Robot Control: Motion Control, Force Control, Impedance Control

CS 6301 Special Topics: Introduction to Robot Manipulation and Navigation
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Motion Control of a Multi-joint Robot

• Dynamics

\[ \tau = M(\theta)\ddot{\theta} + h(\theta, \dot{\theta}) \]

\[ n \times n \]

• Decentralized control
  • Each joint is controlled independently
  • When dynamics are decoupled (approximately)

• Centralized control
  • Full state information for each of the n joints is available to calculate the controls for each joint
Centralized Multi-joint Control

• Computed torque controller

\[\tau = \tilde{M}(\theta) \left( \ddot{\theta}_d + K_p \theta_e + K_i \int \theta_e(t) dt + K_d \dot{\theta}_e \right) + \tilde{h}(\theta, \dot{\theta})\]

\(K_p, K_i, K_d\) positive-definite matrices        We choose the gain matrices as \(k_p I, k_i I, \text{ and } k_d I\)

• PID control and gravity compensation

\[\tau = K_p \theta_e + K_i \int \theta_e(t) dt + K_d \dot{\theta}_e + \tilde{g}(\theta)\]

When the model is not good
Task-Space Motion Control

• Motion as a trajectory of the end-effector in the task space

\[ (X(t), V_b(t)) \quad X \in SE(3) \quad [V_b] = X^{-1} \dot{X} \]

• Option 1: convert the trajectory to joint space
  
  • Forward kinematics
  
  \[ X = T(\theta) \quad V_b = J_b(\theta)\dot{\theta} \quad \dot{V} = \dot{J}(\theta)\dot{\theta} + J(\theta)\ddot{\theta} \]

  • Inverse kinematics

  \[ \theta(t) = T^{-1}(X(t)), \]
  
  \[ \dot{\theta}(t) = J_b^\dagger(\theta(t))V_b(t), \]
  
  \[ \ddot{\theta}(t) = J_b^\dagger(\theta(t)) \left( \dot{V}_b(t) - J_b(\theta(t))\dot{\theta}(t) \right) \]

  may require significant computing power
Task-Space Motion Control

- Task-space dynamics

\[ \dot{\mathbf{v}} = J(\theta)\dot{\theta} \]
\[ \ddot{\mathbf{v}} = \dot{J}(\theta)\dot{\theta} + J(\theta)\ddot{\theta} \]

\[ \dot{\theta} = J^{-1}\mathbf{v}, \]
\[ \ddot{\theta} = J^{-1}\dot{\mathbf{v}} - J^{-1}\dot{\mathbf{j}}J^{-1}\mathbf{v} \]

Dynamics

\[ \mathbf{\tau} = M(\theta)\ddot{\theta} + h(\theta, \dot{\theta}) \]

\[ \mathbf{\tau} = M(\theta) \left( J^{-1}\dot{\mathbf{v}} - J^{-1}\dot{\mathbf{j}}J^{-1}\mathbf{v} \right) + h(\theta, J^{-1}\mathbf{v}) \]

\[ \mathbf{\tau} = J^T(\theta)\mathbf{F} \quad \text{End-effector wrench} \]

\[ J^{-T}\mathbf{\tau} = J^{-T}MJ^{-1}\dot{\mathbf{v}} - J^{-T}MJ^{-1}\dot{\mathbf{j}}J^{-1}\mathbf{v} \]
\[ + J^{-T}h(\theta, J^{-1}\mathbf{v}). \]
Task-Space Motion Control

• Task-space dynamics

\[ \mathcal{F} = \Lambda(\theta) \dot{\mathbf{V}} + \eta(\theta, \mathbf{V}) \]

\[ \Lambda(\theta) = J^{-T} M(\theta) J^{-1}, \]

\[ \eta(\theta, \mathbf{V}) = J^{-T} h(\theta, J^{-1} \mathbf{V}) - \Lambda(\theta) \dot{J} J^{-1} \mathbf{V}. \]
Task-Space Motion Control

- Option 2: task-space dynamics
  \[ F_b = \Lambda(\theta)\dot{\nu}_b + \eta(\theta, \nu_b) \]

- Joint forces and torques
  \[ \tau = J_b^T(\theta)F_b \]

- Computed torque controller
  \[
  \tau = \tilde{M}(\theta) \left( \ddot{\theta} + K_p \theta_e + K_i \int \theta_e(t)dt + K_d \dot{\theta} \right) + \tilde{h}(\theta, \dot{\theta})
  
  \text{Joint space}
  
  \tau = J_b^T(\theta) \left( \tilde{\Lambda}(\theta) \left( \frac{d}{dt} ([\text{Ad}_{X^{-1}} X_d] \nu_d) + K_p X_e + K_i \int X_e(t)dt + K_d \nu_e \right) + \tilde{h}(\theta, \nu_b) \right)
  
  \text{Task space}
  \]
Force Control

• When the task is to apply forces and torques to the environment

• The manipulator dynamics with applied wrench

\[ M(\theta)\ddot{\theta} + c(\theta, \dot{\theta}) + g(\theta) + b(\dot{\theta}) + J^T(\theta)\mathcal{F}_{\text{tip}} = \tau \]

- Centripetal & Coriolis
- Gravity
- Friction
- Wrench applied to the environment

• The Robot moves slowly (or not at all) during a force control task
  • Ignore the acceleration and the velocity terms

\[ g(\theta) + J^T(\theta)\mathcal{F}_{\text{tip}} = \tau \]
Force Control

• Without direct measurements of the force-torque at the robot end-effector, by using joint-angle feedback

The force-control law

\[ \tau = \tilde{g}(\theta) + J^T(\theta)F_d \]

A model of the gravitational torques

Desired wrench
Force Control

• Use a six-axis force-torque sensor between the arm and the end-effector to measure the end-effector wrench $\mathbf{F}_{\text{tip}}$
Force Control

- A PI force controller with a feedforward term and gravity compensation

\[
\tau = \ddot{g}(\theta) + J^T(\theta) \left( F_d + K_{fp}F_e + K_{fi} \int F_e(t)dt \right)
\]

\[
F_e = F_d - F_{tip}
\]

- Adding velocity damping

\[
\tau = \ddot{g}(\theta) + J^T(\theta) \left( F_d + K_{fp}F_e + K_{fi} \int F_e(t)dt - K_{damp} \gamma \right)
\]
Impedance Control

• Robot impedance characterizes the change in endpoint motion as a function of disturbance forces.

\[ R = \frac{F}{V} \]

• Ideal motion control
  • High impedance, little change in motion due to force disturbances

• Ideal force control
  • Low impedance, little change in force due to motion disturbances
Impedance Control

• Impedance control is an approach to dynamic control relating force and position

• The robot end-effector is asked to render particular mass, spring, and damper properties

https://youtu.be/XwiX2vv14Qs
Impedance Control

• The dynamics for a one dof robot rendering an impedance

\[ m \ddot{x} + b \dot{x} + k x = f \]

- Mass
- Damping
- Stiffness
- Force

High impedance: \( b \) or \( k \) is large
Impedance Control

- Goal: implement the task-space behavior

\[ M \ddot{x} + B \dot{x} + K x = f_{\text{ext}} \]

\[ x \in \mathbb{R}^n \] Task-space configuration in a minimum set of coordinates

\( M, B, \text{ and } K \) Positive-definite virtual mass, damping, and stiffness matrices

\( f_{\text{ext}} \) Force applied to the robot
Two ways of Impedance Control

• Impedance controlled
  • The robot senses the endpoint motion
  • Commands joint torques and forces to create
  • Displays the force to the user

\[ M\ddot{x} + B\dot{x} + Kx = f_{\text{ext}} \]

\[ x(t) - f_{\text{ext}} \]

• Admittance controlled
  • The robot senses \( f_{\text{ext}} \)
  • Controls its motion in response
Impedance Control

• Impedance-Control Algorithm

\[ \tau = J^T(\theta) \left( \tilde{\Lambda}(\theta)\ddot{x} + \tilde{\eta}(\theta, \dot{x}) - \left( M\ddot{x} + B\dot{x} + Kx \right) \right) \]

\[ M\ddot{x} + B\dot{x} + Kx = f_{ext} \]

• Admittance-Control Algorithm

\[ \ddot{x}_d = M^{-1}(f_{ext} - B\dot{x} - Kx) \]

\[ \dot{x} = J(\theta)\dot{\theta} \]

\[ \ddot{\theta}_d = J^\dagger(\theta)(\ddot{x}_d - J(\theta)\dot{\theta}) \]

Use inverse dynamics to calculate the commanded joint forces and torques
Summary

- Task-space motion control
- Force control
- Impedance control
Further Reading