Robot Control: Force Control, Hybrid Motion-Force Control, Impedance Control

CS 6301 Special Topics: Introduction to Robot Manipulation and Navigation

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Robot Control

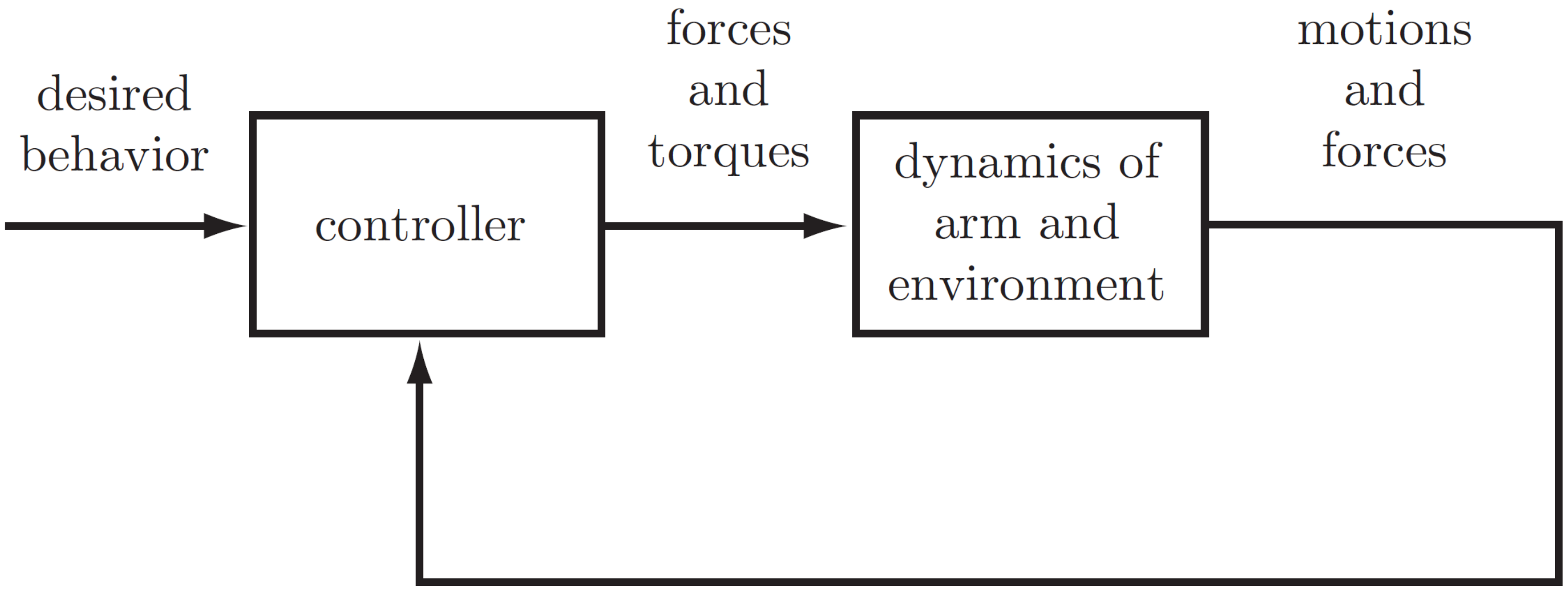
- Convert task specifications to force and torques at the actuators

- Types
  - Motion control
  - Force control
  - Hybrid motion-force control
  - Impedance control

- Feedback control
  - Use sensors for position, velocity and force
  - Compare with the desired behavior to compute the control signals

Control System Overview

- A simplified system



Force Control

- When the task is to apply forces and torques to the environment

- The manipulator dynamics with applied wrench

$$M(\theta)\ddot{\theta} + c(\theta, \dot{\theta}) + g(\theta) + b(\dot{\theta}) + J^{\mathrm{T}}(\theta)\mathcal{F}_{\mathrm{tip}} = \tau$$

Gravity

Friction

Wrench applied to the environment

- The Robot moves slowly (or not at all) during a force control task
  - Ignore the acceleration and the velocity terms

$$g(\theta) + J^{\mathrm{T}}(\theta)\mathcal{F}_{\mathrm{tip}} = \tau$$

Force Control

- Without direct measurements of the force-torque at the robot end-effector, by using joint-angle feedback

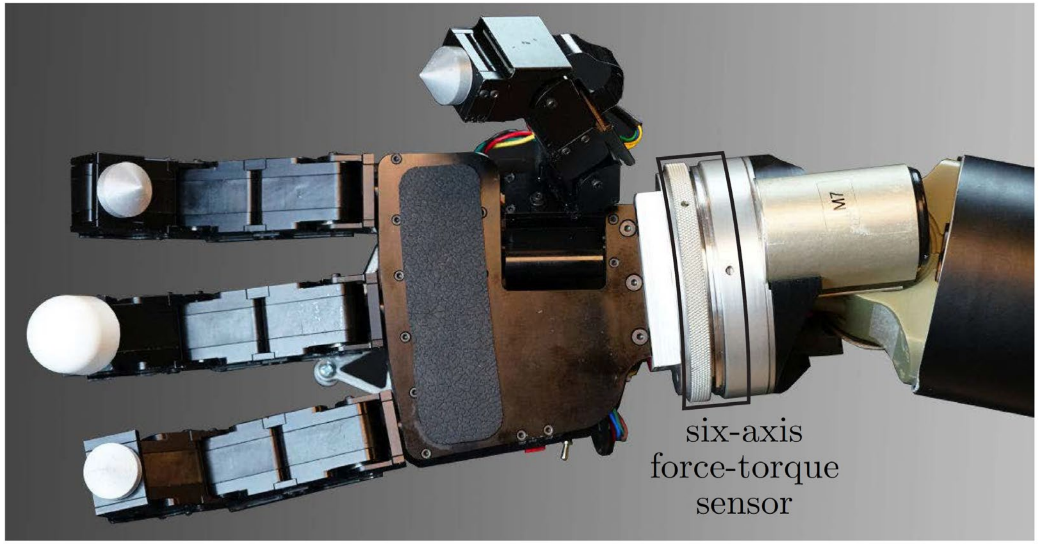
The force-control law $$\tau = \tilde{g}(\theta) + J^{\mathrm{T}}(\theta)\mathcal{F}_d$$

A model of the gravitational torques

Desired wrench

Force Control

- Use a six-axis force-torque sensor between the arm and the end-effector to measure the end-effector wrench $\mathcal{F}_{\text{tip}}$



Force Control

- A PI force controller with a feedforward term and gravity compensation

$$\tau = \tilde{g}(\theta) + J^{\mathrm{T}}(\theta)\left(\mathcal{F}_d + K_{fp}\mathcal{F}_e + K_{fi}\int \mathcal{F}_e(\mathrm{t})d\mathrm{t}\right)$$

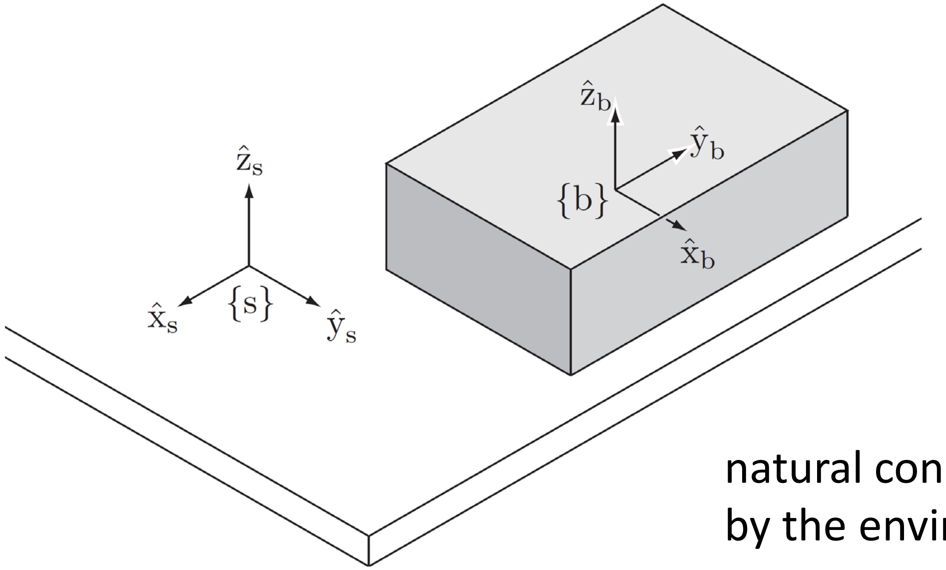
$$\mathcal{F}_e = \mathcal{F}_d - \mathcal{F}_{\mathrm{tip}}$$

- Adding velocity damping

$$\tau = \tilde{g}(\theta) + J^{\mathrm{T}}(\theta)\left(\mathcal{F}_d + K_{fp}\mathcal{F}_e + K_{fi}\int \mathcal{F}_e(\mathrm{t})d\mathrm{t} - K_{\mathrm{damp}}\mathcal{V}\right)$$

Hybrid Motion-Force Control

- Generating controlled forces and motions jointly
- Example: a robot erasing a frictionless chalkboard



- Configuration of the eraser $X(t) \in SE(3)$

body-frame twist $\mathcal{V}_b = (\omega_x, \omega_y, \omega_z, v_x, v_y, v_z)$

body-frame wrench $\mathcal{F}_b = (m_x, m_y, m_z, f_x, f_y, f_z)$

- To maintain contact with the board

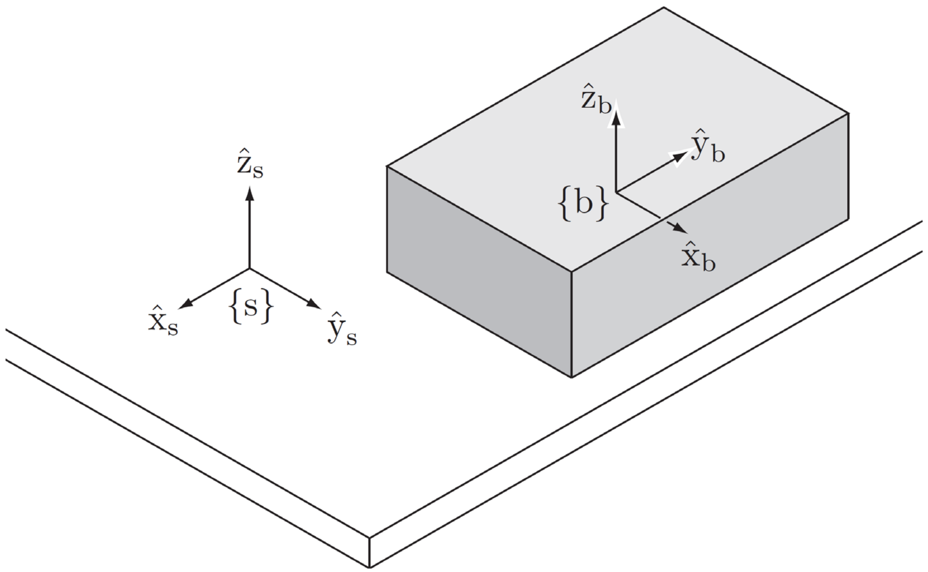
natural constraints specified by the environments

$$\omega_x = 0, \quad \omega_y = 0, \quad v_z = 0.$$

$$m_z = f_x = f_y = 0$$

Hybrid Motion-Force Control

- Example: a robot erasing a frictionless chalkboard



natural constraint	artificial constraint
$\omega_x = 0$	$m_x = 0$
$\omega_y = 0$	$m_y = 0$
$m_z = 0$	$\omega_z = 0$
$f_x = 0$	$v_x = k_1$
$f_y = 0$	$v_y = 0$
$v_z = 0$	$f_z = k_2 < 0$

Cause the eraser to move

A Hybrid Motion-Force Controller

- Express k nature constraints on the velocity in the task space as the Pfaffian constraints

$$A(\theta)\mathcal{V} = 0$$

$$A(\theta) \in \mathbb{R}^{k\times 6} \qquad \mathcal{V} \in \mathbb{R}^6$$

- Task-space dynamics of the robot

$$\mathcal{F} = \Lambda(\theta)\dot{\mathcal{V}} + \eta(\theta, \mathcal{V})$$

- Constrained dynamics

$$\mathcal{F} = \Lambda(\theta)\dot{\mathcal{V}} + \eta(\theta, \mathcal{V}) + \underbrace{A^{\mathrm{T}}(\theta)\lambda}_{\mathcal{F}_{\mathrm{tip}}}$$

Lagrange multipliers

$$\lambda \in \mathbb{R}^k$$

A Hybrid Motion-Force Controller

- The nature constraints should be satisfied at all times $A(\theta)\mathcal{V} = 0$

Take derivative $A(\theta)\dot{\mathcal{V}} + \dot{A}(\theta)\mathcal{V} = 0$

$$\mathcal{F} = \Lambda(\theta)\dot{\mathcal{V}} + \eta(\theta, \mathcal{V}) + \underbrace{A^{\mathrm{T}}(\theta)\lambda}_{\mathcal{F}_{\mathrm{tip}}}$$

$$\lambda = (A\Lambda^{-1}A^{\mathrm{T}})^{-1}(A\Lambda^{-1}(\mathcal{F} - \eta) - A\dot{\mathcal{V}})$$ $$\mathcal{F}_{\mathrm{tip}} = A^{\mathrm{T}}(\theta)\lambda$$

Constrained dynamics

$$P(\theta)\mathcal{F} = P(\theta)(\Lambda(\theta)\dot{\mathcal{V}} + \eta(\theta, \mathcal{V}))$$

$$P = I - A^{\mathrm{T}}(A\Lambda^{-1}A^{\mathrm{T}})^{-1}A\Lambda^{-1}$$

Rank n – k, move the end-effector tangent to the constraints

A Hybrid Motion-Force Controller

$$\tau = J_b^{\mathrm{T}}(\theta)\Bigg(\underbrace{P(\theta)\left(\tilde{\Lambda}(\theta)\left(\frac{d}{dt}([\mathrm{Ad}_{X^{-1}X_d}]\mathcal{V}_d) + K_p X_e + K_i \int X_e(\mathrm{t})d\mathrm{t} + K_d \mathcal{V}_e\right)\right)}_{\text{motion control}}$$

$$+ \underbrace{(I - P(\theta))\left(\mathcal{F}_d + K_{fp}\mathcal{F}_e + K_{fi}\int \mathcal{F}_e(\mathrm{t})d\mathrm{t}\right)}_{\text{force control}}$$

$$+ \underbrace{\tilde{\eta}(\theta, \mathcal{V}_b)}_{\text{Coriolis and gravity}}\Bigg). \quad (11.61)$$

$$P = I - A^{\mathrm{T}}(A\Lambda^{-1}A^{\mathrm{T}})^{-1}A\Lambda^{-1}$$

Impedance Control

- Robot impedance characterizes the change in endpoint motion as a function of disturbance forces.

- Ideal motion control
  - High impedance, little change in motion due to force disturbances

- Ideal force control
  - Low impedance, little change in force due to motion disturbances

Impedance Control

- Impedance control is an approach to dynamic control relating force and position

- The robot end-effector is asked to render particular mass, spring, and damper properties

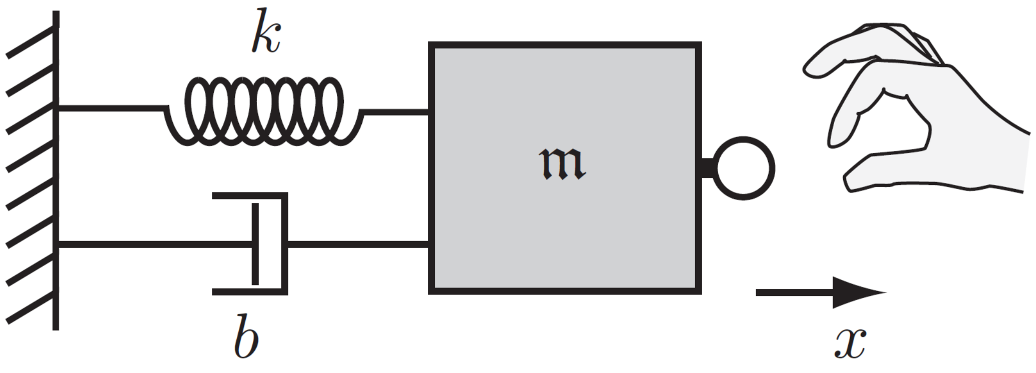
Impedance Control

- The dynamics for a one dof robot rendering an impedance

$$\mathfrak{m}\ddot{x} + b\dot{x} + kx = f$$

mass — damping — stiffness — force

High impedance: b or k is large



Impedance Control

- Goal: implement the task-space behavior

$$M\ddot{x} + B\dot{x} + Kx = f_{\text{ext}}$$

$x \in \mathbb{R}^n$ Task-space configuration in a minimum set of coordinates

$M, B, \text{ and } K$ Positive-definite virtual mass, damping, and stiffness matrices

f_{ext} Force applied to the robot

Impedance Control

$$M\ddot{x} + B\dot{x} + Kx = f_{\text{ext}}$$

- Impedance controlled
  - The robot senses the endpoint motion $x(t)$
  - Commands joint torques and forces to create $-f_{\text{ext}}$
  - Displays the force to the user

- Admittance controlled
  - The robot senses f_{ext}
  - Controls its motion in response

Impedance Control

- Impedance-Control Algorithm

$$M\ddot{x} + B\dot{x} + Kx = f_{\text{ext}}$$

$$\tau = J^{\mathrm{T}}(\theta)\left(\underbrace{\tilde{\Lambda}(\theta)\ddot{x} + \tilde{\eta}(\theta,\dot{x})}_{\text{arm dynamics compensation}} - \underbrace{(M\ddot{x} + B\dot{x} + Kx)}_{f_{\text{ext}}}\right)$$

- Admittance-Control Algorithm

$$\ddot{x}_d = M^{-1}(f_{\text{ext}} - B\dot{x} - Kx)$$

$$\dot{x} = J(\theta)\dot{\theta}$$

$$\ddot{\theta}_d = J^{\dagger}(\theta)(\ddot{x}_d - \dot{J}(\theta)\dot{\theta})$$

Use inverse dynamics used to calculate the commanded joint forces and torques

Summary

- Force control
- Hybrid motion-force control
- Impedance control

Further Reading

- Chapter 11 in Kevin M. Lynch and Frank C. Park. Modern Robotics: Mechanics, Planning, and Control. 1st Edition, 2017.