# Configuration Space

CS 6301 Special Topics: Introduction to Robot Manipulation and Navigation

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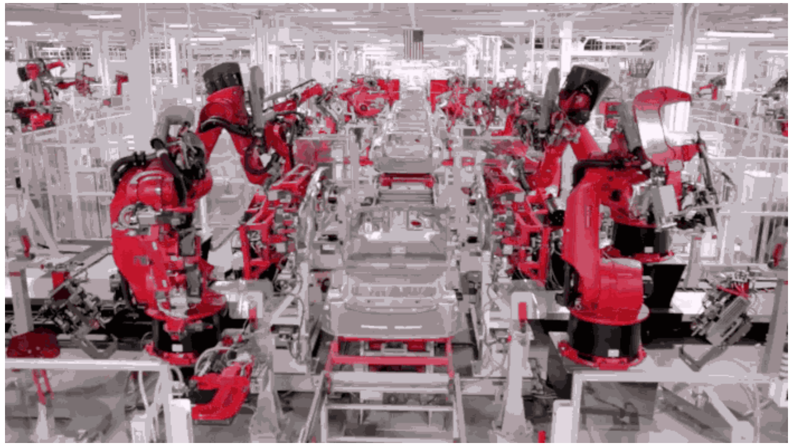
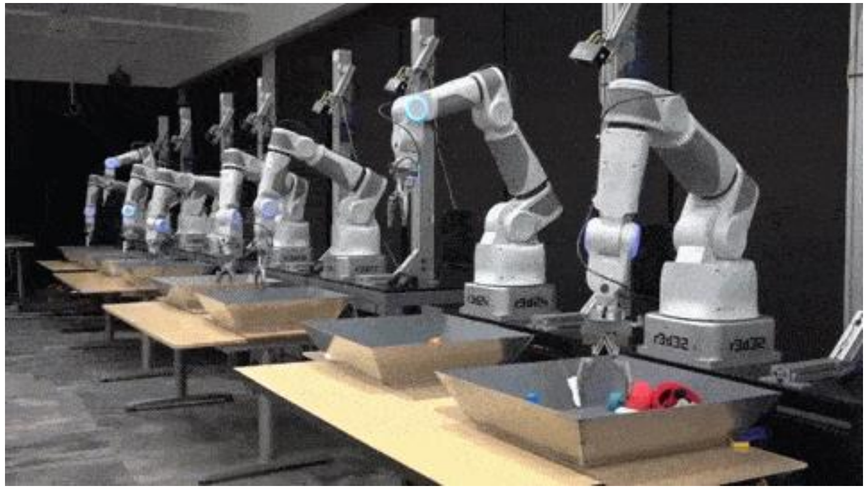
The University of Texas at Dallas

# Robotics

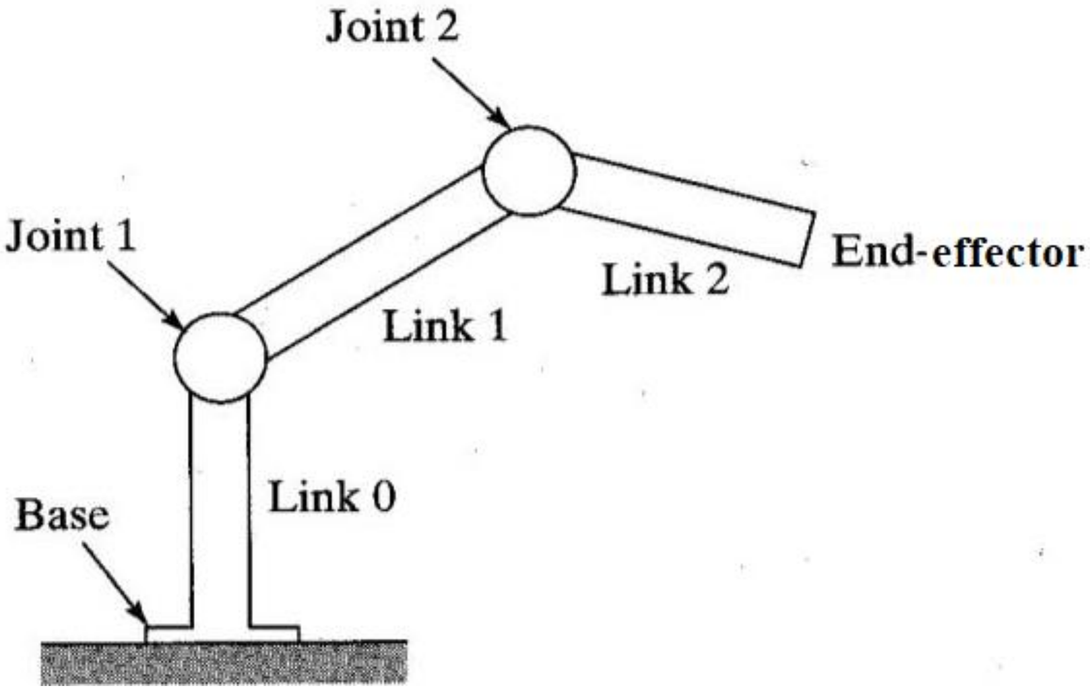


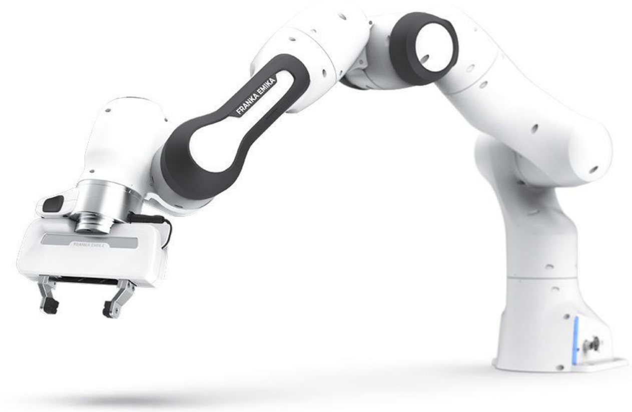



What is the common phenomenon in these robots? Motion

# Robot Mechanisms

- Links and Joints

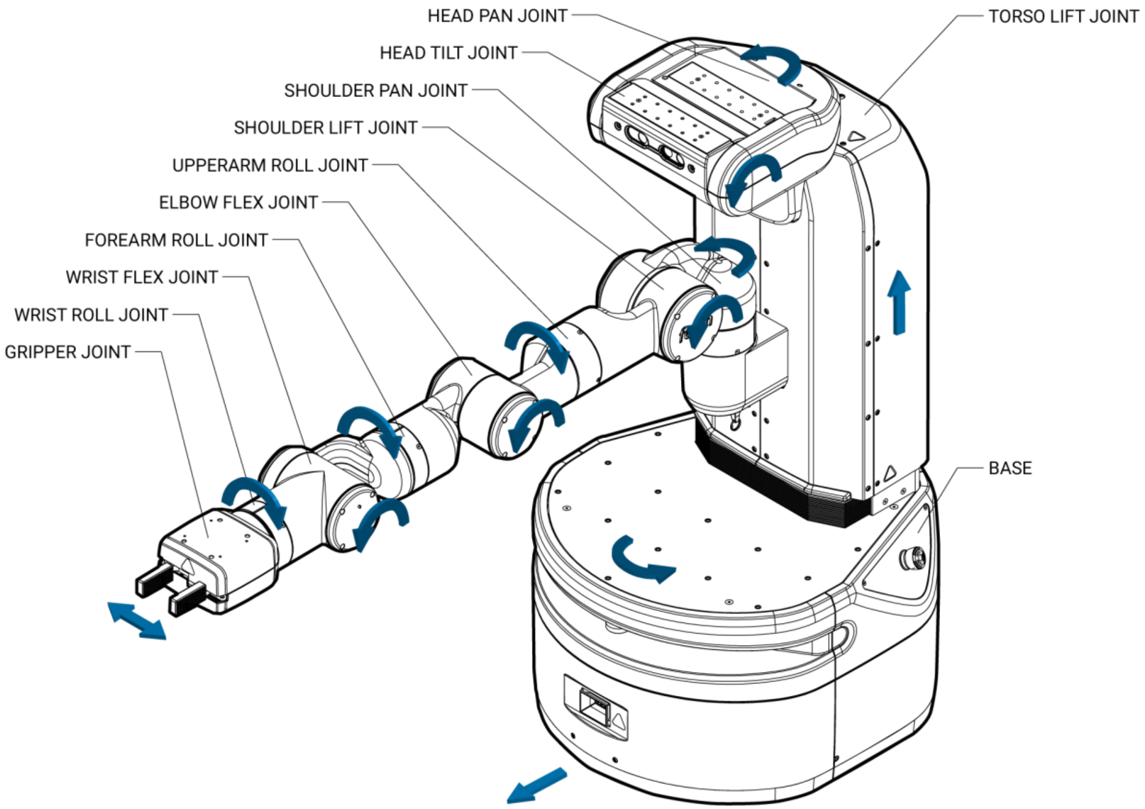


Franka Emika

# Robot Mechanisms

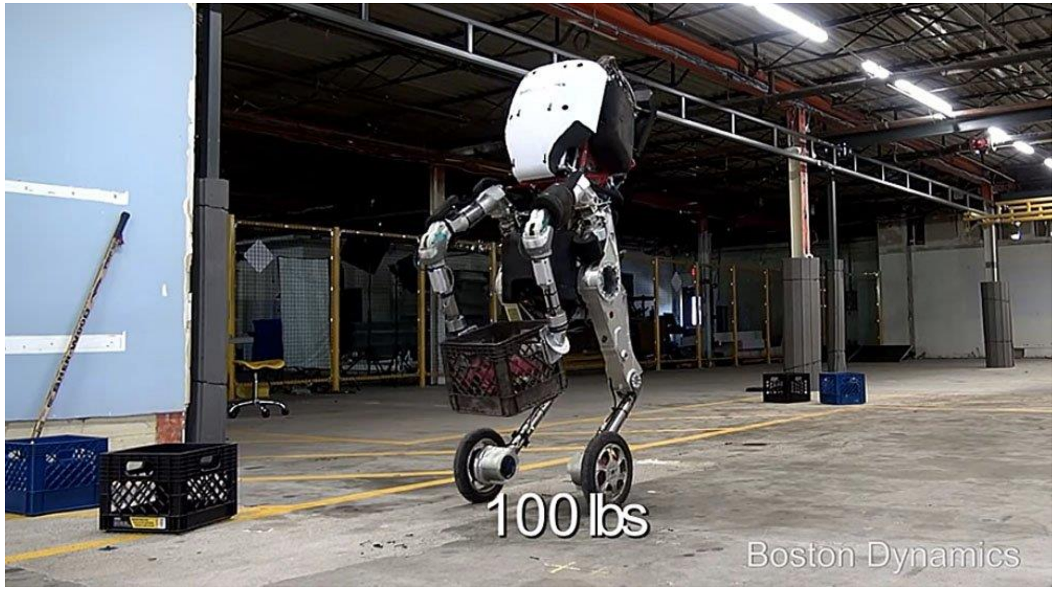
- Links and Joints

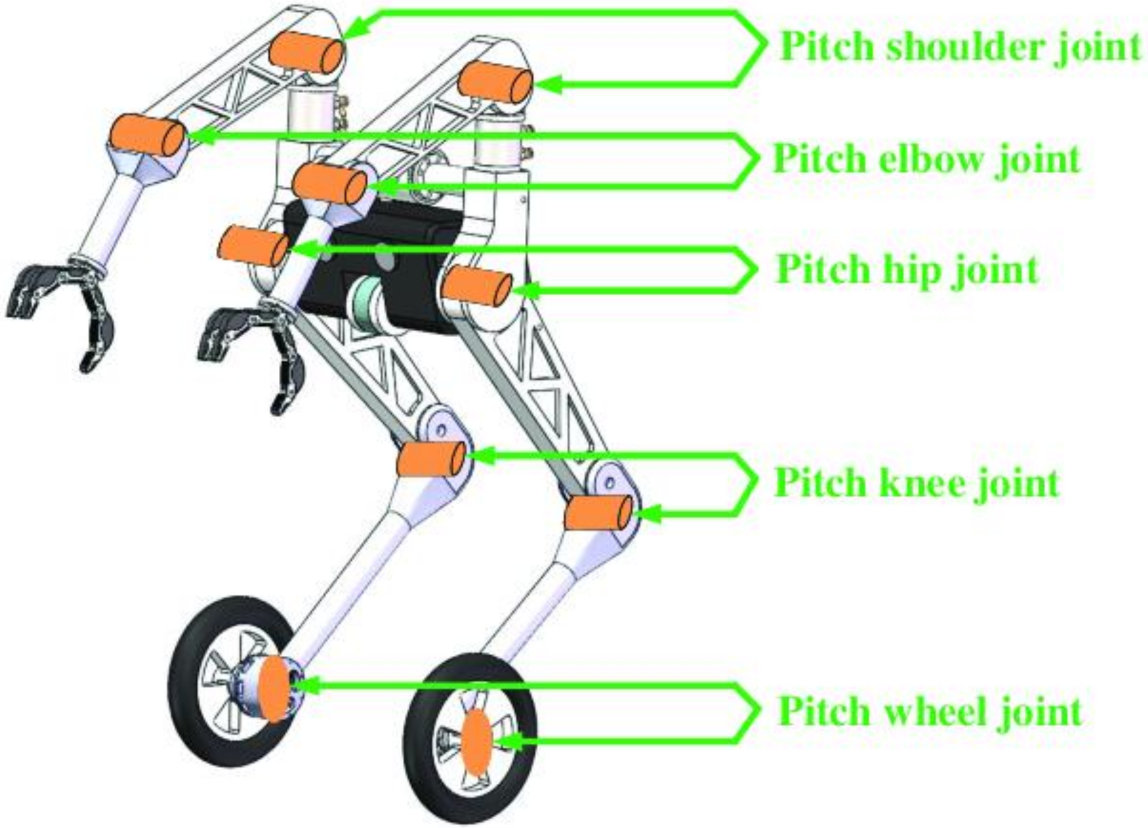




Fetch Mobile Manipulator

# Robot Mechanisms

- Links and Joints

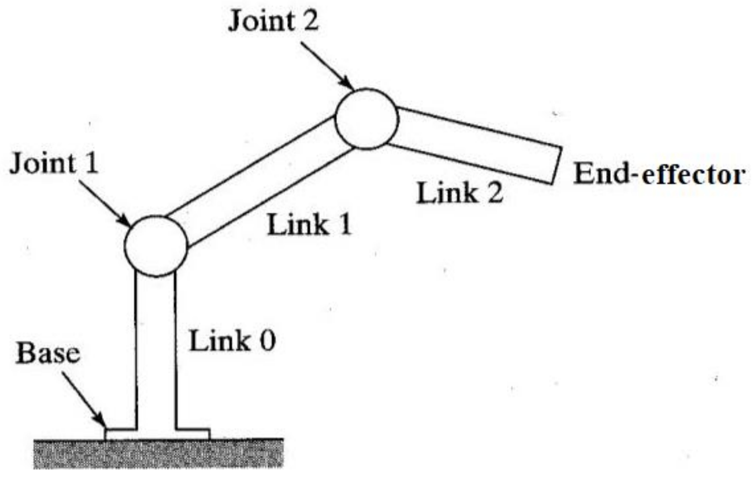


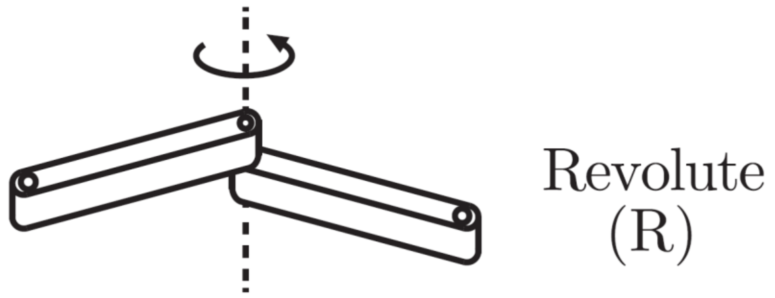

https://thenewstack.io/boston-dynamics-agile-wheel-legged-humanoid-robot-performs-incredible-stunts/

# Robot Joints

- Every joint connects exactly two links

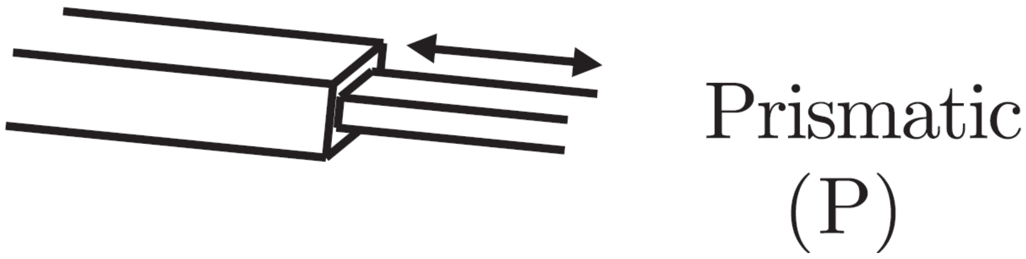



- Revolute joint (R)
  - Hinge joint
  - Allows rotation motion about the joint axis

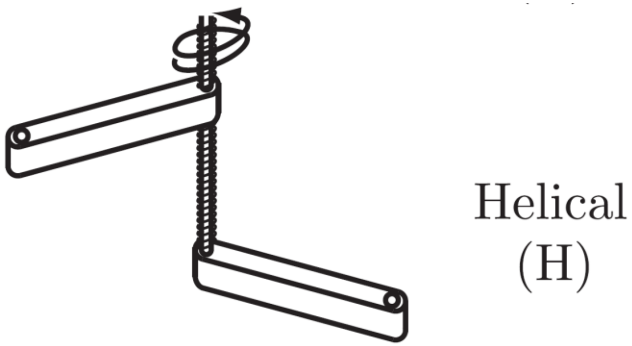


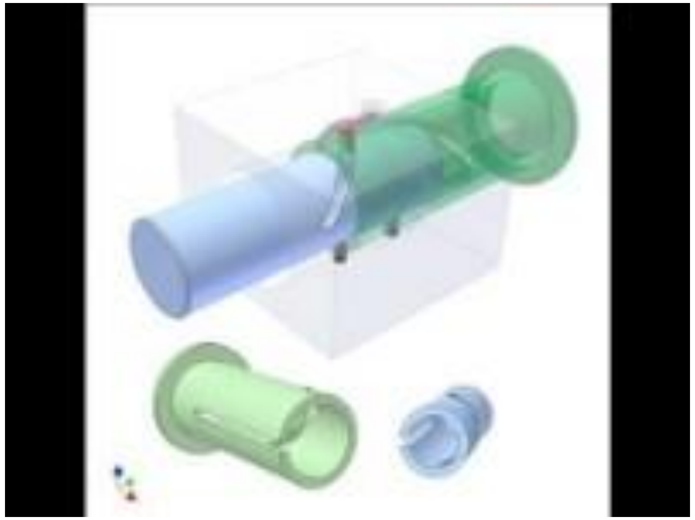
# Robot Joints

- Prismatic Joint (P)
  - Sliding joint or linear joint
  - Allows translational motion along the direction of the joint axis



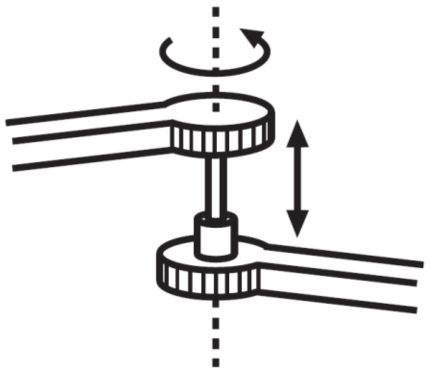

- Helical Joint (H)
  - Screw joint
  - Allows rotation and translatio about a screw axis



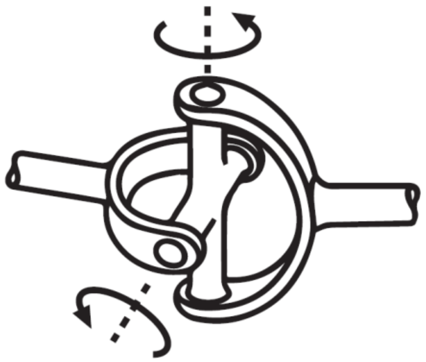
# Robot Joints

- Cylindrical joint (C)
  - Allows independent translations and rotations about a single fixed joint axis



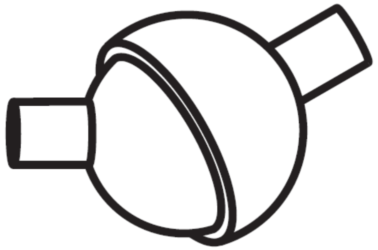


- Universal joint (U)
  - A pair of revolute joints with orthogonal joint axes



# Robot Joints

- Spherical joint (S)
  - Ball-and-socket joint

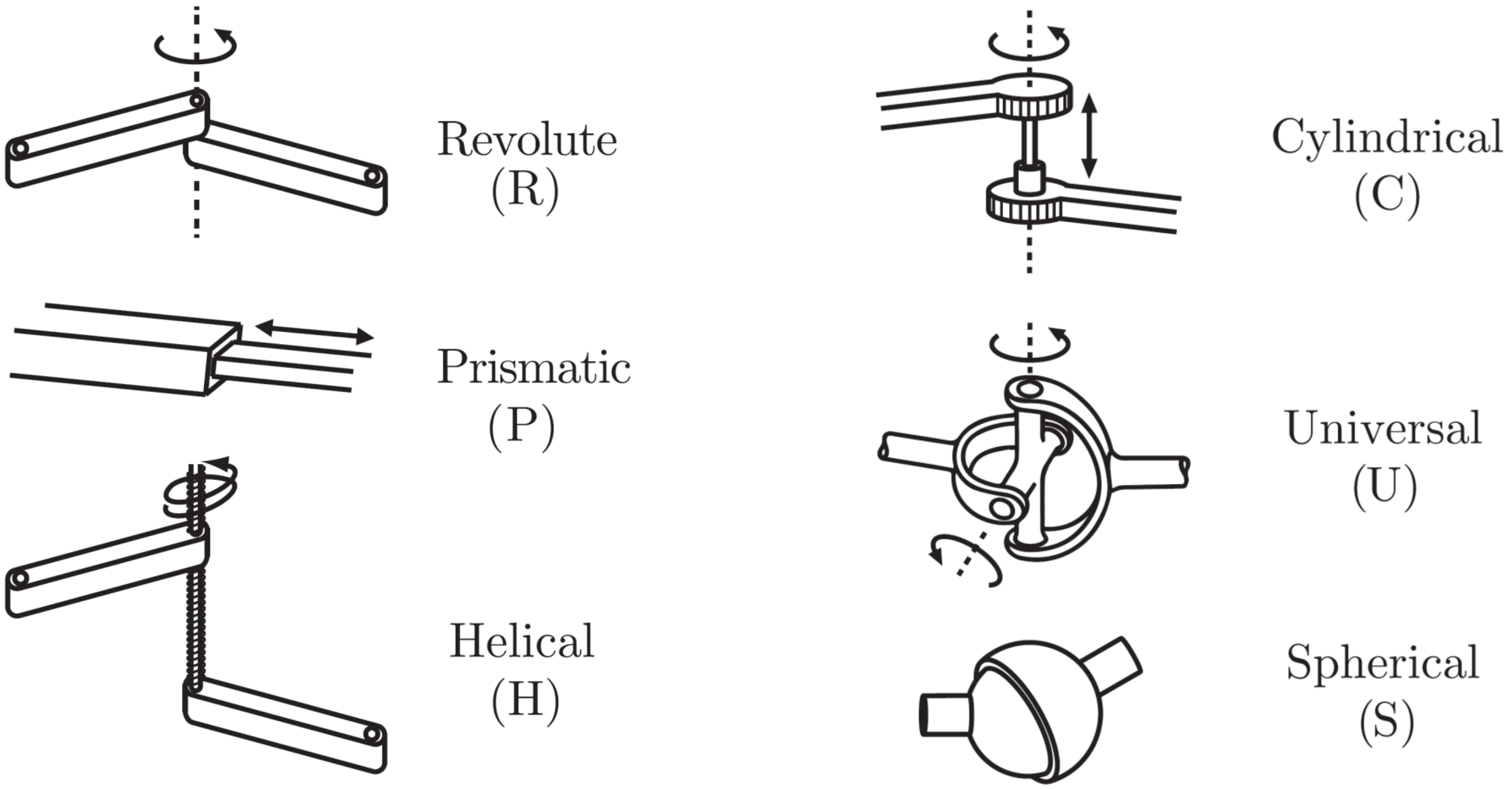


Spherical
(S)

https://youtu.be/kztZu3uTyvM

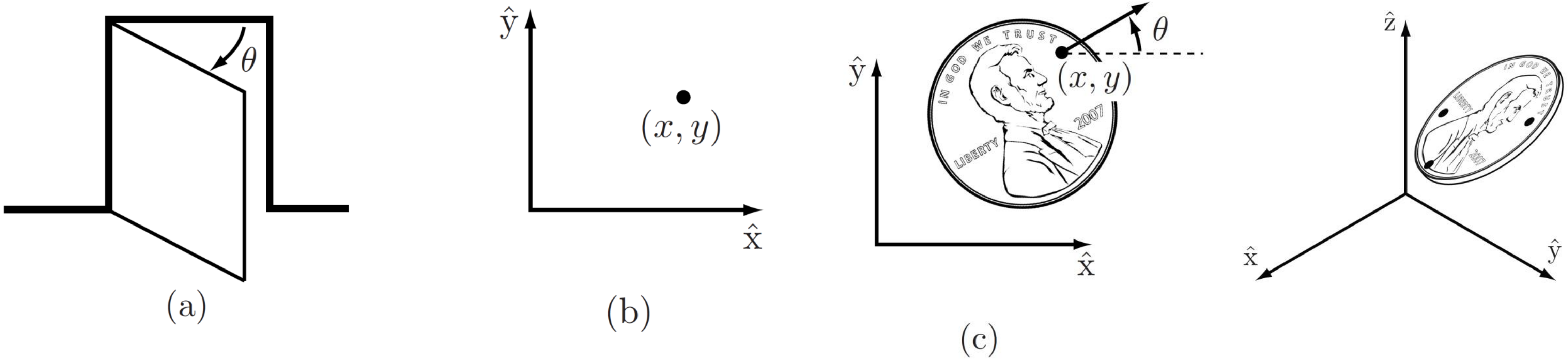
# Robot Joints

- Every joint connects exactly two links



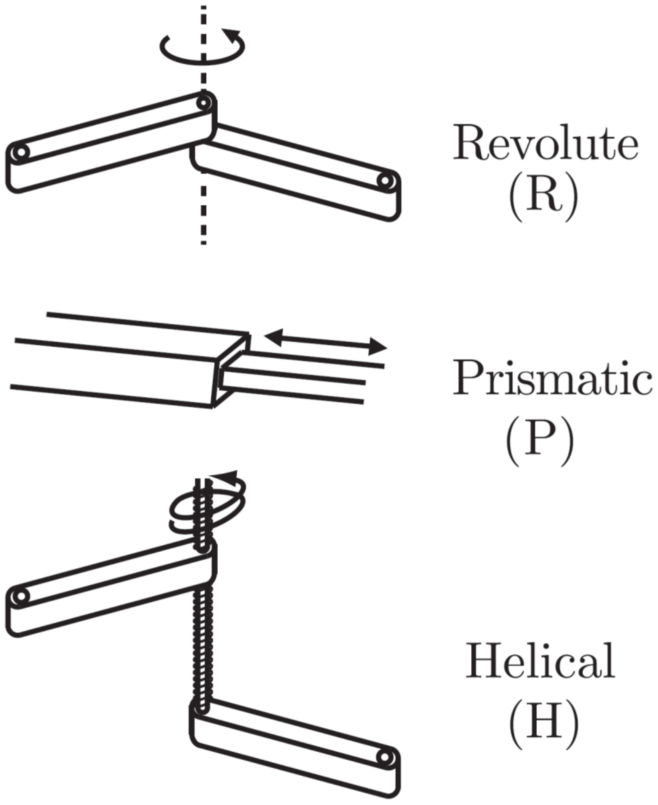
# Degrees of Freedom

- Maximum number of logically independent values
- Specify the position of a rigid body



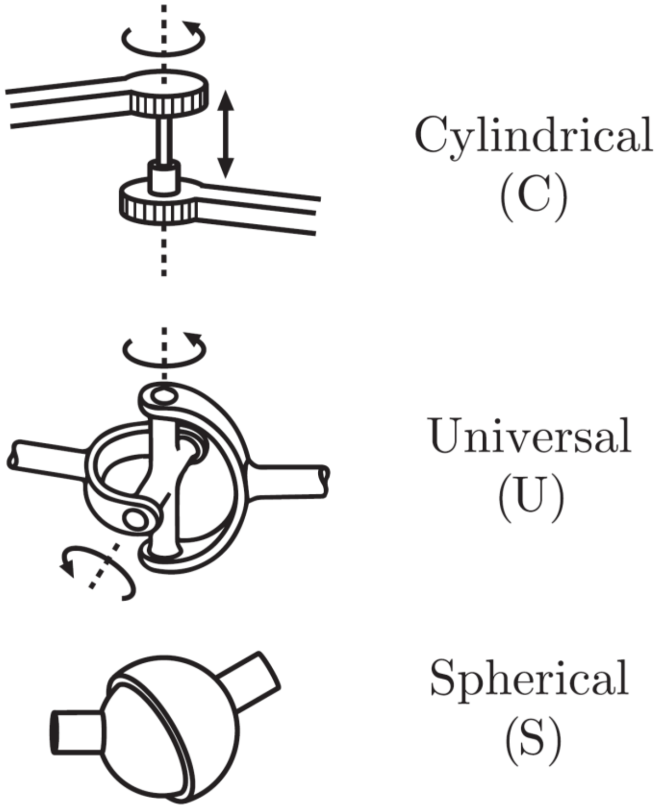
# Degrees of Freedom of Robot Joints

- Revolute joint
  - 1 DOF
- Prismatic joint
  - 1 DOF
- Helical joint
  - 1 DOF

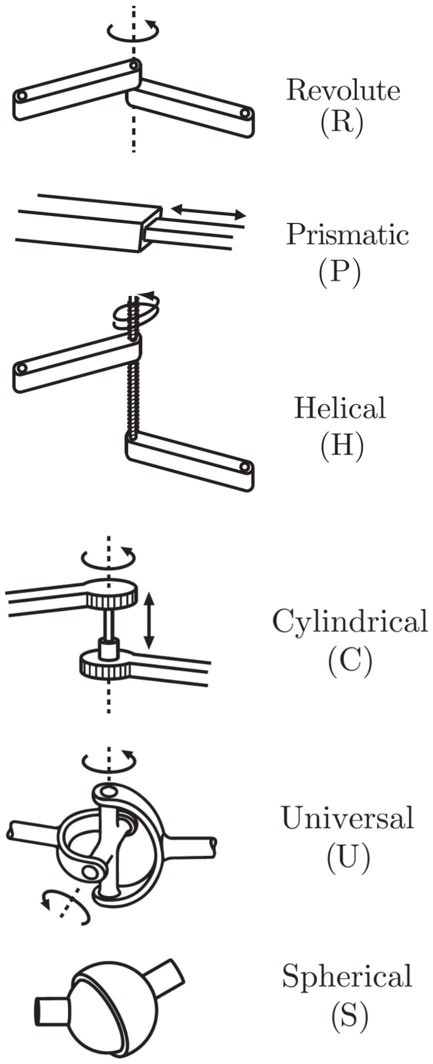


# Degrees of Freedom of Robot Joints

- Cylindrical joint
  - 2 DOF
- Universal joint
  - 2 DOF
- Spherical joint
  - 3 DOF

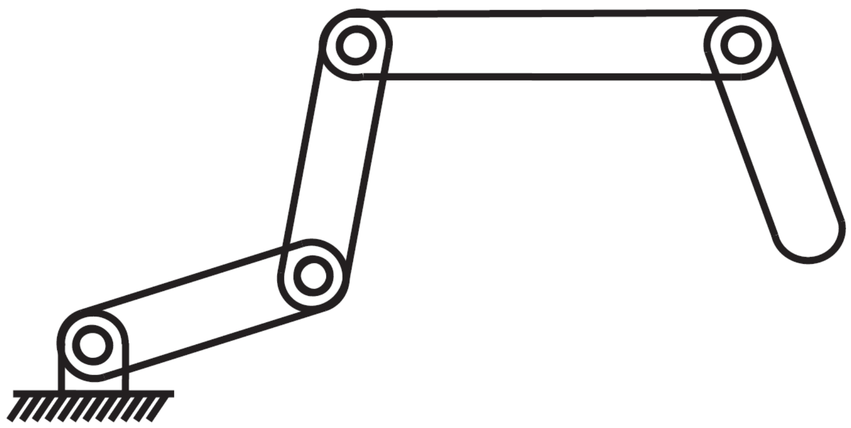


# Degrees of Freedom of Robot Joints

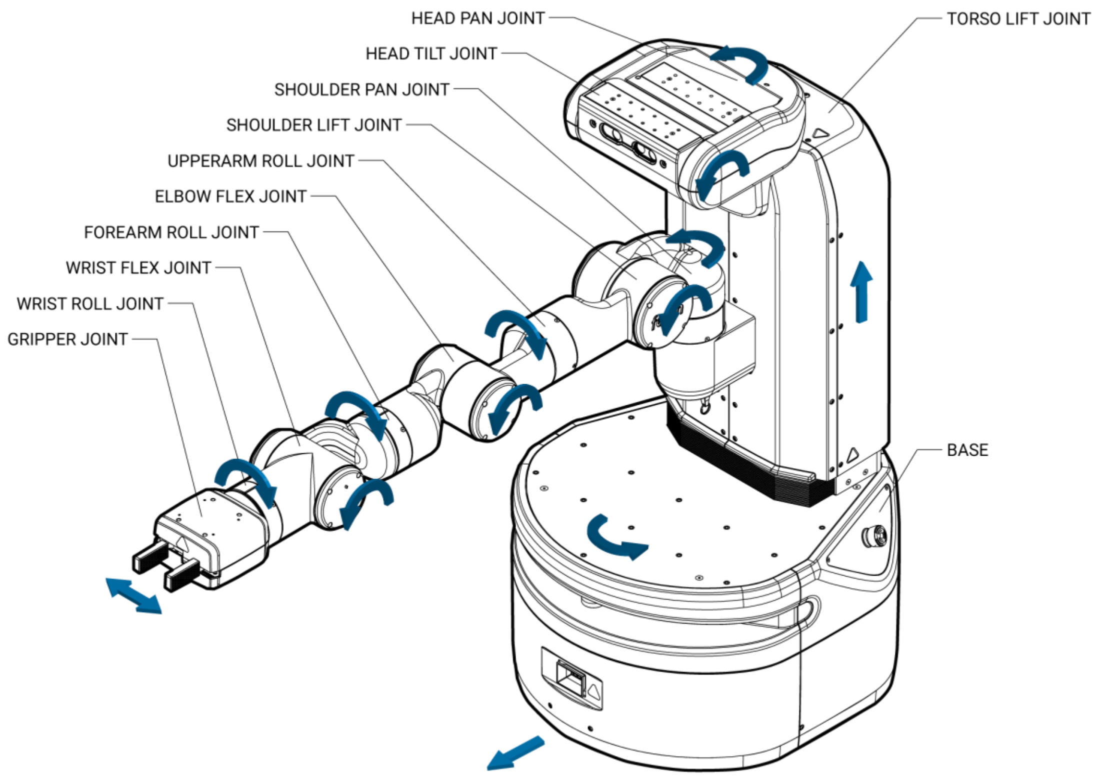



| Joint type | dof $f$ | Constraints $c$ between two planar rigid bodies | Constraints $c$ between two spatial rigid bodies |
|---|---|---|---|
| Revolute (R) | 1 | 2 | 5 |
| Prismatic (P) | 1 | 2 | 5 |
| Helical (H) | 1 | N/A | 5 |
| Cylindrical (C) | 2 | N/A | 4 |
| Universal (U) | 2 | N/A | 4 |
| Spherical (S) | 3 | N/A | 3 |

# Degrees of Freedom of a Robot



- 4 revolute joints
- 4 DOFs




- 7 revolute joints for the arm
- 7 DOFs

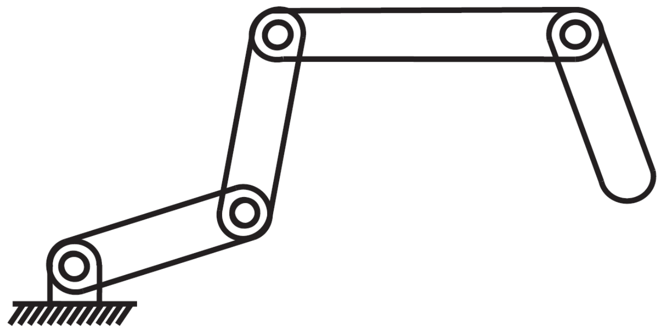
# Configuration Space of a Robot

- The configuration of a robot is a complete specification of the position of every point of the robot.

- The minimum number n of real-valued coordinates needed to represent the configuration is the number of degrees of freedom (DOF) of the robot.

- The n-dimensional space containing all possible configurations of the robot is called the configuration space (C-space).

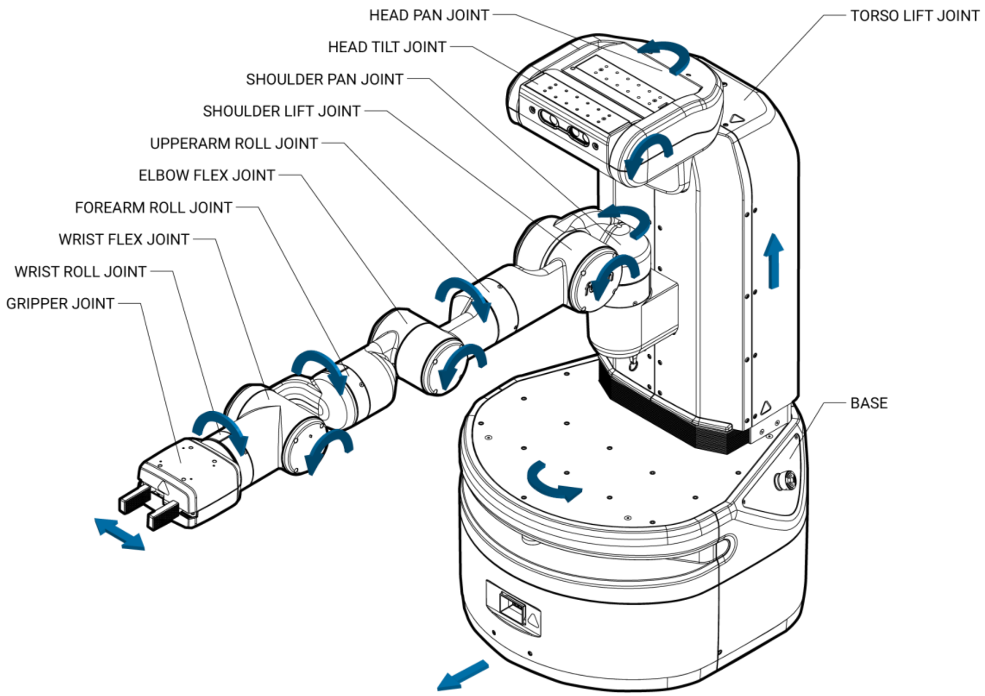
- The configuration of a robot is represented by a point in its C-space.



- 4 revolute joints
- 4 DOFs

# Configuration Space of a Robot

- The configuration space of the Fetch arm is a 7D space
- Each value in the 7D vector indicates the value of the revolute joint



# Grübler’s Formula

- The number of degrees of freedom of a mechanism with links and joints can be calculated using Grübler's formula

$$\text{degrees of freedom} = (\text{sum of freedoms of the bodies}) - (\text{number of independent constraints})$$

- Consider the following setting
  - A robot with N links, J joints (consider ground as one link)
  - Each link has m DOF (planar link? spatial link?)
  - Number of freedoms by joint i $f_i$
  - Number of constraints by joint i $c_i$

$$f_i + c_i = m$$

# Grübler's Formula

$$\text{dof} = \underbrace{m(N-1)}_{\text{rigid body freedoms}} - \underbrace{\sum_{i=1}^{J} c_i}_{\text{joint constraints}}$$

Ground is regarded as a link

$$= m(N-1) - \sum_{i=1}^{J} (m - f_i)$$

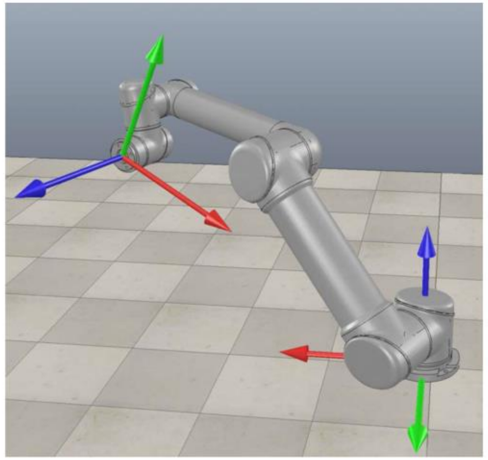
$$= m(N-1-J) + \sum_{i=1}^{J} f_i.$$

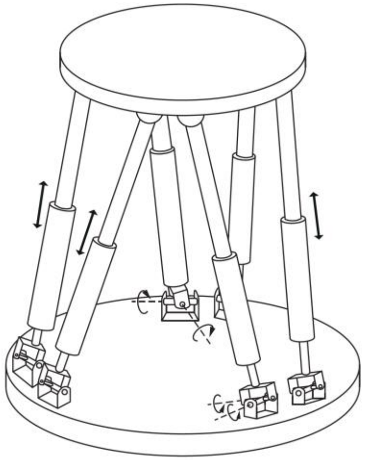
Assume all joint constraints are independent.

# Open-Chain vs. Closed-Chain

- Open-chain mechanisms: without a closed loop
- Closed-chain mechanisms: with a closed loop

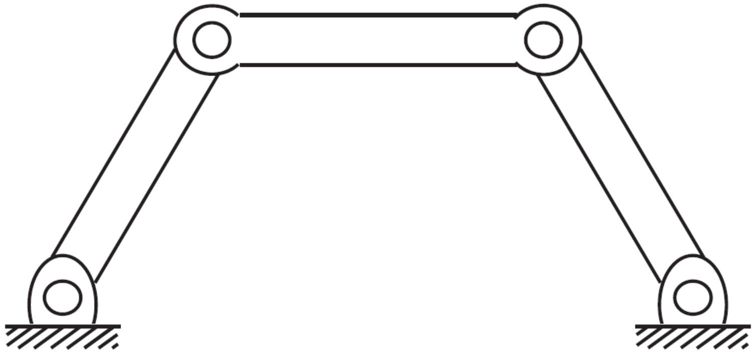
- Examples
  - A person standing with both feet





Stewart-Gough platform

# Grübler's Formula



The planar four-bar linkage

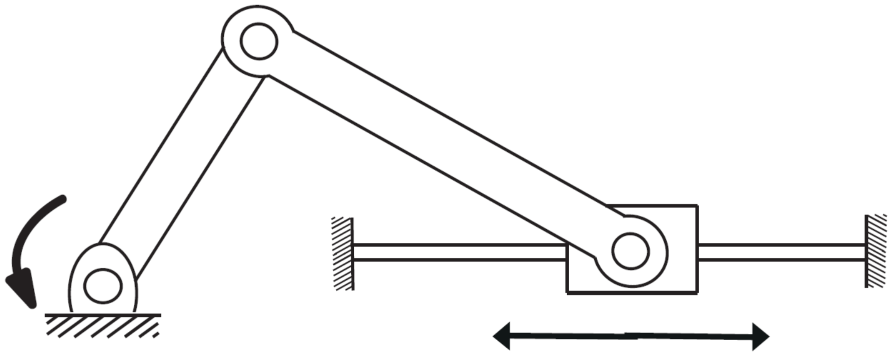
- How many links?
  - 4 (one is ground)

- Each link has m DOF. What is m?
  - m=3

DOF $= m(N - 1 - J) + \sum_{i=1}^{J} f_i$

$$= 3(4 - 1 - 4) + \sum_{i=1}^{4} 1$$

# Grübler's Formula



Slider-crank mechanism
(planar)

- How many links?
  - 4 (one is ground)
- Each link has m DOF. What is m?
  - m=3
- How many joints?
  - 3 revolute joints, 1 prismatic joint

$$\text{DOF} = m(N - 1 - J) + \sum_{i=1}^{J} f_i$$

$$= 3(4 - 1 - 4) + \sum_{i=1}^{4} 1$$

# Configuration Space Topology

- Configuration specifies the position of a robot
- For a robot with n joints, the configuration is a vector in $\mathbb{R}^n$
  - C-space
- Joints may have limits, upper bound and lower bound
- Topology: shape of the space
  - Consider all the feasible points in the configuration space

# Configuration Space Topology

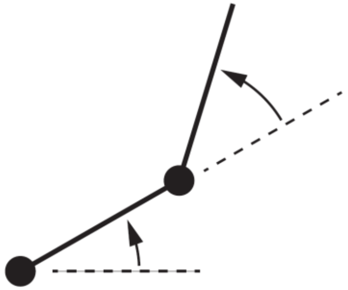
- n-dimensional Euclidean space $\mathbb{R}^n$
- n-dimensional sphere in a (n+1)-dimensional Euclidean space $S^n$
  - Two-dimensional surface of a sphere in three-dimensional space $S^2$

$S^2$

- The C-space can have different representations, but its shape is the same
  - A point on a circle, angle $\theta$, coordinates (x, y) $x^2 + y^2 = 1$

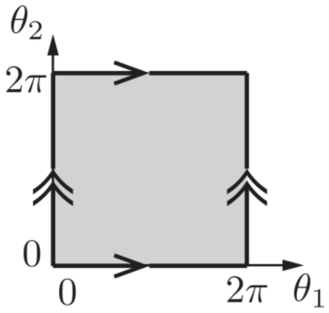
# Configuration Space Topology

- C-space as Cartesian product
  - A rigid body in the plane $\mathbb{R}^2 \times S^1$
  - A PR robot (Prismatic-Revolute) $\mathbb{R}^1 \times S^1$
    - Ignore joint limits
  - A 2R robot $S^1 \times S^1 = T^2$ n-dimensional surface of a torus in an (n+1)-dimensional space



2R robot arm

$T^2 = S^1 \times S^1$




$[0, 2\pi) \times [0, 2\pi)$

sample representation

# Configuration Space Topology

- C-space of a planar rigid body with a 2R robot arm

$$\mathbb{R}^2 \times S^1 \times T^2 = \mathbb{R}^2 \times T^3$$

- C-space of a rigid body in 3D space
  - 3D translation
  - 3D rotation

$$\mathbb{R}^3 \times S^2 \times S^1$$

# Configuration Space Representation

- Explicit parameterization
  - Use n coordinates for n-dimensional space

  - A sphere: latitude-longitude
    - Singularities at North Pole and South Pole
    - Problem with the representation, not the topology
    - Infinity velocity problem $\sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}$
- Deal with singularities
  - Use more than one coordinate chart (each covers a portion of the space)
  - Implicit representations
    - Sphere $(x, y, z)$ $x^2 + y^2 + z^2 = 1$ More numbers than DOF
    - Rotation matrix for 3D rotations

# Summary

- Robot links and joints
- Degrees of freedom of joints and robots
- Grübler's Formula
- Configuration space

# Further Reading


- Chapter 2 in Kevin M. Lynch and Frank C. Park. Modern Robotics: Mechanics, Planning, and Control. 1st Edition, 2017 http://hades.mech.northwestern.edu/images/7/7f/MR.pdf
- T. Lozano-Perez. Spatial planning: a configuration space approach. A.I. Memo 605, MIT Artificial Intelligence Laboratory, 1980. http://people.csail.mit.edu/tlp/
- W. M. Boothby. An Introduction to Differentiable Manifolds and Riemannian Geometry. Academic Press, 2002.