Configuration Space

CS 6301 Special Topics: Introduction to Robot Manipulation and Navigation
Professor Yu Xiang
The University of Texas at Dallas
Robotics

What is the common phenomenon in these robots? Motion
Robot Mechanisms

• Links and Joints

Franka Emika
Robot Mechanisms

• Links and Joints

Fetch Mobile Manipulator
Robot Mechanisms

• Links and Joints

Robot Joints

• Every joint connects exactly two links

• Revolute joint (R)
  • Hinge joint
  • Allows rotation motion about the joint axis
Robot Joints

- **Prismatic Joint (P)**
  - Sliding joint or linear joint
  - Allows translational motion along the direction of the joint axis

- **Helical Joint (H)**
  - Screw joint
  - Allows rotation and translational motion about a screw axis
Robot Joints

• Cylindrical joint (C)
  • Allows independent translations and rotations about a single fixed joint axis

• Universal joint (U)
  • A pair of revolute joints with orthogonal joint axes
Robot Joints

- Spherical joint (S)
  - Ball-and-socket joint

https://youtu.be/kztZu3uTvYM
Robot Joints

• Every joint connects exactly two links
Degrees of Freedom

• Maximum number of logically independent values

• Specify the position of a rigid body
Degrees of Freedom of Robot Joints

- Revolute joint
  - 1 DOF

- Prismatic joint
  - 1 DOF

- Helical joint
  - 1 DOF
Degrees of Freedom of Robot Joints

• Cylindrical joint
  • 2 DOF

• Universal joint
  • 2 DOF

• Spherical joint
  • 3 DOF
## Degrees of Freedom of Robot Joints

<table>
<thead>
<tr>
<th>Joint type</th>
<th>dof $f$</th>
<th>Constraints $c$ between two planar rigid bodies</th>
<th>Constraints $c$ between two spatial rigid bodies</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revolute (R)</td>
<td>1</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>Prismatic (P)</td>
<td>1</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>Helical (H)</td>
<td>1</td>
<td>N/A</td>
<td>5</td>
</tr>
<tr>
<td>Cylindrical (C)</td>
<td>2</td>
<td>N/A</td>
<td>4</td>
</tr>
<tr>
<td>Universal (U)</td>
<td>2</td>
<td>N/A</td>
<td>4</td>
</tr>
<tr>
<td>Spherical (S)</td>
<td>3</td>
<td>N/A</td>
<td>3</td>
</tr>
</tbody>
</table>
Degrees of Freedom of a Robot

- 4 revolute joints
- 4 DOFs
- 7 revolute joints for the arm
- 7 DOFs
Configuration Space of a Robot

• The configuration of a robot is a complete specification of the position of every point of the robot.

• The minimum number $n$ of real-valued coordinates needed to represent the configuration is the number of degrees of freedom (DOF) of the robot.

• The $n$-dimensional space containing all possible configurations of the robot is called the configuration space (C-space).

• The configuration of a robot is represented by a point in its C-space.

• 4 revolute joints
• 4 DOFs
Configuration Space of a Robot

• The configuration space of the Fetch arm is a 7D space

• Each value in the 7D vector indicates the value of the revolute joint
Grübler’s Formula

• The number of degrees of freedom of a mechanism with links and joints can be calculated using Grübler's formula

\[ \text{degrees of freedom} = (\text{sum of freedoms of the bodies}) - (\text{number of independent constraints}) \]

• Consider the following setting
  • A robot with \( N \) links, \( J \) joints (consider ground as one link)
  • Each link has \( m \) DOF (planar link? spatial link?)
  • Number of freedoms by joint \( i \) \( f_i \)
  • Number of constraints by joint \( i \) \( c_i \)

\[ f_i + c_i = m \]
Grübler’s Formula

\[
dof = m(N - 1) - \sum_{i=1}^{J} c_i
\]

rigid body freedoms

\[
= m(N - 1) - \sum_{i=1}^{J} (m - f_i)
\]

joint constraints

\[
= m(N - 1 - J) + \sum_{i=1}^{J} f_i.
\]

Ground is regarded as a link

Assume all joint constraints are independent.
Open-Chain vs. Closed-Chain

• Open-chain mechanisms: without a closed loop
• Closed-chain mechanisms: with a closed loop

• Examples
  • A person standing with both feet

Stewart-Gough platform
Grübler’s Formula

• How many links?
  • 4 (one is ground)

• Each link has m DOF. What is m?
  • m=3

\[
\text{DOF} = m(N - 1 - J) + \sum_{i=1}^{J} f_i
\]

\[
= 3(4 - 1 - 4) + \sum_{i=1}^{4} 1
\]

The planar four-bar linkage
Grübler’s Formula

How many links?
- 4 (one is ground)

Each link has m DOF. What is m?
- m = 3

How many joints?
- 3 revolute joints, 1 prismatic joint

\[
\text{DOF} = m(N - 1 - J) + \sum_{i=1}^{J} f_i = 3(4 - 1 - 4) + \sum_{i=1}^{4} 1
\]
Configuration Space Topology

- Configuration specifies the position of a robot

- For a robot with n joints, the configuration is a vector in $\mathbb{R}^n$
  - C-space

- Joints may have limits, upper bound and lower bound

- Topology: shape of the space
  - Consider all the feasible points in the configuration space
Configuration Space Topology

• n-dimensional Euclidean space \( \mathbb{R}^n \)

• n-dimensional sphere in a \((n+1)\)-dimensional Euclidean space \( S^n \)
  • Two-dimensional surface of a sphere in three-dimensional space \( S^2 \)

• The C-space can have different representations, but its shape is the same
  • A point on a circle, angle \( \theta \), coordinates \((x, y)\) \( x^2 + y^2 = 1 \)
Configuration Space Topology

• C-space as Cartesian product
  • A rigid body in the plane \( \mathbb{R}^2 \times S^1 \)

• A PR robot (Prismatic-Revolute) \( \mathbb{R}^1 \times S^1 \)
  • Ignore joint limits

• A 2R robot \( S^1 \times S^1 = T^2 \)
  n-dimensional surface of a torus in an (n+1)-dimensional space

sample representation
Configuration Space Topology

• C-space of a planar rigid body with a 2R robot arm

\[ \mathbb{R}^2 \times S^1 \times T^2 = \mathbb{R}^2 \times T^3 \]

• C-space of a rigid body in 3D space
  • 3D translation
  • 3D rotation

\[ \mathbb{R}^3 \times S^2 \times S^1 \]
Configuration Space Representation

- Explicit parameterization
  - Use n coordinates for n-dimensional space

- A sphere: latitude-longitude
  - Singularities at North Pole and South Pole
  - Problem with the representation, not the topology
  - Infinity velocity problem $\sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}$

- Deal with singularities
  - Use more than one coordinate chart (each covers a portion of the space)
  - Implicit representations
    - Sphere \((x, y, z)\) \(x^2 + y^2 + z^2 = 1\)
    - Rotation matrix for 3D rotations

More numbers than DOF
Summary

• Robot links and joints

• Degrees of freedom of joints and robots

• Grübler’s Formula

• Configuration space
Further Reading

  http://hades.mech.northwestern.edu/images/7/7f/MR.pdf

  http://people.csail.mit.edu/tlp/