Robot Control: Motion Control with Velocities, Forces or Torques

CS 6301 Special Topics: Introduction to Robot Manipulation and Navigation

Professor Yu Xiang

The University of Texas at Dallas

Motion Control with Velocity Inputs

- Motion control with velocity inputs
  - Given a desired trajectory of a robot in joint space or in task space

$$\theta_d(t) \qquad X_d(t)$$

- Proportional controller or P controller

$$\dot{\theta}(t) = K_p(\theta_d(t) - \theta(t)) = K_p\theta_e(t)$$

Control gain

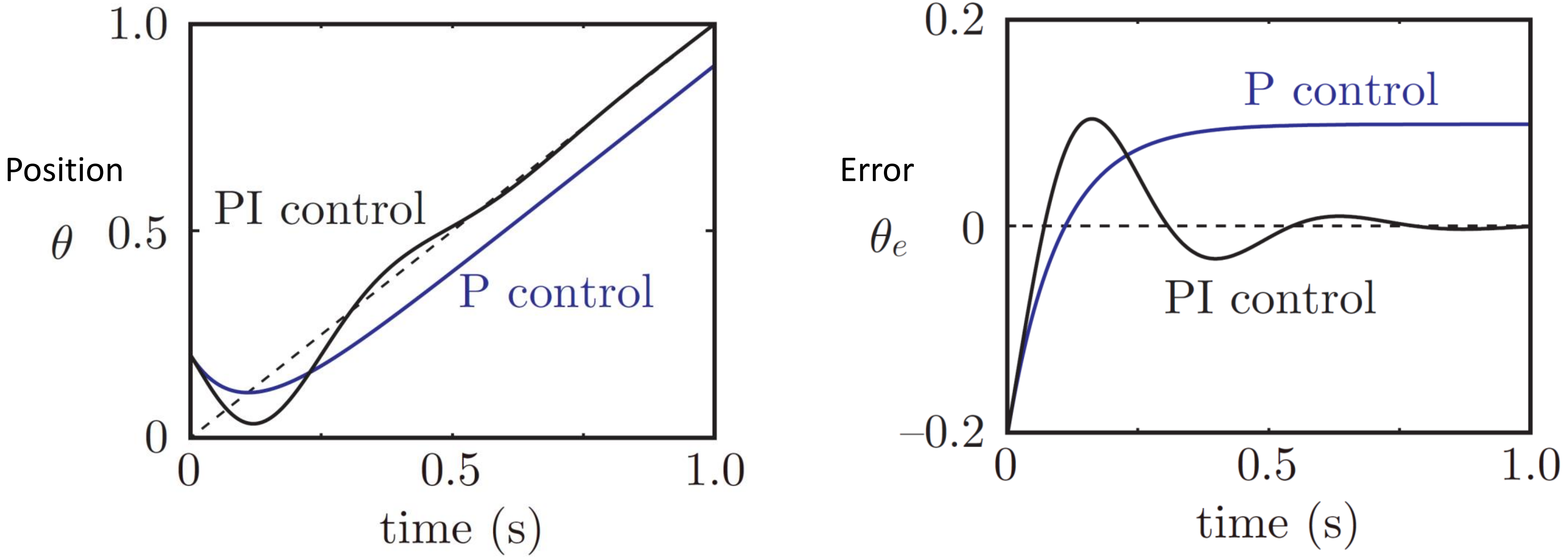
$$K_p > 0$$

- Proportional-integral controller or PI controller

$$\dot{\theta}(t) = K_p\theta_e(t) + K_i \int_0^t \theta_e(\mathrm{t})\ dt$$

Comparison between P Controller and PI Controller

$$\dot{\theta}_d(t) = c$$

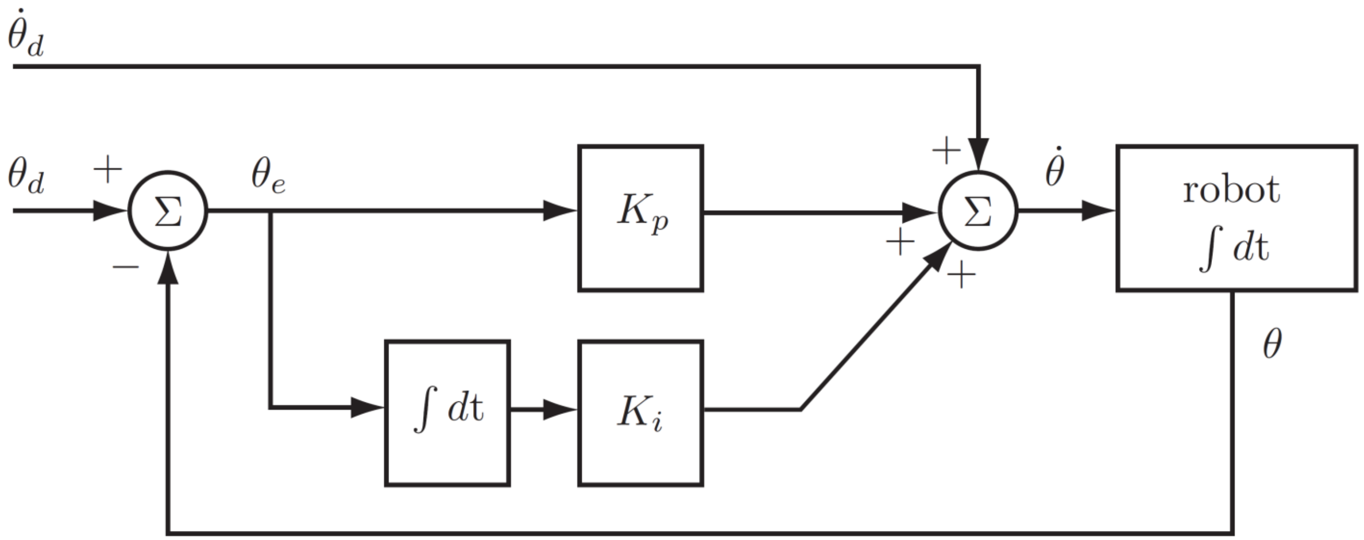


Reference trajectory (dashed)

Feedforward Plus Feedback Control

- Feedback control: an error is required before the joint begins to move
- Feedforward plus feedback control: Initiate motion before any error accumulates

$$\dot{\theta}(t) = \dot{\theta}_d(t) + K_p\theta_e(t) + K_i \int_0^t \theta_e(\mathrm{t})\, d\mathrm{t}$$



Preferred control law for producing a commanded velocity to the joint

Motion Control of Multi-Joint Robots

- Reference position $\theta_d(t)$ and actual position $\theta(t)$ n dimensional vector
- Gains K_p K_i $n \times n$ matrix

$$k_p I \qquad k_i I$$

Control law $\dot{\theta}(t) = \dot{\theta}_d(t) + K_p\theta_e(t) + K_i \int_0^t \theta_e(\mathrm{t})\ dt$

Task-Space Motion Control

- Consider the configuration of the end-effector $X(t) \in SE(3)$
- Recall body twist $T^{-1}\dot{T} = [\mathcal{V}_b] = \begin{bmatrix} [\omega_b] & v_b \\ 0 & 0 \end{bmatrix} \in se(3)$
- End-effector twist $\mathcal{V}_b(t) \quad [\mathcal{V}_b] = X^{-1}\dot{X}$
- Desired motion is given by $X_d(t) \quad [\mathcal{V}_d] = X_d^{-1}\dot{X}_d$

Task-Space Motion Control

$$[\mathrm{Ad}_T] = \begin{bmatrix} R & 0 \\ [p]R & R \end{bmatrix} \in \mathbb{R}^{6\times 6}$$

- Joint-space control law

$$\dot{\theta}(t) = \dot{\theta}_d(t) + K_p\theta_e(t) + K_i \int_0^t \theta_e(\mathrm{t})\, dt$$

- A task-space control law

$$\mathcal{V}_b(t) = [\mathrm{Ad}_{X^{-1}X_d}]\mathcal{V}_d(t) + K_pX_e(t) + K_i \int_0^t X_e(\mathrm{t})\, dt$$

$$[X_e] = \log(X^{-1}X_d) \qquad K_p, K_i \in \mathbb{R}^{6\times 6} \qquad X_{sb} \quad X_{sd}$$

Commanded joint velocities $\dot{\theta} = J_b^{\dagger}(\theta)\mathcal{V}_b$

Motion Control with Velocity Inputs

- Motion control with velocity inputs
  - Given a desired trajectory of a robot in joint space or in task space
  - Direct control of the joint velocities

- Limited to applications with low or predictable force-torque requirements

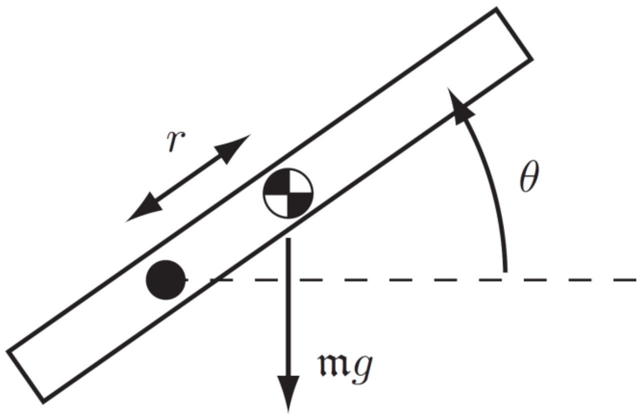
- Do not make use of a dynamic model of the robot

Motion Control with Torque or Force Inputs

- Controller generates joint torques and forces to track a desired trajectory
- Motion Control of a single joint

Dynamics $\tau = M\ddot{\theta} + \mathfrak{m}gr\cos\theta$

Scalar inertia mass



Friction torque $\tau_{\text{fric}} = b\dot{\theta}$

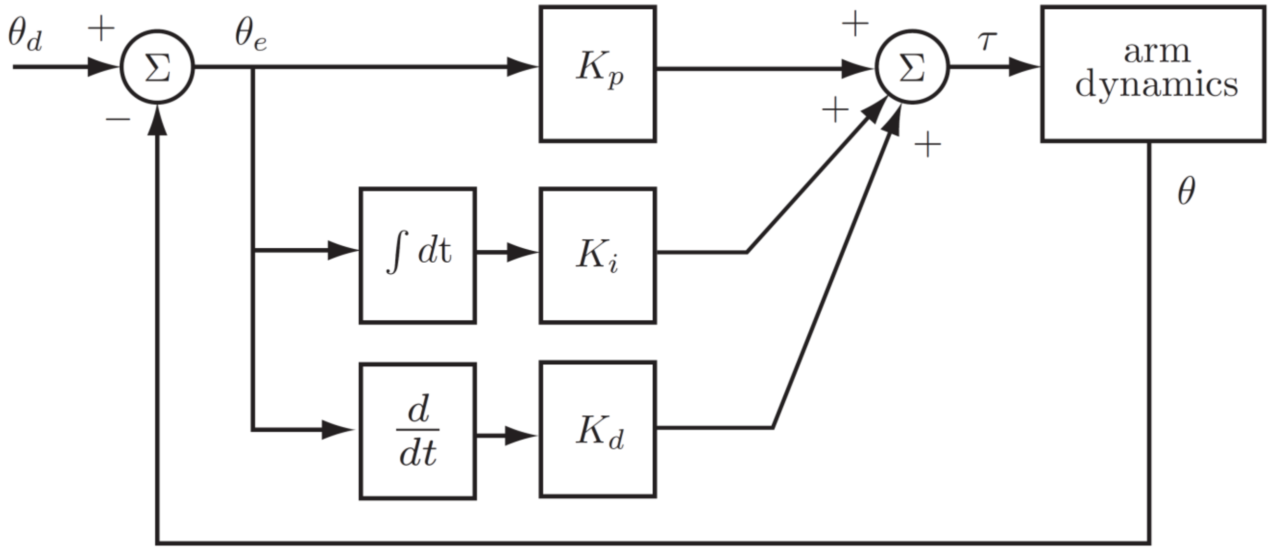
$$\tau = M\ddot{\theta} + \mathfrak{m}gr\cos\theta + b\dot{\theta}$$

$$\tau = M\ddot{\theta} + h(\theta, \dot{\theta})$$

Motion Control of a Single Joint

- Feedback control: PID control
  - Proportional-Integral-Derivative control

$$\tau = K_p\theta_e + K_i \int \theta_e(\mathrm{t})dt + K_d\dot{\theta}_e \qquad \theta_e = \theta_d - \theta$$



PD Control

- Dynamics $\tau = M\ddot{\theta} + \mathfrak{m}gr\cos\theta + b\dot{\theta}$
- PD control law $K_p(\theta_d - \theta) + K_d(\dot{\theta}_d - \dot{\theta})$

Assume $g = 0$

$$M\ddot{\theta} + b\dot{\theta} = K_p(\theta_d - \theta) + K_d(\dot{\theta}_d - \dot{\theta})$$

Control objective: constant θ_d $\dot{\theta}_d = \ddot{\theta}_d = 0$

$$\theta_e = \theta_d - \theta \qquad \dot{\theta}_e = -\dot{\theta} \qquad \ddot{\theta}_e = -\ddot{\theta}$$

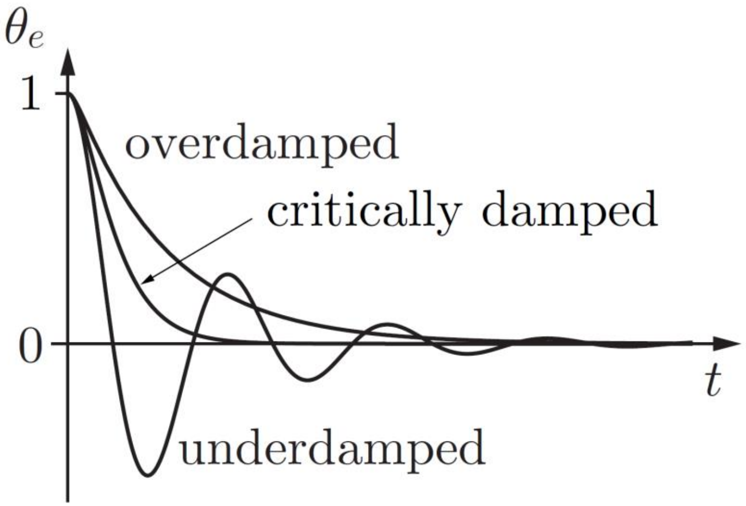
Error dynamics $$M\ddot{\theta}_e + (b + K_d)\dot{\theta}_e + K_p\theta_e = 0$$

PD Control

- Standard second-order form

$$M\ddot{\theta}_e + (b + K_d)\dot{\theta}_e + K_p\theta_e = 0$$

$$\ddot{\theta}_e + \frac{b + K_d}{M}\dot{\theta}_e + \frac{K_p}{M}\theta_e = 0 \quad \rightarrow \quad \ddot{\theta}_e + 2\zeta\omega_n\dot{\theta}_e + \omega_n^2\theta_e = 0$$



$$\zeta = \frac{b + K_d}{2\sqrt{K_p M}} \qquad \omega_n = \sqrt{\frac{K_p}{M}}$$

Critically damped: $\zeta = 1$

PD Control

- When g > 0, the error dynamics

$$M\ddot{\theta}_e + (b + K_d)\dot{\theta}_e + K_p\theta_e = \mathfrak{m}gr\cos\theta$$

When the joint comes to rest at a configuration θ , $K_p\theta_e = \mathfrak{m}gr\cos\theta$

the final error θ_e is nonzero when $\theta_d \neq \pm\pi/2$

Non-zero steady-state error

PID Control

- Setpoint error dynamics

$$M\ddot{\theta}_e + (b + K_d)\dot{\theta}_e + K_p\theta_e + K_i \int \theta_e(\mathrm{t})dt = \tau_{\text{dist}}$$

Disturbance torque

$$\mathfrak{m}gr\cos\theta$$

Taking derivatives

$$M\theta_e^{(3)} + (b + K_d)\ddot{\theta}_e + K_p\dot{\theta}_e + K_i\theta_e = \dot{\tau}_{\text{dist}}$$

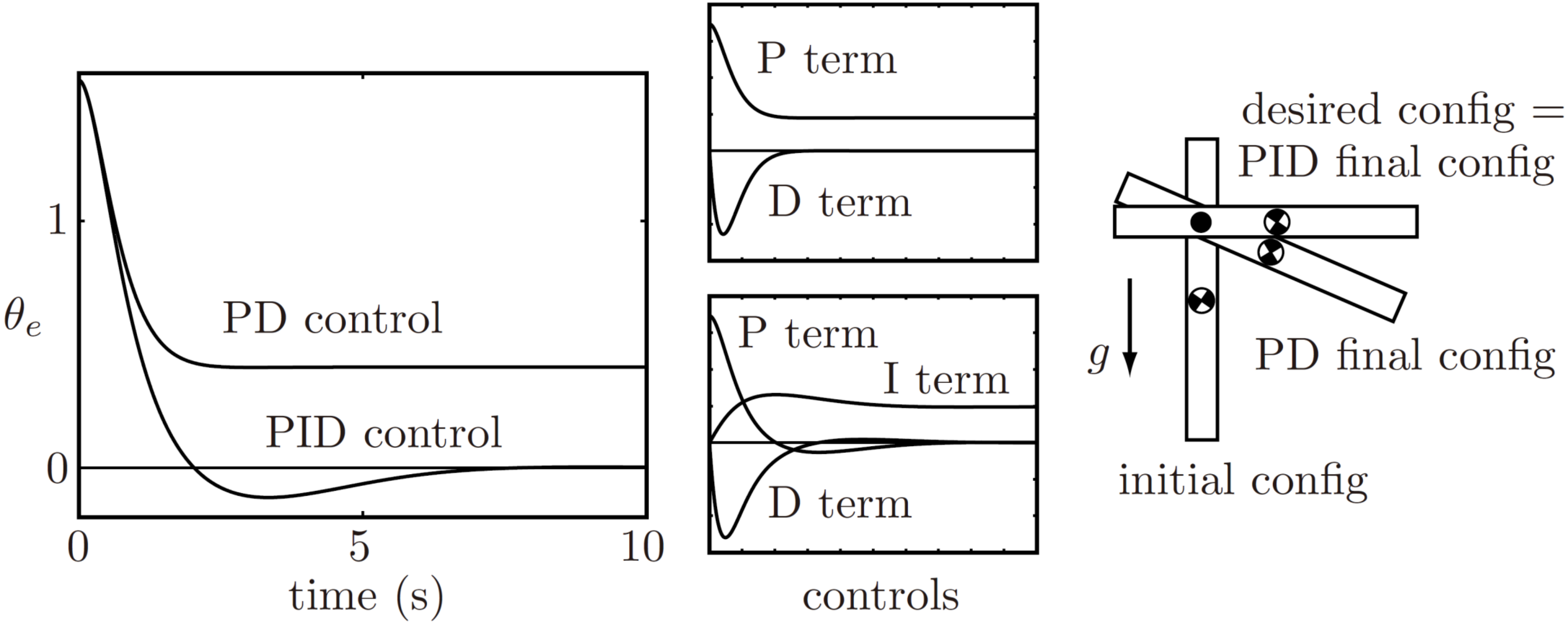
Third-Order Error Dynamics

$$s^3 + \frac{b + K_d}{M}s^2 + \frac{K_p}{M}s + \frac{K_i}{M} = 0$$

If τ_{dist}
Constant

If all roots have a negative real part, then the error dynamics is stable, and θ_e converges to zero

PID Control


1
θ_e
0
PD control
PID control
0
5
10
time (s)
P term
D term
P term
I term
D term
controls
desired config =
PID final config
g
PD final config
initial config

PID Control

```
time = 0                        // dt = servo cycle time
eint = 0                        // error integral
qprev = senseAngle              // initial joint angle q
loop
  [qd,qdotd] = trajectory(time) // from trajectory generator

  q = senseAngle                // sense actual joint angle
  qdot = (q - qprev)/dt         // simple velocity calculation
  qprev = q

  e = qd - q
  edot = qdotd - qdot
  eint = eint + e*dt

  tau = Kp*e + Kd*edot + Ki*eint
  commandTorque(tau)

  time = time + dt
end loop
```

Feedforward Control

- Uses the dynamics of the robot
- The controller's model of the dynamics

$$\tau = \tilde{M}(\theta)\ddot{\theta} + \tilde{h}(\theta, \dot{\theta})$$

$\tilde{M}(\theta) = M(\theta)$ and $\tilde{h}(\theta, \dot{\theta}) = h(\theta, \dot{\theta})$ if the model is perfect

- Given θ_d, $\dot{\theta}_d$, and $\ddot{\theta}_d$

Feedforward torque $\tau(t) = \tilde{M}(\theta_d(t))\ddot{\theta}_d(t) + \tilde{h}(\theta_d(t), \dot{\theta}_d(t))$

The dynamics model of the controller cannot be perfect in practice

Feedforward Plus Feedback Linearization

- Goal: achieve the following error dynamics

$$\ddot{\theta}_e + K_d\dot{\theta}_e + K_p\theta_e + K_i\int \theta_e(\mathrm{t})d\mathrm{t} = c$$

A PID controller can achieve exponential decay of the trajectory error

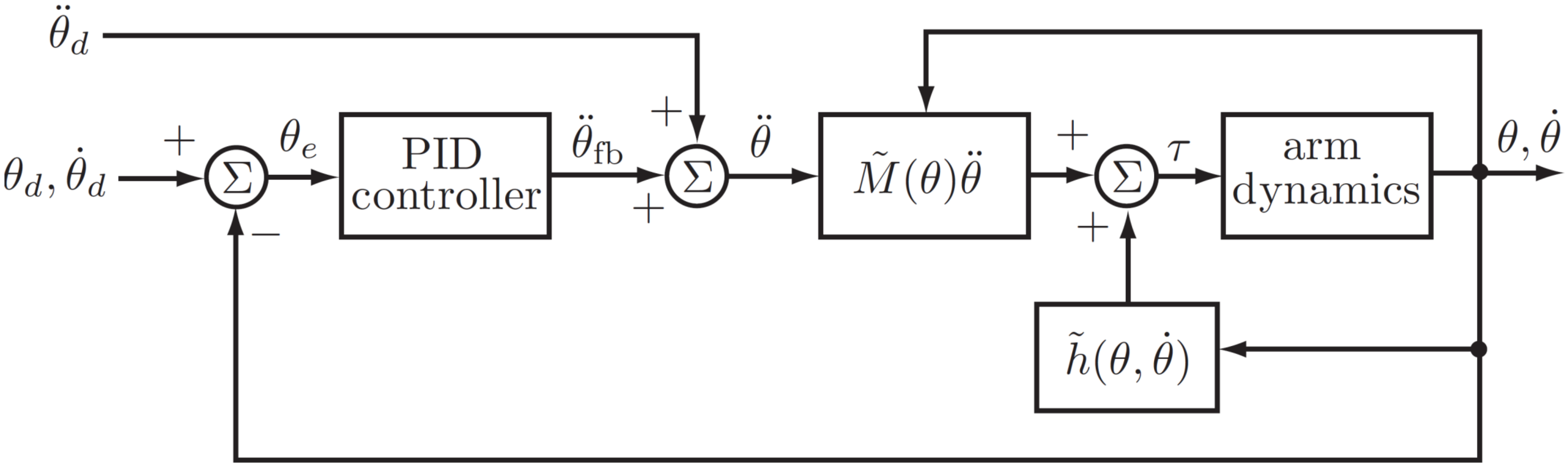
- We first choose $\ddot{\theta} = \ddot{\theta}_d - \ddot{\theta}_e$ $\quad$ $\ddot{\theta} = \ddot{\theta}_d + K_d\dot{\theta}_e + K_p\theta_e + K_i\int \theta_e(\mathrm{t})d\mathrm{t}$

- Feedforward plus feedback linearizing controller (inverse dynamics controller, computed torque controller)

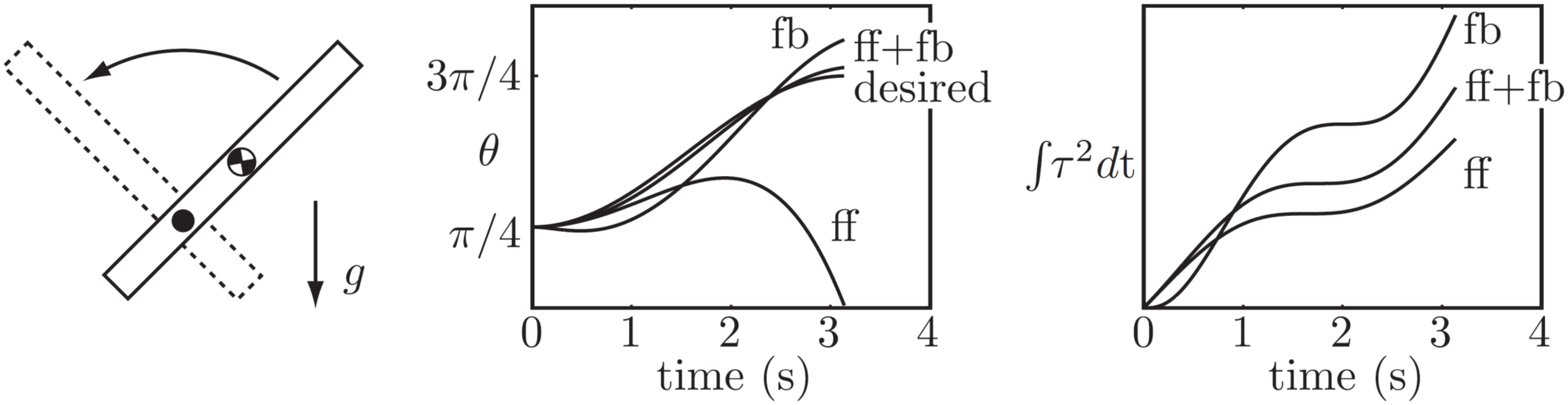
$$\tau = \tilde{M}(\theta)\left(\ddot{\theta}_d + K_p\theta_e + K_i\int \theta_e(\mathrm{t})d\mathrm{t} + K_d\dot{\theta}_e\right) + \tilde{h}(\theta, \dot{\theta})$$

Feedforward Plus Feedback Linearization

$$\tau = \tilde{M}(\theta)\left(\ddot{\theta}_d + K_p\theta_e + K_i\int\theta_e(\mathrm{t})dt + K_d\dot{\theta}_e\right) + \tilde{h}(\theta,\dot{\theta})$$



Feedforward Plus Feedback Linearization



Summary

- Motion control with velocities
  - P controller
  - PI controller
  - Feedforward plus feedback controller

- Motion control with torque or force Inputs
  - PID control
  - Computed torque control

Further Reading

- Chapter 11 in Kevin M. Lynch and Frank C. Park. Modern Robotics: Mechanics, Planning, and Control. 1st Edition, 2017.