Robot Control: Motion Control with Velocities

CS 6301 Special Topics: Introduction to Robot Manipulation and Navigation

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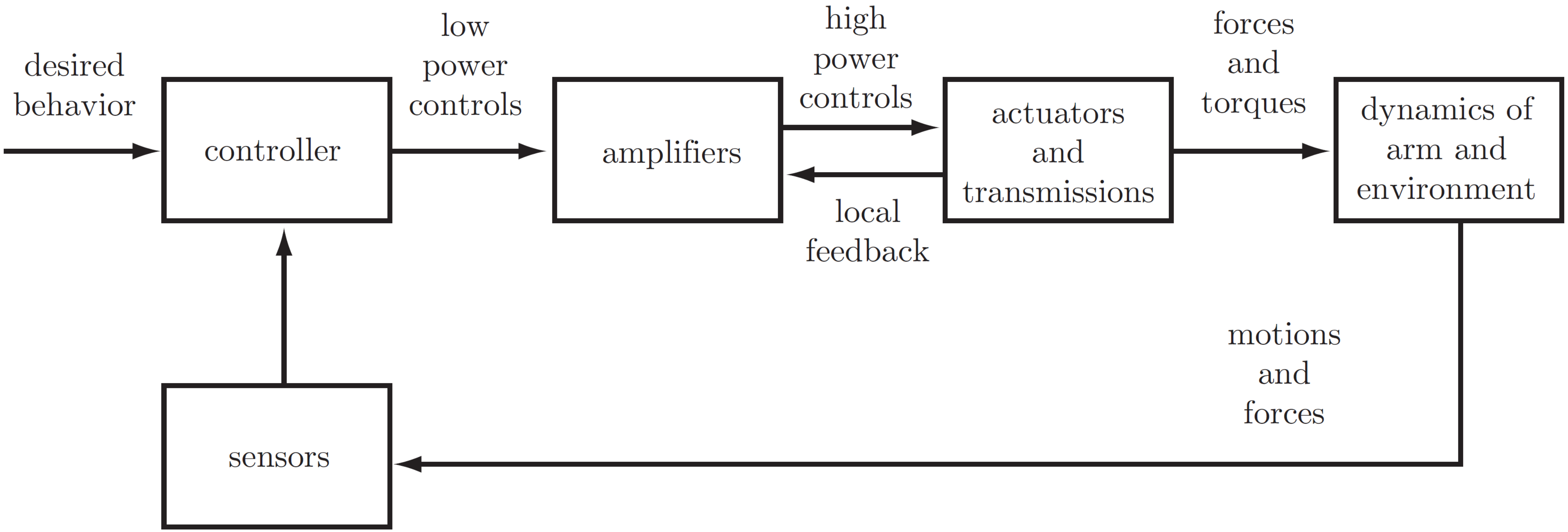
Robot Control

- Convert task specifications to force and torques at the actuators

- Types
  - Motion control
  - Force control
  - Hybrid motion-force control
  - Impedance control

- Feedback control
  - Use sensors for position, velocity and force
  - Compare with the desired behavior to compute the control signals

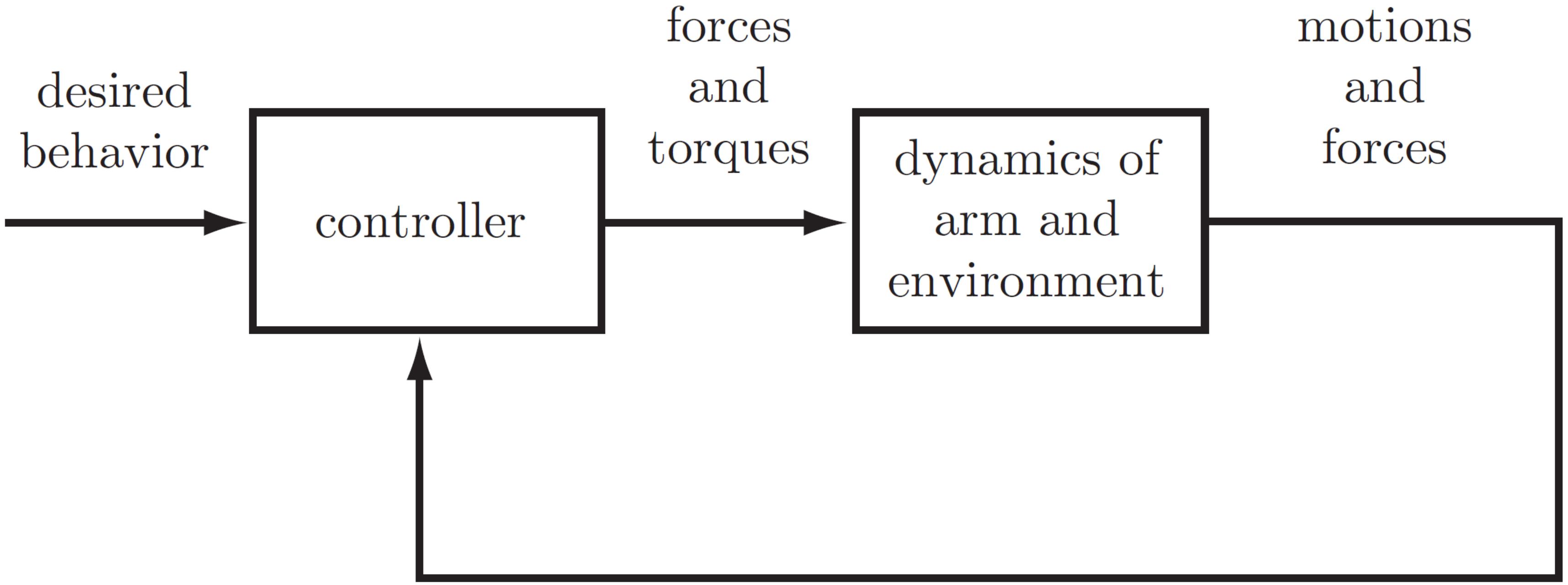
Control System Overview



- Potentiometers, encoders, or resolvers for joint position and angle sensing
- Tachometers for joint velocity sensing
- Joint force-torque sensors
- Multi-axis force-torque sensors at the “wrist" between the end of the arm and the end-effector

Control System Overview

- A simplified system



desired behavior

controller

forces and torques

dynamics of arm and environment

motions and forces

Controlled Dynamics of a Single Joint

- Desired joint position $\theta_d(t)$
- The current joint position $\theta(t)$
- Joint error $\theta_e(t) = \theta_d(t) - \theta(t)$

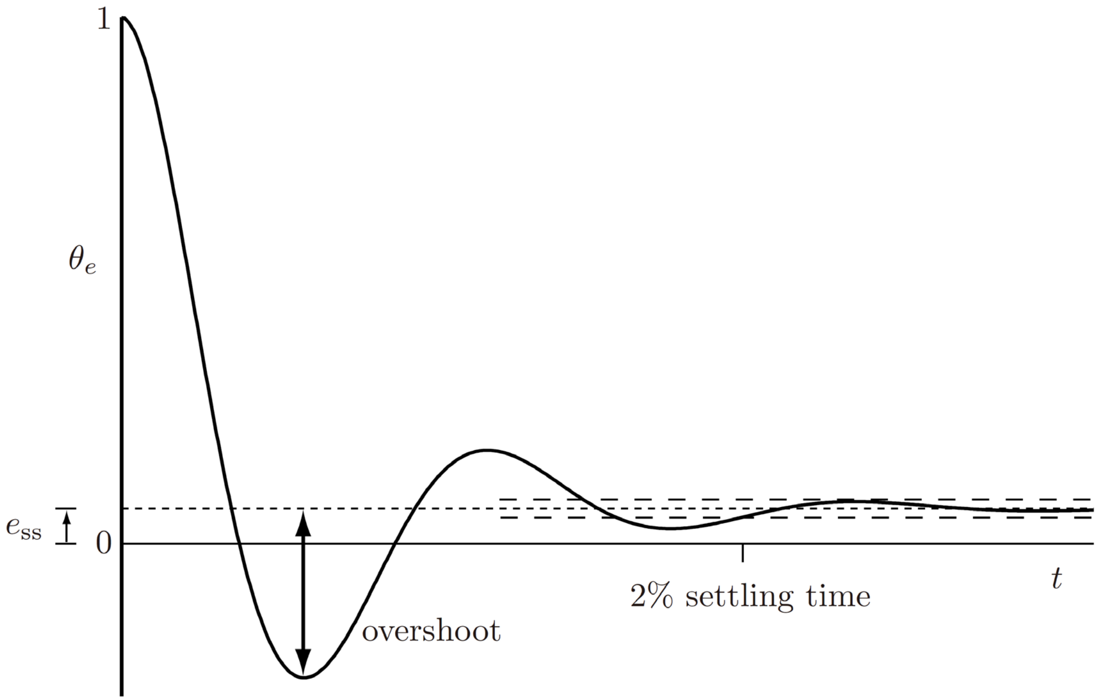
- Error dynamics: the differential equation governing the evolution of the joint error

- Feedback controller: create an error dynamics to make $\theta_e(t)$ become zero or a small value when t increases

Error Response

- How well a controller works?
  - Specify a nonzero initial error $\theta_e(0)$ and see how the controller reduces the error

- Error response $\theta_e(t), t > 0$
  - Initial conditions $\theta_e(0) = 1$ $\dot{\theta}_e(0) = \ddot{\theta}_e(0) = \cdots = 0$
  - Steady-state error $\theta_e(t)$ as $t \rightarrow \infty$

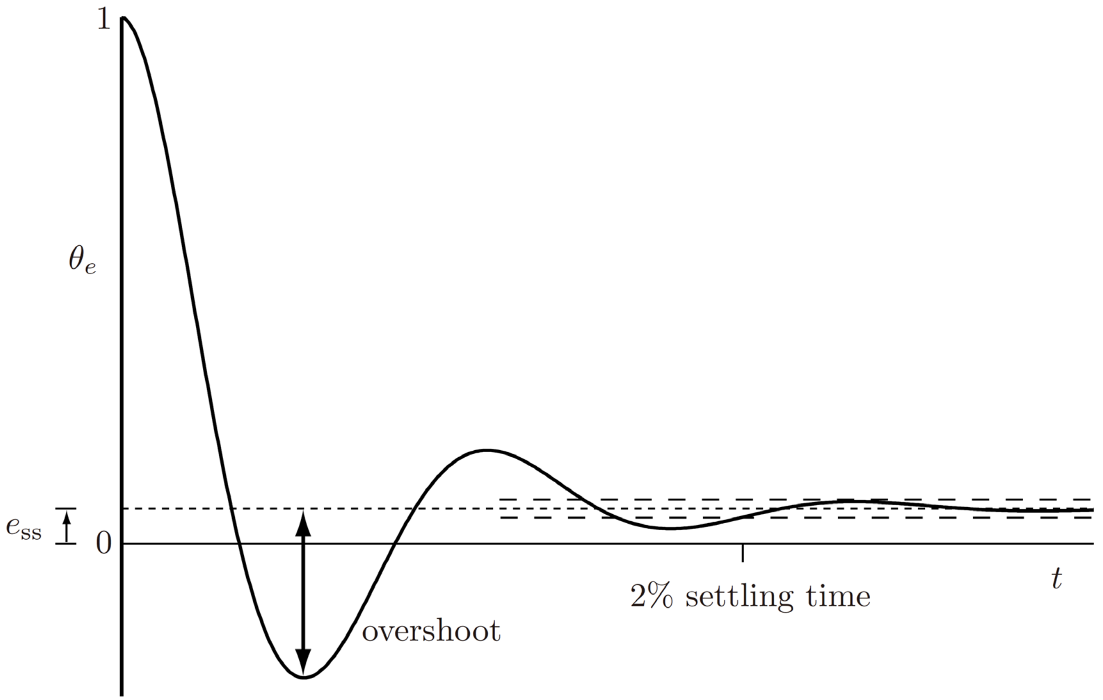


Error Response

- (2%) Settling time: first time T such that $|\theta_e(t) - e_{\mathrm{ss}}| \leq 0.02(\theta_e(0) - e_{\mathrm{ss}})$ for all $t \geq T$

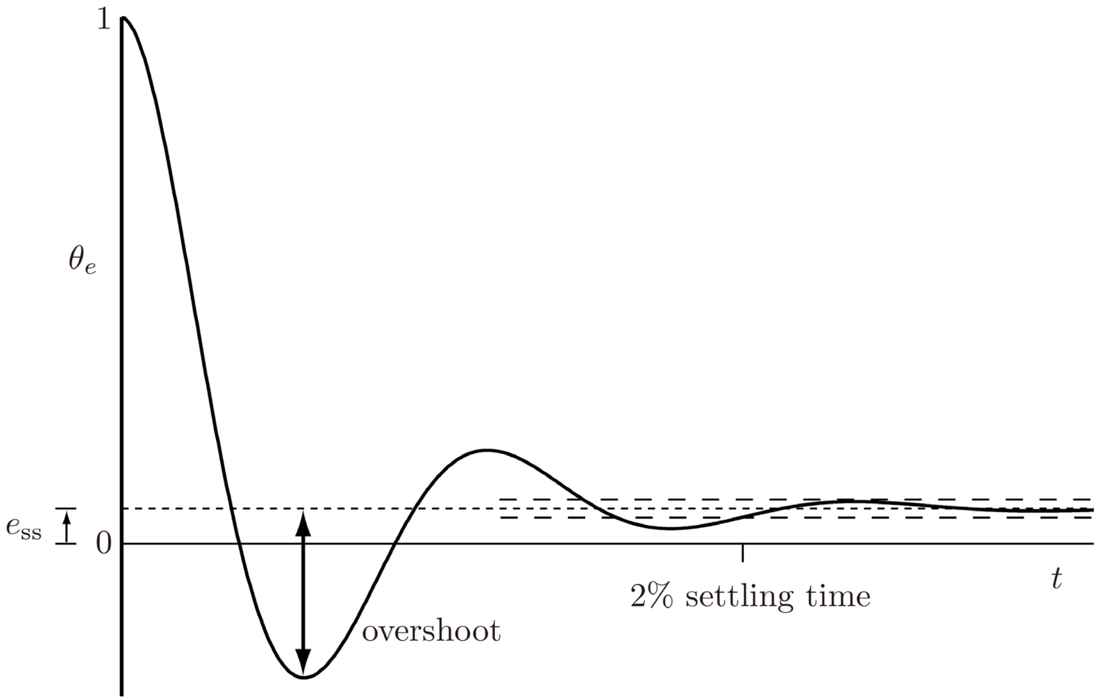
- Overshoot

$$\text{overshoot} = \left| \frac{\theta_{e,\min} - e_{\mathrm{ss}}}{\theta_e(0) - e_{\mathrm{ss}}} \right| \times 100\%$$



Error Response

- A good error response
  - Little or no steady-state error
  - Little or no overshoot
  - A short 2% settling time



Motion Control with Velocity Inputs

- Typically, we assume direct control of the forces or torques at robot joints

- In some cases, we can assume that there is direct control of the joint velocities
  - The velocity of a joint is determined directly by the frequency of the pulse train sent to the stepper motor

- Motion control with velocity inputs
  - Given a desired trajectory of a robot in joint space or in task space

$$\theta_d(t) \qquad X_d(t)$$

Motion Control of a Single Joint

- Feedforward control or open-loop control
  - Given a desired joint trajectory $\theta_d(t)$
  - Choose the velocity command $\dot{\theta}(t) = \dot{\theta}_d(t)$
  - Cons: accumulating position errors

- Feedback control
  - Measure the joint position continuously for feedback

Motion Control of a Single Joint

- Proportional controller or P controller

$$\dot{\theta}(t) = K_p(\theta_d(t) - \theta(t)) = K_p\theta_e(t)$$

Control gain

$$K_p > 0$$

- When $\theta_d(t)$ is a constant $\dot{\theta}_d(t) = 0$

Setpoint control

  - Error dynamics

$$\dot{\theta}_e(t) = \cancelto{0}{\dot{\theta}_d(t)} - \dot{\theta}(t)$$

$$\dot{\theta}_e(t) = -K_p\theta_e(t) \quad \rightarrow \quad \dot{\theta}_e(t) + K_p\theta_e(t) = 0$$

First-Order Error Dynamics

$$\dot{\theta}_e(t) + \frac{1}{\mathfrak{t}}\theta_e(t) = 0$$

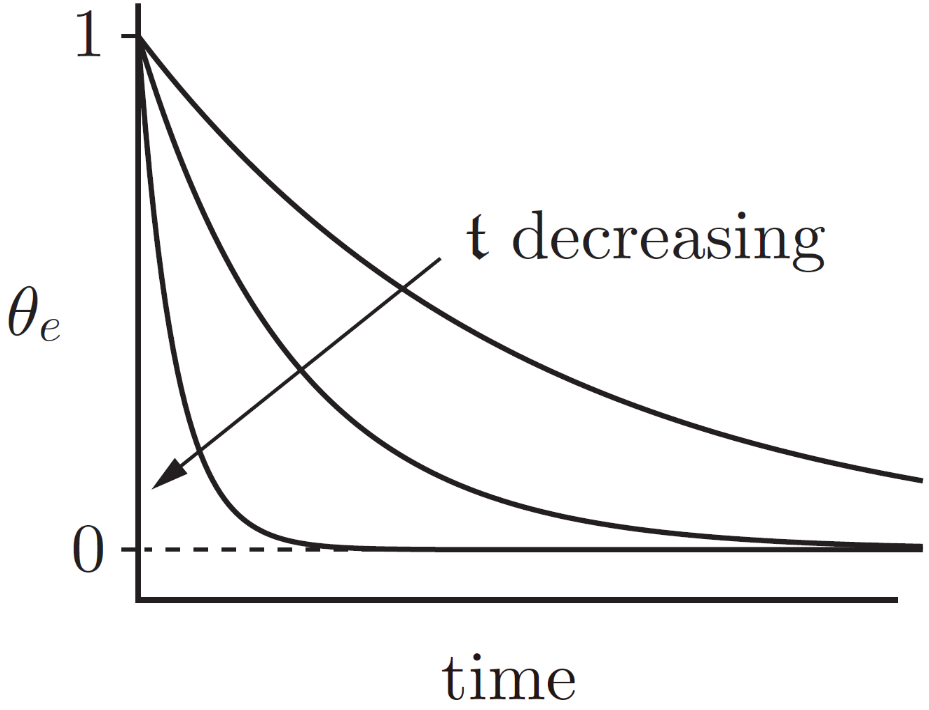
time constant $\mathfrak{t}$

Solution $\theta_e(t) = e^{-t/\mathfrak{t}}\theta_e(0)$

Setpoint control

$$\mathfrak{t} = 1/K_p$$

- 0 steady state error
- No overshoot
- 2% settling time $4/K_p$



P Controller

$$\dot{\theta}(t) = K_p(\theta_d(t) - \theta(t)) = K_p \theta_e(t)$$

- When $\theta_d(t)$ is not constant but $\dot{\theta}_d(t)$ is constant $\dot{\theta}_d(t) = c$
- Error dynamics

$$\dot{\theta}_e(t) = \dot{\theta}_d(t) - \dot{\theta}(t) = c - K_p \theta_e(t)$$

Solution

$$\theta_e(t) = \frac{c}{K_p} + \left(\theta_e(0) - \frac{c}{K_p}\right) e^{-K_p t} \longrightarrow c/K_p$$

steady-state error

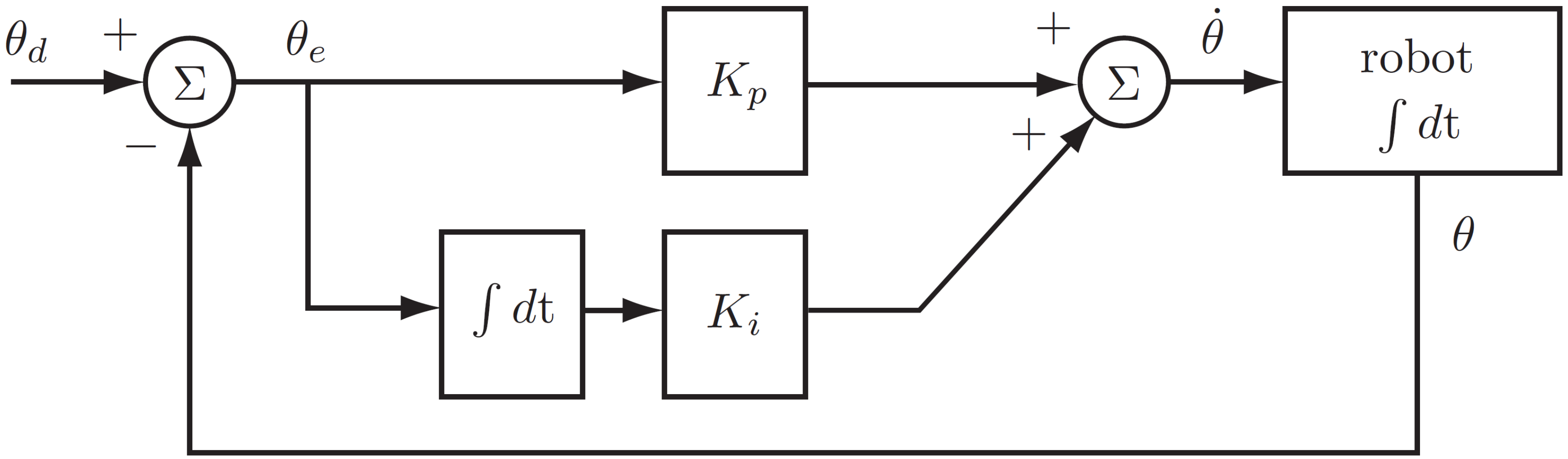
We cannot make K_p arbitrarily large

PI Controller

- A proportional-integral controller

$$\dot{\theta}(t) = K_p\theta_e(t) + K_i \int_0^t \theta_e(\mathrm{t})\ dt$$

Time-integral of the error



PI Controller

- Error dynamics for a constant $\dot{\theta}_d(t) = c$ $\dot{\theta}(t) = K_p \theta_e(t) + K_i \int_0^t \theta_e(\mathrm{t})\, dt$

$$\dot{\theta}_e(t) = \dot{\theta}_d(t) - \dot{\theta}(t)$$

$$\dot{\theta}_e(t) + K_p \theta_e(t) + K_i \int_0^t \theta_e(\mathrm{t})\, d\mathrm{t} = c$$

$$\ddot{\theta}_e(t) + K_p \dot{\theta}_e(t) + K_i \theta_e(t) = 0$$

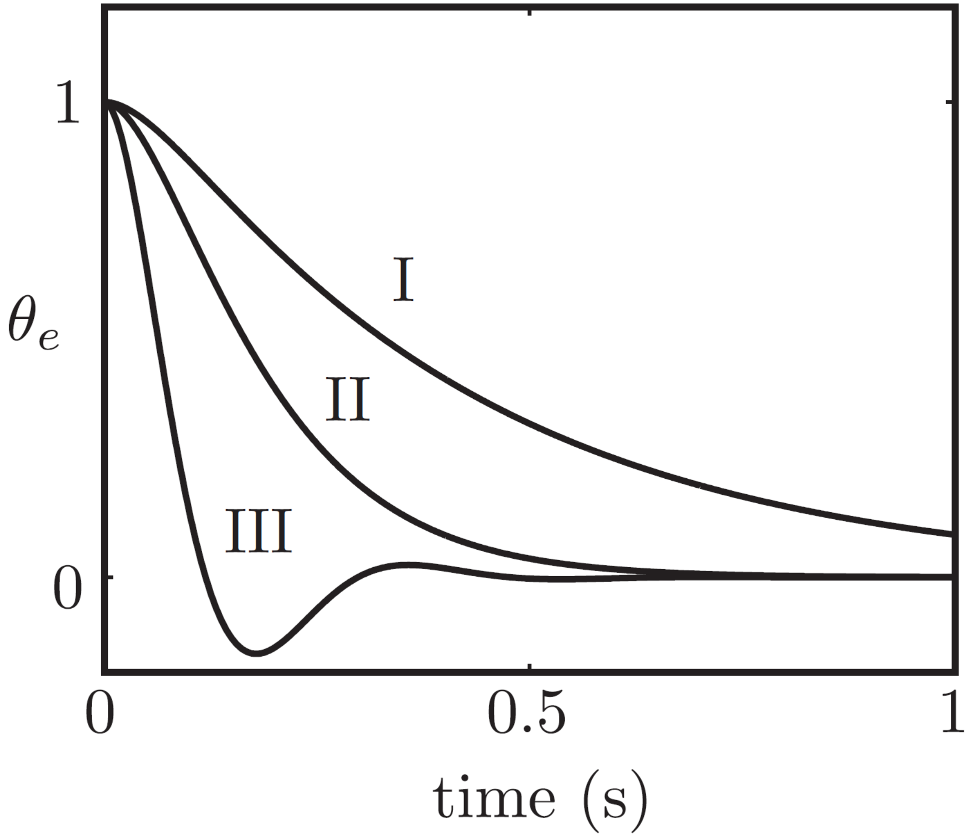
Second-Order Error Dynamics

Standard second-order form $\ddot{\theta}_e(t) + 2\zeta\omega_n \dot{\theta}_e(t) + \omega_n^2 \theta_e(t) = 0$

natural frequency ω_n damping ratio ζ $\omega_n = \sqrt{K_i}$ $\zeta = K_p/(2\sqrt{K_i})$

PI Controller

Error response

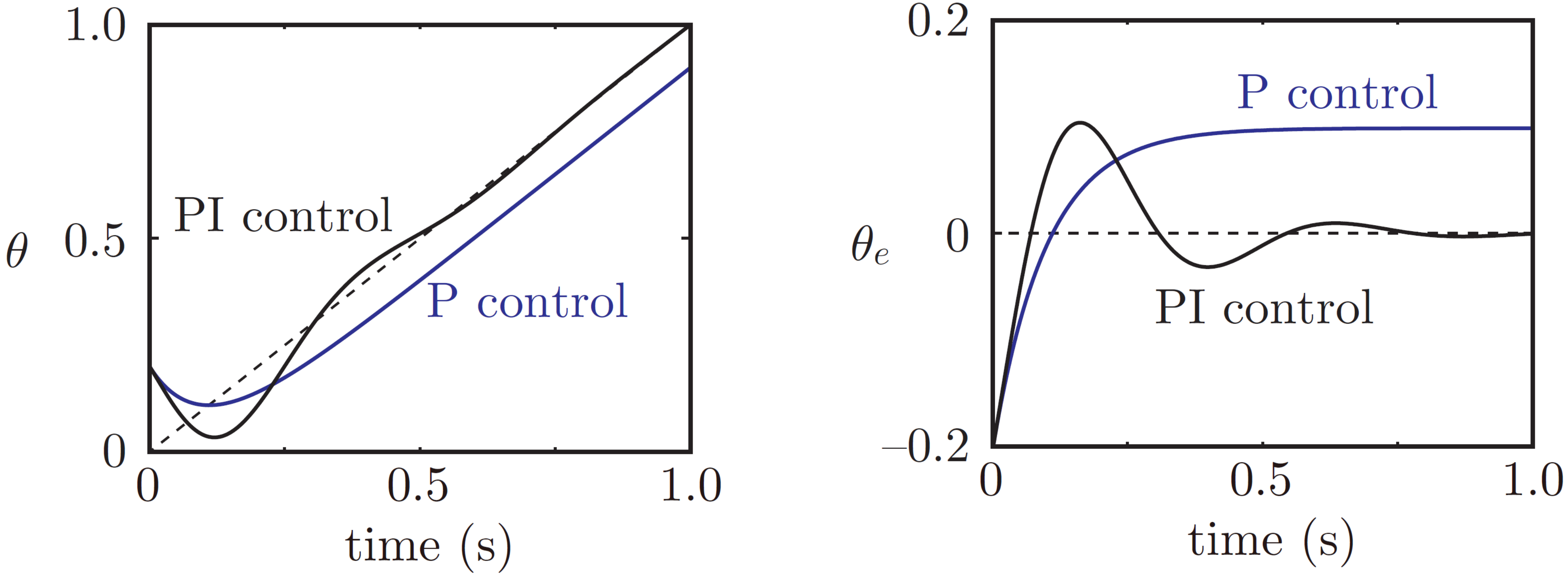


$$K_p = 20 \qquad \zeta = K_p/(2\sqrt{K_i})$$

- Overdamped $\zeta = 1.5,\ K_i = 44.4,\ \text{case I}$
- Critically damped $\zeta = 1,\ K_i = 100,\ \text{case II}$
- Underdamped $\zeta = 0.5,\ K_i = 400,\ \text{case III}$

Comparison between P Controller and PI Controller

$$\dot{\theta}_d(t) = c$$

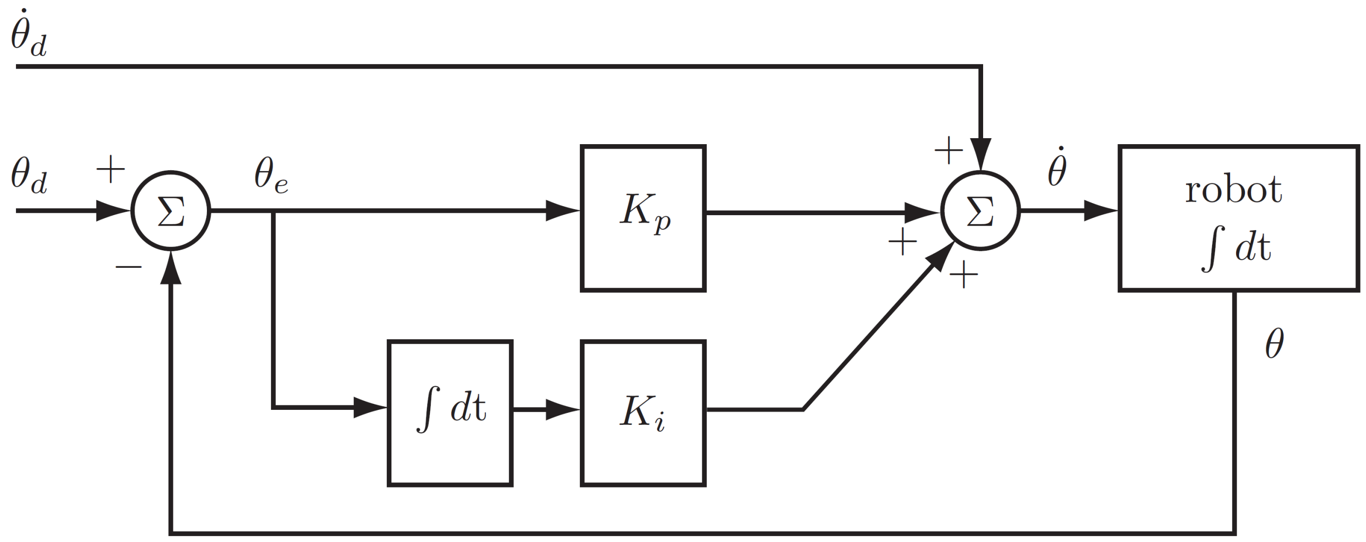


Reference trajectory (dashed)

Feedforward Plus Feedback Control

- Feedback control: an error is required before the joint begins to move
- Feedforward plus feedback control: Initiate motion between any error accumulates

$$\dot{\theta}(t) = \dot{\theta}_d(t) + K_p\theta_e(t) + K_i \int_0^t \theta_e(\mathrm{t})\ d\mathrm{t}$$



Preferred control law for producing a commanded velocity to the joint

Motion Control of Multi-Joint Robots

- Reference position $\theta_d(t)$ and actual position $\theta(t)$ n dimensional vector
- Gains K_p K_i $n \times n$ matrix

$$k_p I \qquad k_i I$$

Control law $$\dot{\theta}(t) = \dot{\theta}_d(t) + K_p \theta_e(t) + K_i \int_0^t \theta_e(\mathrm{t})\, dt$$

Task-Space Motion Control

- Consider the configuration of the end-effector $X(t) \in SE(3)$
- End-effector twist $\mathcal{V}_b(t) \quad [\mathcal{V}_b] = X^{-1}\dot{X}$
- Desired motion is given by $X_d(t) \quad [\mathcal{V}_d] = X_d^{-1}\dot{X}_d$

- A task-space control law

$$\mathcal{V}_b(t) = [\mathrm{Ad}_{X^{-1}X_d}]\mathcal{V}_d(t) + K_p X_e(t) + K_i \int_0^t X_e(\mathrm{t})\, dt$$

$$\dot{\theta} = J_b^{\dagger}(\theta)\mathcal{V}_b$$

$$[X_e] = \log(X^{-1}X_d) \qquad K_p, K_i \in \mathbb{R}^{6\times 6} \qquad X_{sb} \quad X_{sd}$$

Summary

- Robot control
    - Error dynamics

- Motion control
    - P controller
    - PI controller
    - Feedforward plus feedback controller

- Task-space motion control

Further Reading

- Chapter 11 in Kevin M. Lynch and Frank C. Park. Modern Robotics: Mechanics, Planning, and Control. 1st Edition, 2017.