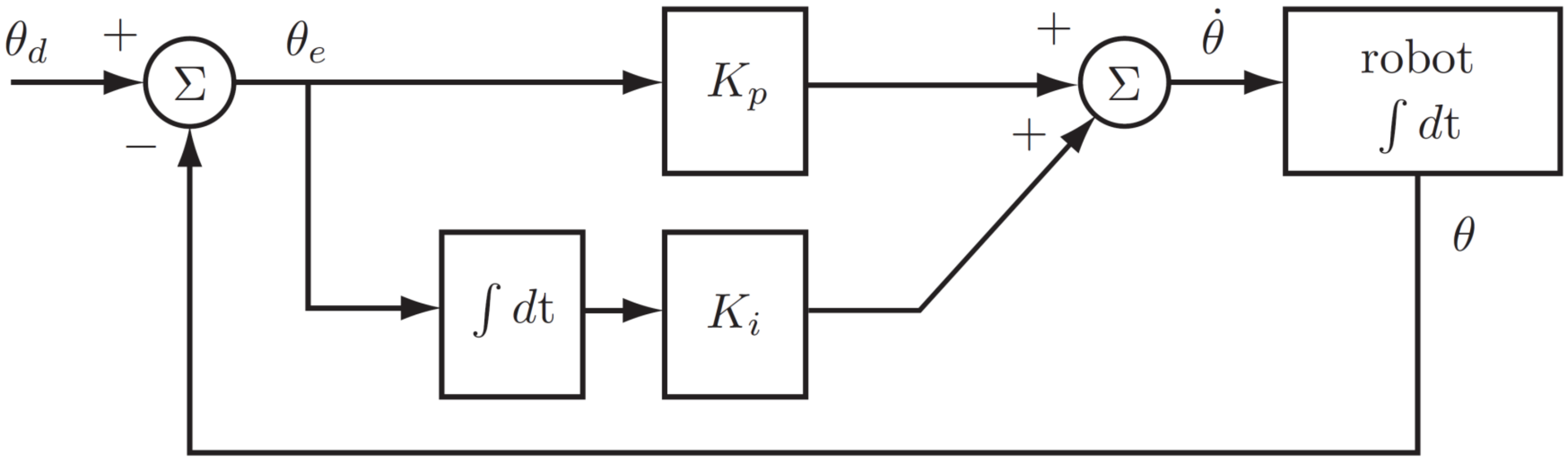
We cannot make K_p arbitrarily large (velocity limit, instability)

PI Controller

- A proportional-integral controller

$$\dot{\theta}(t) = K_p \theta_e(t) + K_i \int_0^t \theta_e(\mathrm{t})\ dt$$

Time-integral of the error



PI Controller

- Error dynamics for a constant $\dot{\theta}_d(t) = c$

$$\dot{\theta}(t) = K_p \theta_e(t) + K_i \int_0^t \theta_e(\mathrm{t})\, d\mathrm{t}$$

$$\dot{\theta}_e(t) = \dot{\theta}_d(t) - \dot{\theta}(t)$$

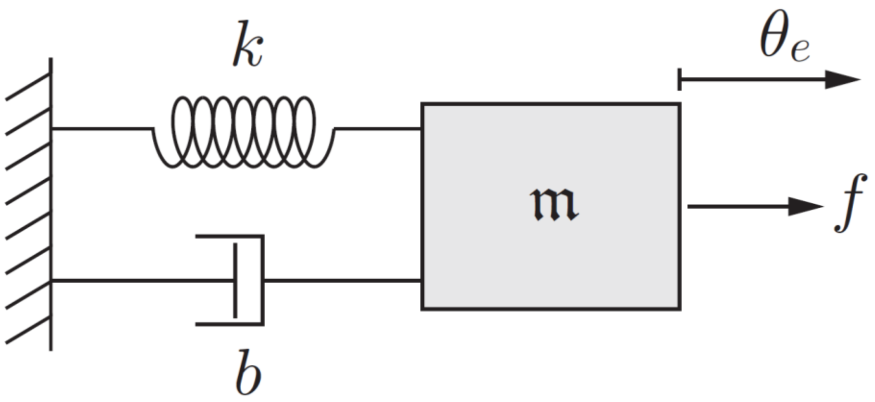
$$\dot{\theta}_e(t) + K_p \theta_e(t) + K_i \int_0^t \theta_e(\mathrm{t})\, d\mathrm{t} = c$$

$$\ddot{\theta}_e(t) + K_p \dot{\theta}_e(t) + K_i \theta_e(t) = 0$$

Second-Order Error Dynamics

PI Controller

- Mass-spring-damper



$$\mathfrak{m}\ddot{\theta}_e + b\dot{\theta}_e + k\theta_e = f$$

$$f = 0 \quad \ddot{\theta}_e(t) + \frac{b}{\mathfrak{m}}\dot{\theta}_e(t) + \frac{k}{\mathfrak{m}}\theta_e(t) = 0$$

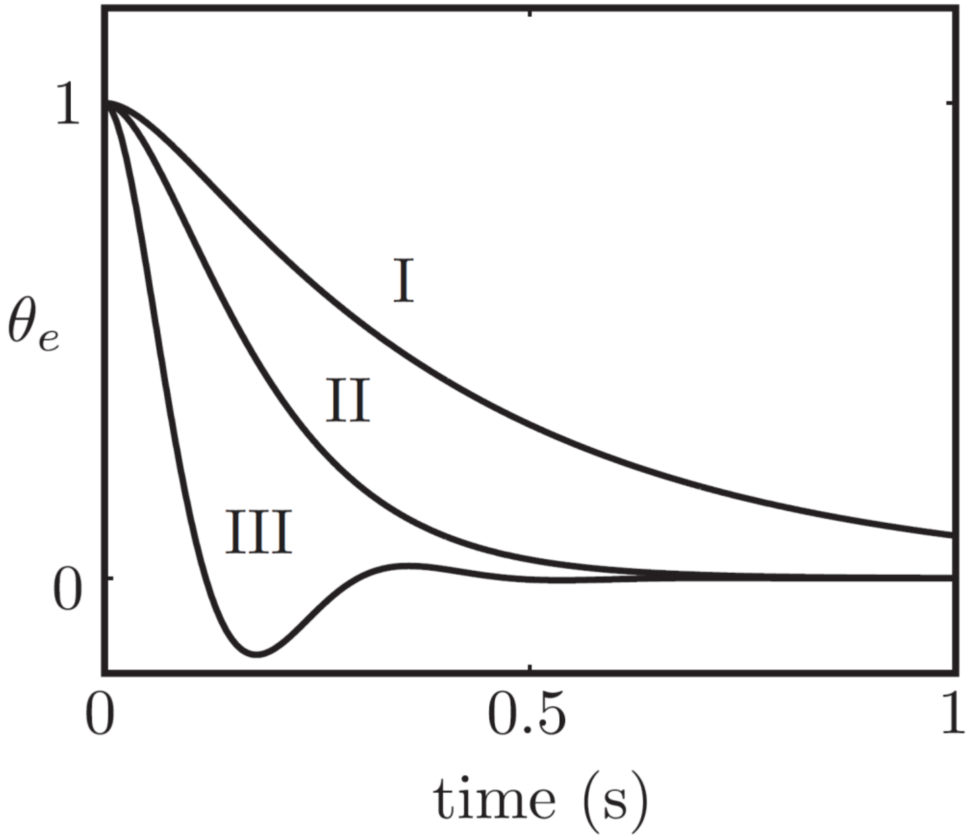
$$\ddot{\theta}_e(t) + K_p\dot{\theta}_e(t) + K_i\theta_e(t) = 0$$

Standard second-order form $\ddot{\theta}_e(t) + 2\zeta\omega_n\dot{\theta}_e(t) + \omega_n^2\theta_e(t) = 0$

natural frequency ω_n damping ratio ζ $\omega_n = \sqrt{K_i}$ $\zeta = K_p/(2\sqrt{K_i})$

PI Controller

Error response



$K_p = 20$ $\quad\quad$ $\zeta = K_p/(2\sqrt{K_i})$

- Overdamped $\zeta = 1.5, \; K_i = 44.4, \; \text{case I}$
- Critically damped $\zeta = 1, \; K_i = 100, \; \text{case II}$
- Underdamped $\zeta = 0.5, \; K_i = 400, \; \text{case III}$

Which one is the best?

Summary

- Robot control
  - Error dynamics

- Motion control
  - P controller
  - PI controller

Further Reading

- Chapter 11 in Kevin M. Lynch and Frank C. Park. Modern Robotics: Mechanics, Planning, and Control. 1st Edition, 2017.