Robot Control: Motion Control, Force Control, Impedance Control

CS 6301 Special Topics: Introduction to Robot Manipulation and Navigation

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Motion Control of a Multi-joint Robot

- Dynamics

$$\tau = M(\theta)\ddot{\theta} + h(\theta, \dot{\theta})$$

$$n \times n$$

- Decentralized control
  - Each joint is controlled independently
  - When dynamics are decoupled (approximately)
- Centralized control
  - Full state information for each of the n joints is available to calculate the controls for each joint

Centralized Multi-joint Control

- Computed torque controller

$$\tau = \tilde{M}(\theta)\left(\ddot{\theta}_d + K_p\theta_e + K_i\int \theta_e(\mathrm{t})dt + K_d\dot{\theta}_e\right) + \tilde{h}(\theta, \dot{\theta})$$

K_p, K_i, K_d positive-definite matrices We choose the gain matrices as $k_pI,\ k_iI,\ \text{and}\ k_dI$

- PID control and gravity compensation When the model is not good

$$\tau = K_p\theta_e + K_i\int \theta_e(\mathrm{t})dt + K_d\dot{\theta}_e + \tilde{g}(\theta)$$

Task-Space Motion Control

- Motion as a trajectory of the end-effector in the task space

$$(X(t), \mathcal{V}_b(t)) \qquad X \in SE(3) \qquad [\mathcal{V}_b] = X^{-1}\dot{X}$$ Twist

- Option 1: convert the trajectory to joint space
  - Forward kinematics $X = T(\theta) \qquad \mathcal{V}_b = J_b(\theta)\dot{\theta} \qquad \dot{\mathcal{V}} = \dot{J}(\theta)\dot{\theta} + J(\theta)\ddot{\theta}$
  - Inverse kinematics

$$\theta(t) = T^{-1}(X(t)),$$

$$\dot{\theta}(t) = J_b^{\dagger}(\theta(t))\mathcal{V}_b(t),$$

$$\ddot{\theta}(t) = J_b^{\dagger}(\theta(t))\left(\dot{\mathcal{V}}_b(t) - \dot{J}_b(\theta(t))\dot{\theta}(t)\right)$$

may require significant computing power

Task-Space Motion Control

- Task-space dynamics

$$\mathcal{V} = J(\theta)\dot{\theta} \qquad \dot{\mathcal{V}} = \dot{J}(\theta)\dot{\theta} + J(\theta)\ddot{\theta}$$

$$\begin{aligned} \dot{\theta} &= J^{-1}\mathcal{V}, \\ \ddot{\theta} &= J^{-1}\dot{\mathcal{V}} - J^{-1}\dot{J}J^{-1}\mathcal{V} \end{aligned}$$

Dynamics $\tau = M(\theta)\ddot{\theta} + h(\theta, \dot{\theta})$

$$\tau = M(\theta)\left(J^{-1}\dot{\mathcal{V}} - J^{-1}\dot{J}J^{-1}\mathcal{V}\right) + h(\theta, J^{-1}\mathcal{V})$$

$\tau = J^{\mathrm{T}}(\theta)\mathcal{F}$ End-effector wrench

$$\begin{aligned} J^{-\mathrm{T}}\tau &= J^{-\mathrm{T}}MJ^{-1}\dot{\mathcal{V}} - J^{-\mathrm{T}}MJ^{-1}\dot{J}J^{-1}\mathcal{V} \\ &\quad + J^{-\mathrm{T}}h(\theta, J^{-1}\mathcal{V}). \end{aligned}$$

Task-Space Motion Control

- Task-space dynamics

$$\mathcal{F} = \Lambda(\theta)\dot{\mathcal{V}} + \eta(\theta, \mathcal{V})$$

$$\begin{aligned} \Lambda(\theta) &= J^{-\mathrm{T}} M(\theta) J^{-1}, \\ \eta(\theta, \mathcal{V}) &= J^{-\mathrm{T}} h(\theta, J^{-1}\mathcal{V}) - \Lambda(\theta)\dot{J}J^{-1}\mathcal{V}. \end{aligned}$$

Task-Space Motion Control

- Option 2: task-space dynamics $\mathcal{F}_b = \Lambda(\theta)\dot{\mathcal{V}}_b + \eta(\theta, \mathcal{V}_b)$

- Joint forces and torques $\tau = J_b^{\mathrm{T}}(\theta)\mathcal{F}_b$

- Computed torque controller

Joint space $$\tau = \tilde{M}(\theta)\left(\ddot{\theta}_d + K_p\theta_e + K_i\int \theta_e(\mathrm{t})dt + K_d\dot{\theta}_e\right) + \tilde{h}(\theta, \dot{\theta})$$

Task space $$\tau = J_b^{\mathrm{T}}(\theta)\left(\tilde{\Lambda}(\theta)\left(\frac{d}{dt}([\mathrm{Ad}_{X^{-1}X_d}]\mathcal{V}_d) + K_pX_e + K_i\int X_e(\mathrm{t})dt + K_d\mathcal{V}_e\right) + \tilde{\eta}(\theta, \mathcal{V}_b)\right)$$

Force Control

- When the task is to apply forces and torques to the environment

- The manipulator dynamics with applied wrench

$$M(\theta)\ddot{\theta} + c(\theta, \dot{\theta}) + g(\theta) + b(\dot{\theta}) + J^{\mathrm{T}}(\theta)\mathcal{F}_{\mathrm{tip}} = \tau$$

Centripetal & Coriolis — Gravity — Friction — Wrench applied to the environment

- The Robot moves slowly (or not at all) during a force control task
  - Ignore the acceleration and the velocity terms

$$g(\theta) + J^{\mathrm{T}}(\theta)\mathcal{F}_{\mathrm{tip}} = \tau$$

Force Control

- Without direct measurements of the force-torque at the robot end-effector, by using joint-angle feedback

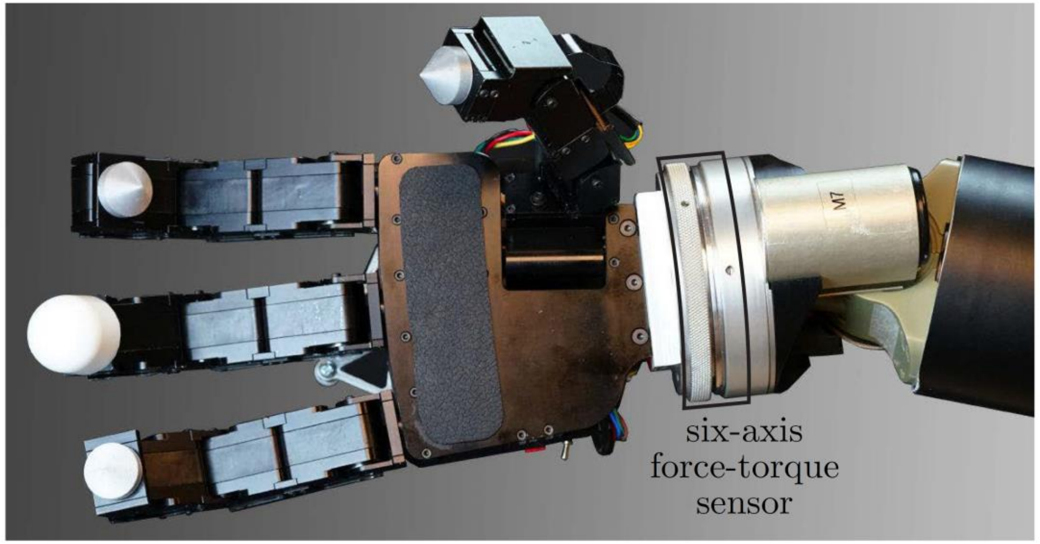
The force-control law $$\tau = \tilde{g}(\theta) + J^{\mathrm{T}}(\theta)\mathcal{F}_d$$

A model of the gravitational torques

Desired wrench

Force Control

- Use a six-axis force-torque sensor between the arm and the end-effector to measure the end-effector wrench $\mathcal{F}_{\text{tip}}$



Force Control

- A PI force controller with a feedforward term and gravity compensation

$$\tau = \tilde{g}(\theta) + J^{\mathrm{T}}(\theta)\left(\mathcal{F}_d + K_{fp}\mathcal{F}_e + K_{fi}\int \mathcal{F}_e(\mathrm{t})d\mathrm{t}\right)$$

$$\mathcal{F}_e = \mathcal{F}_d - \mathcal{F}_{\mathrm{tip}}$$

- Adding velocity damping

$$\tau = \tilde{g}(\theta) + J^{\mathrm{T}}(\theta)\left(\mathcal{F}_d + K_{fp}\mathcal{F}_e + K_{fi}\int \mathcal{F}_e(\mathrm{t})d\mathrm{t} - K_{\mathrm{damp}}\mathcal{V}\right)$$

Impedance Control

- Robot impedance characterizes the change in endpoint motion as a function of disturbance forces.

$$\text{Impedance} \quad R = \frac{F}{V} \quad \begin{matrix}\text{Force} \\ \text{Velocity}\end{matrix}$$

- Ideal motion control
  - High impedance, little change in motion due to force disturbances

- Ideal force control
  - Low impedance, little change in force due to motion disturbances

Impedance Control

- Impedance control is an approach to dynamic control relating force and position
- The robot end-effector is asked to render particular mass, spring, and damper properties

https://youtu.be/XwiX2vv14Qs

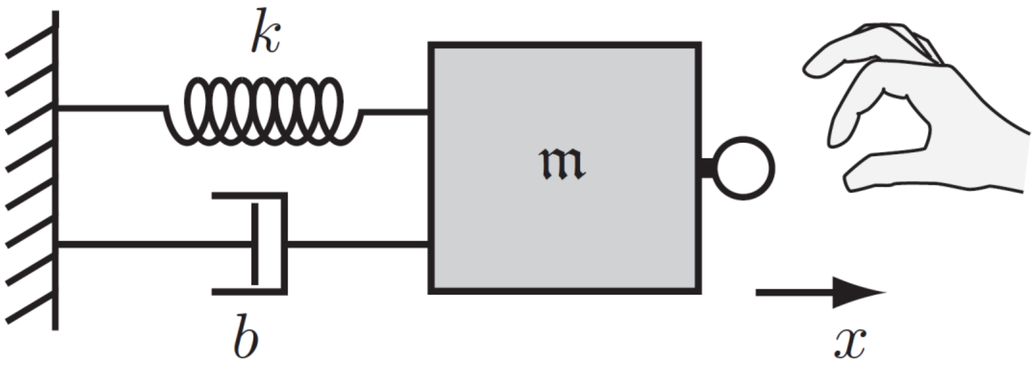
Impedance Control

- The dynamics for a one dof robot rendering an impedance

$$\mathfrak{m}\ddot{x} + b\dot{x} + kx = f$$

mass — damping — stiffness — force

High impedance: b or k is large



Impedance Control

- Goal: implement the task-space behavior

$$M\ddot{x} + B\dot{x} + Kx = f_{\text{ext}}$$

$x \in \mathbb{R}^n$ Task-space configuration in a minimum set of coordinates

$M, B, \text{ and } K$ Positive-definite virtual mass, damping, and stiffness matrices

f_{ext} Force applied to the robot

Two ways of Impedance Control

$$M\ddot{x} + B\dot{x} + Kx = f_{\mathrm{ext}}$$

- Impedance controlled
  - The robot senses the endpoint motion $x(t)$
  - Commands joint torques and forces to create $-f_{\mathrm{ext}}$
  - Displays the force to the user
- Admittance controlled
  - The robot senses f_{ext}
  - Controls its motion in response

Impedance Control

- Impedance-Control Algorithm

$$M\ddot{x} + B\dot{x} + Kx = f_{\text{ext}}$$

$$\tau = J^{\mathrm{T}}(\theta)\left(\underbrace{\tilde{\Lambda}(\theta)\ddot{x} + \tilde{\eta}(\theta, \dot{x})}_{\text{arm dynamics compensation}} - \underbrace{(M\ddot{x} + B\dot{x} + Kx)}_{f_{\text{ext}}}\right)$$

- Admittance-Control Algorithm

$$\ddot{x}_d = M^{-1}(f_{\text{ext}} - B\dot{x} - Kx)$$

$$\dot{x} = J(\theta)\dot{\theta}$$

$$\ddot{\theta}_d = J^{\dagger}(\theta)(\ddot{x}_d - \dot{J}(\theta)\dot{\theta})$$

Use inverse dynamics to calculate the commanded joint forces and torques

Summary

- Task-space motion control
- Force control
- Impedance control

Further Reading

- Chapter 11 in Kevin M. Lynch and Frank C. Park. Modern Robotics: Mechanics, Planning, and Control. 1st Edition, 2017.