Intensity Surfaces and Gradients

CS 4391 Introduction Computer Vision

Professor Yu Xiang

The University of Texas at Dallas

Some slides of this lecture are courtesy Robert Collins (PSU)

Image Data

width

height

$H \times W \times 3$

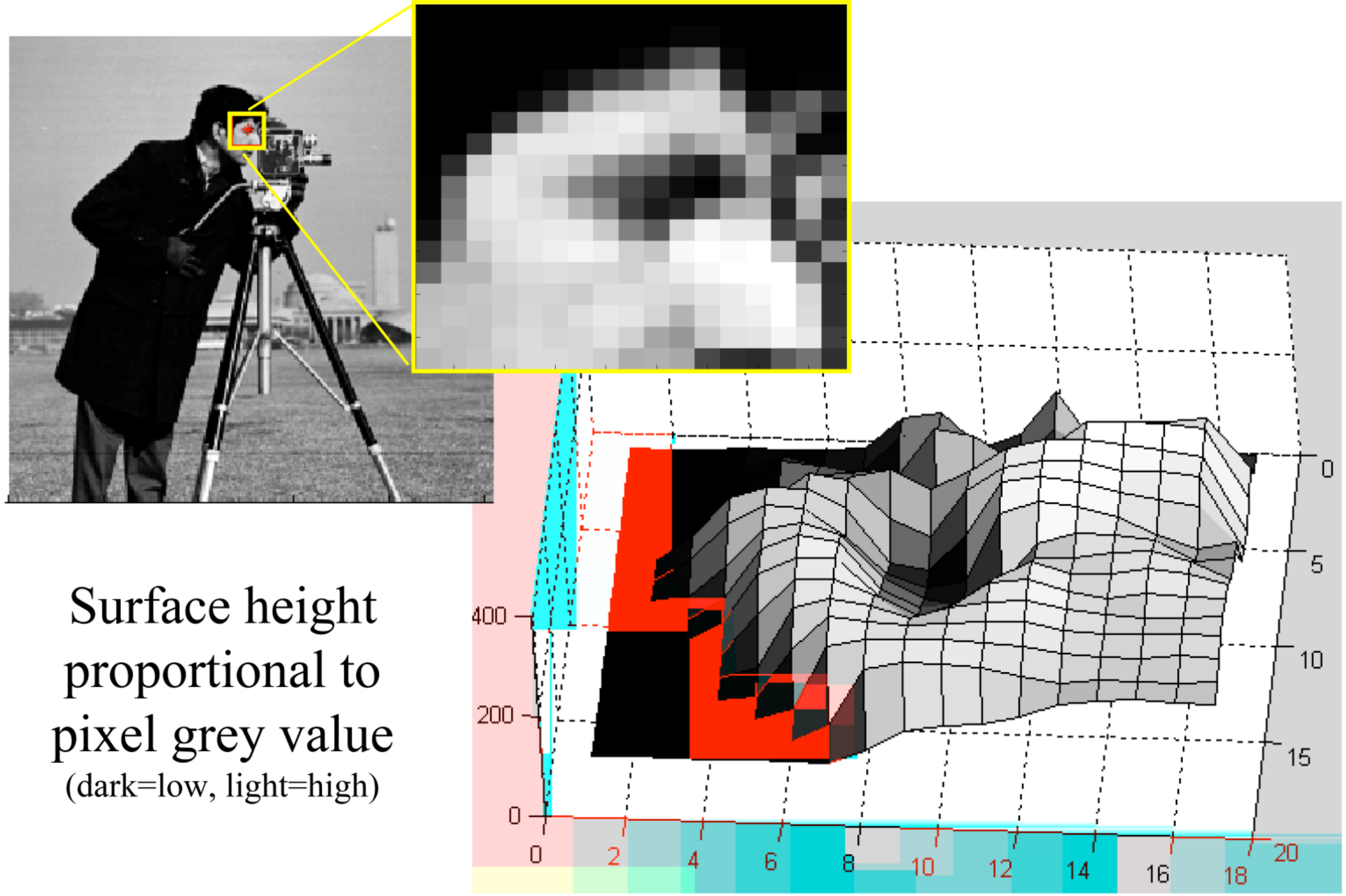
$H \times W$

Grayscale
[0, 255]

RGB color space
[0, 255]

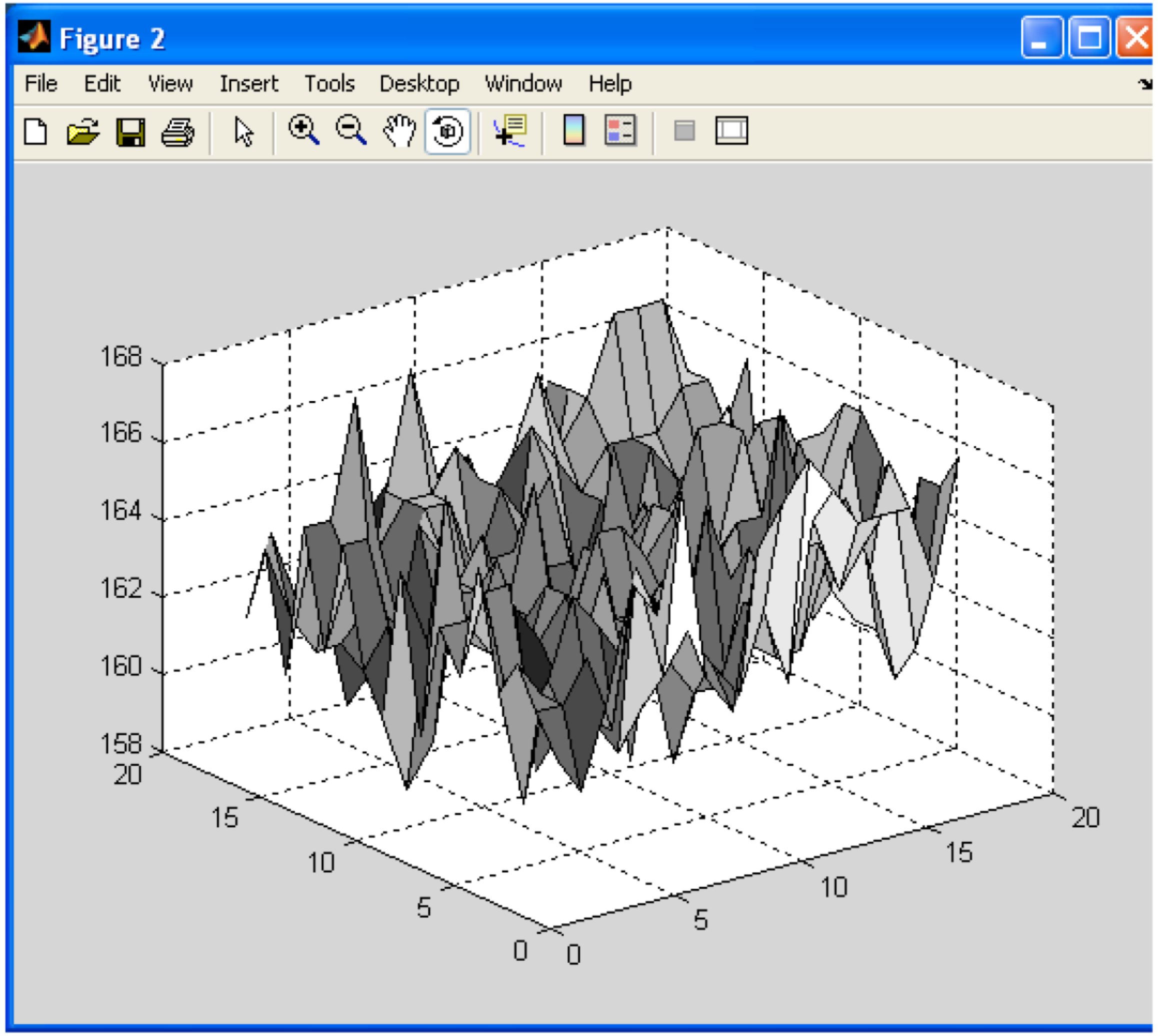
```
0.2989 * R + 0.5870 * G + 0.1140 * B
```

Images as Surfaces



Surface height proportional to pixel grey value
(dark=low, light=high)

Examples



$\text{Mean} = 164 \quad \text{Std} = 1.8$

Examples

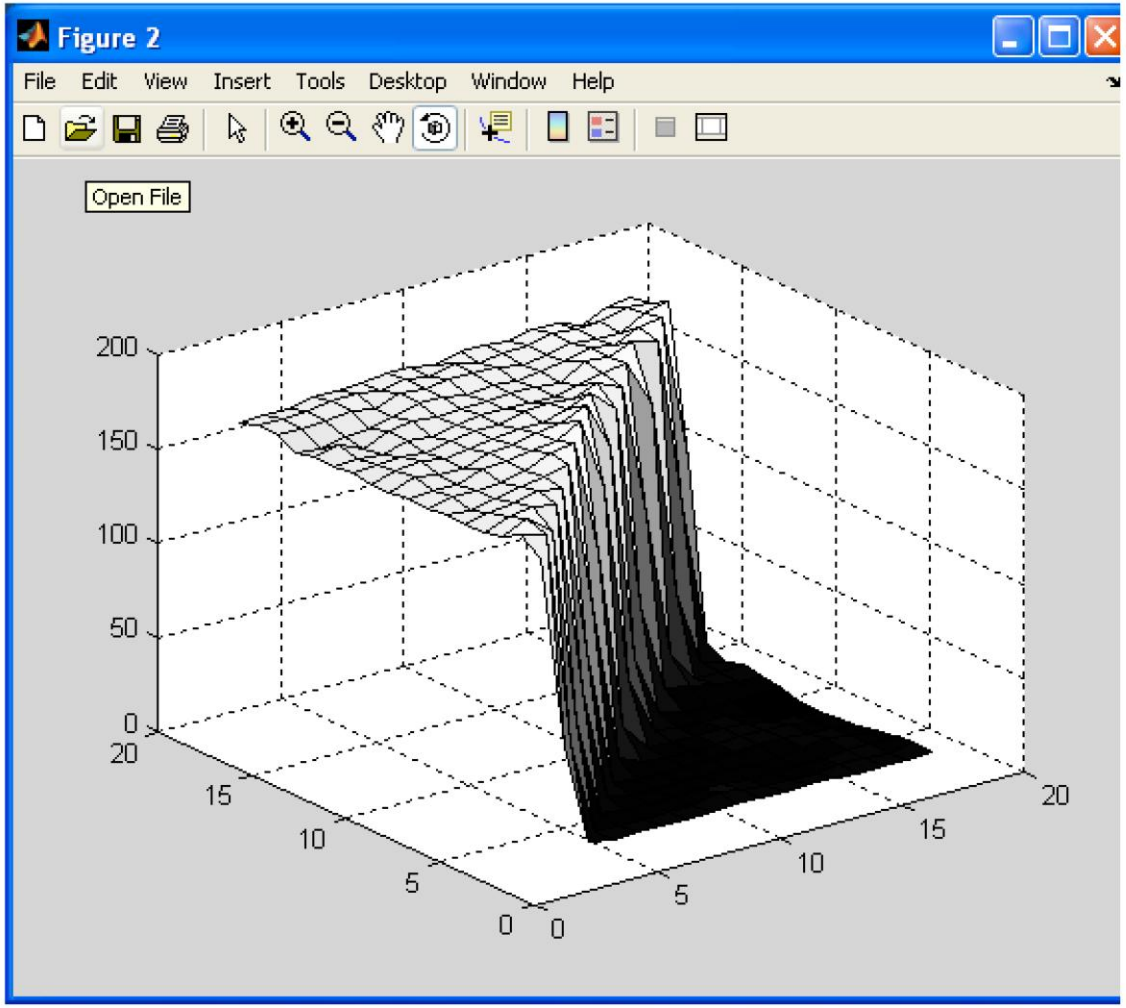


Figure 2
File Edit View Insert Tools Desktop Window Help
Open File
200
150
100
50
0
20
15
10
5
0
0
5
10
15
20

Examples

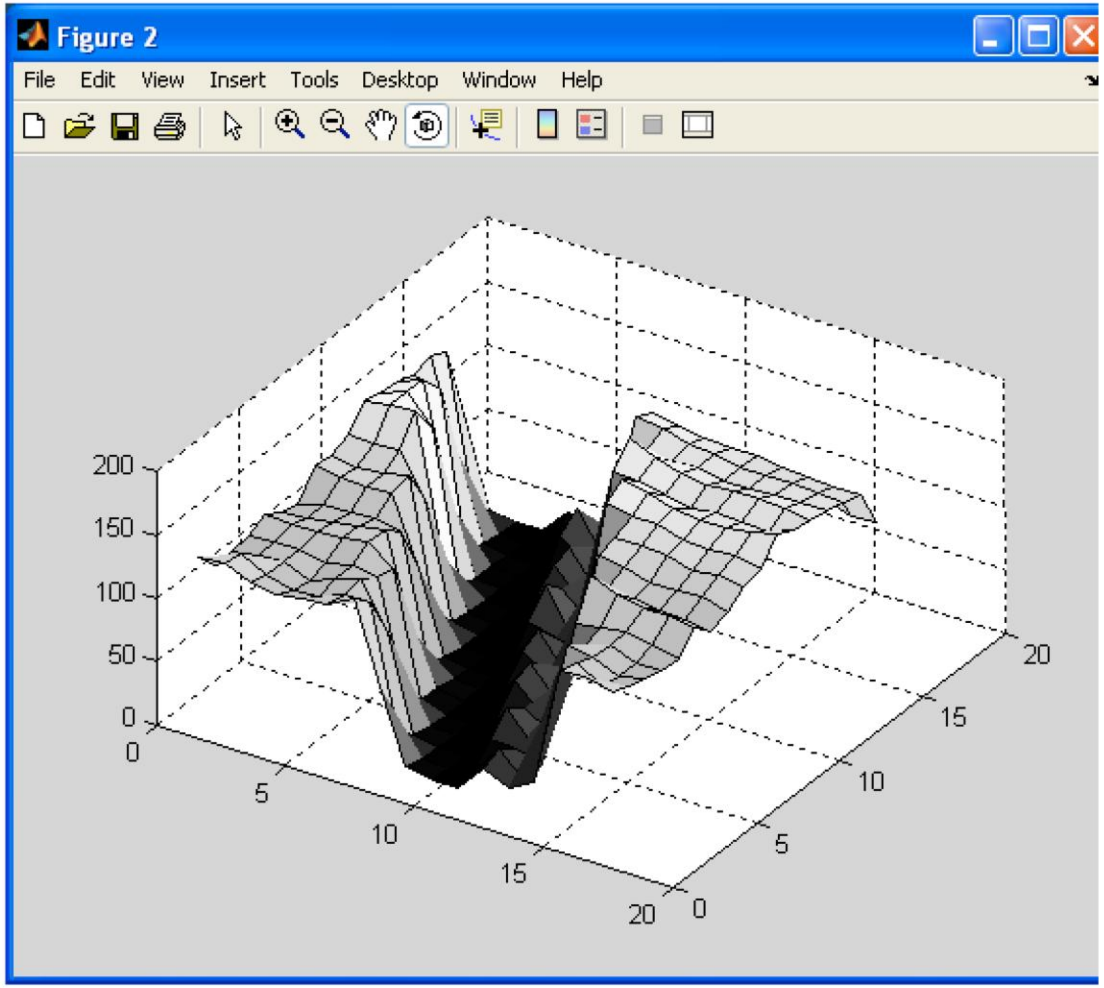


Figure 2
File Edit View Insert Tools Desktop Window Help
200
150
100
50
0
0
5
10
15
20
0
5
10
15
20

Examples

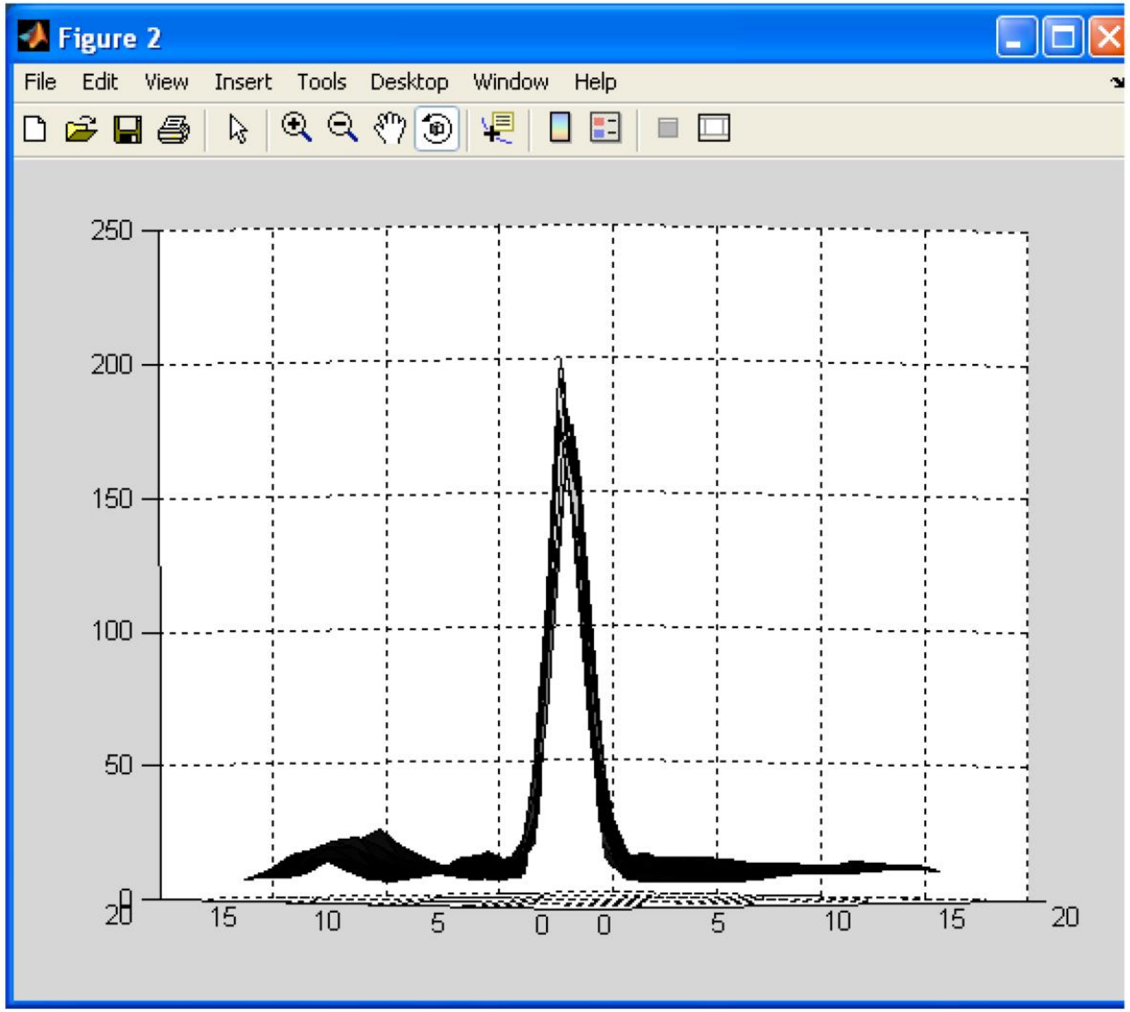
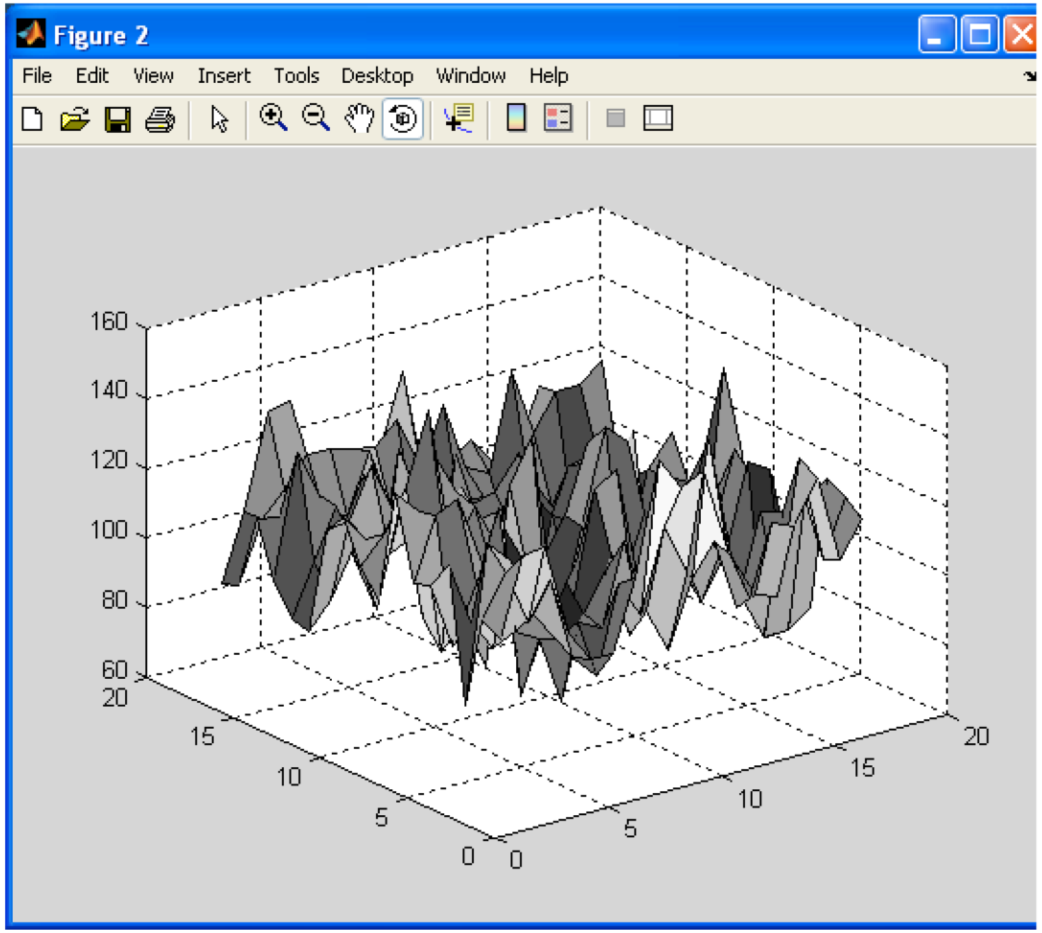


Figure 2
File Edit View Insert Tools Desktop Window Help
250
200
150
100
50
0
20
15
10
5
0
0
5
10
15
20

Examples



Mean = 111 Std = 15.4

Examples

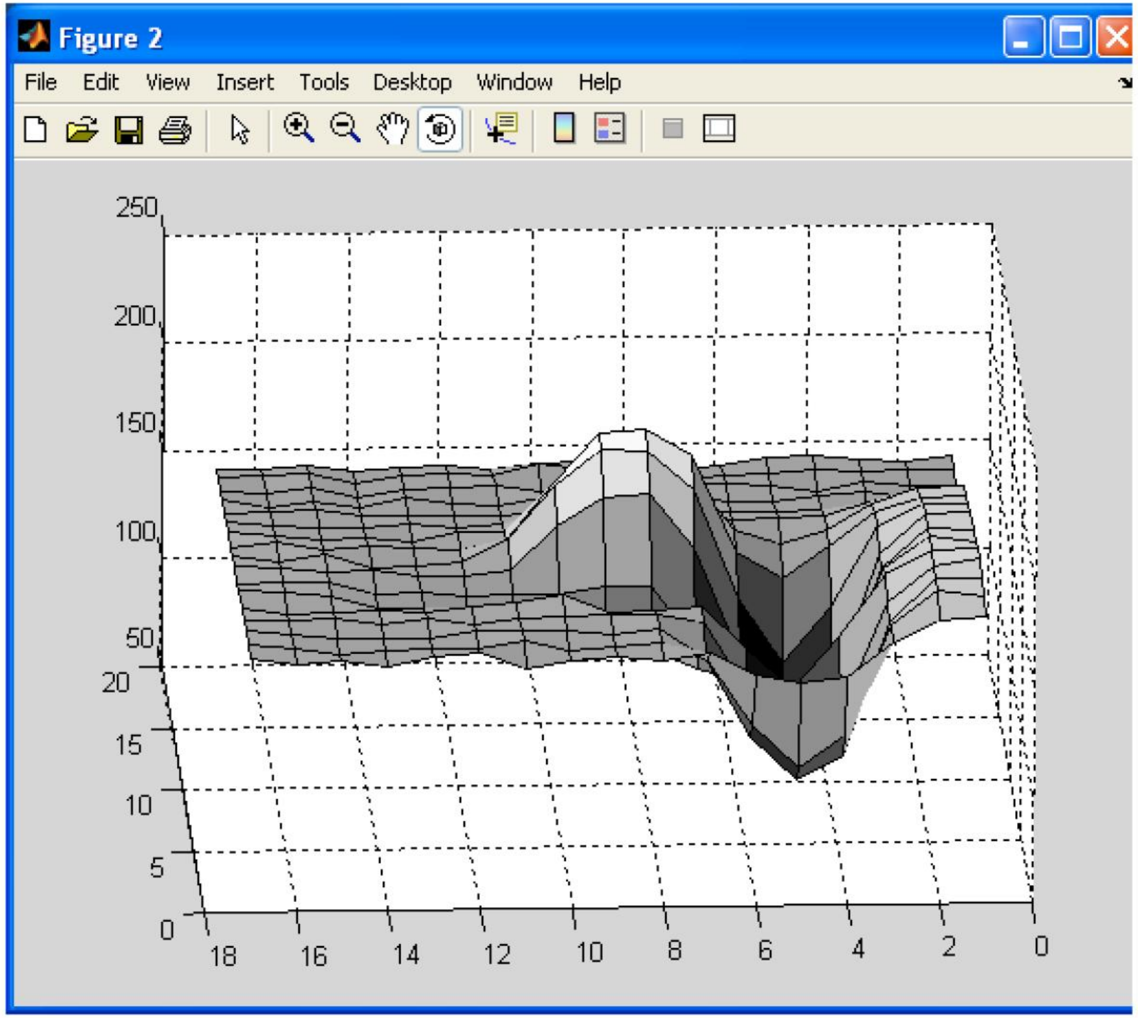
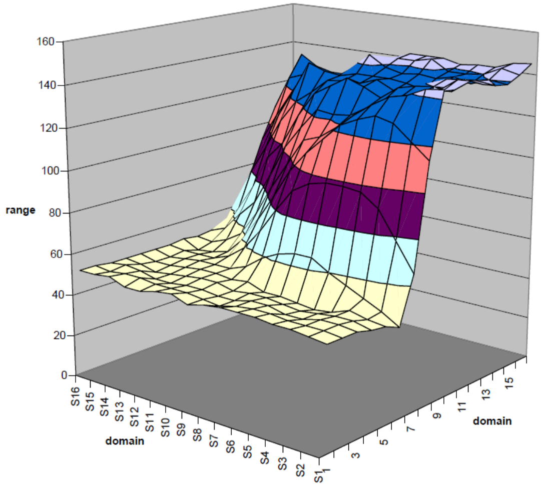


Figure 2
File Edit View Insert Tools Desktop Window Help
250
200
150
100
50
20
15
10
5
0
18 16 14 12 10 8 6 4 2 0

Images as Functions

45	60	98	127	132	133	137	133
46	65	98	123	126	128	131	133
47	65	96	115	119	123	135	137
47	63	91	107	113	122	138	134
50	59	80	97	110	123	133	134
49	53	68	83	97	113	128	133
50	50	58	70	84	102	116	126
50	50	52	58	69	86	101	120



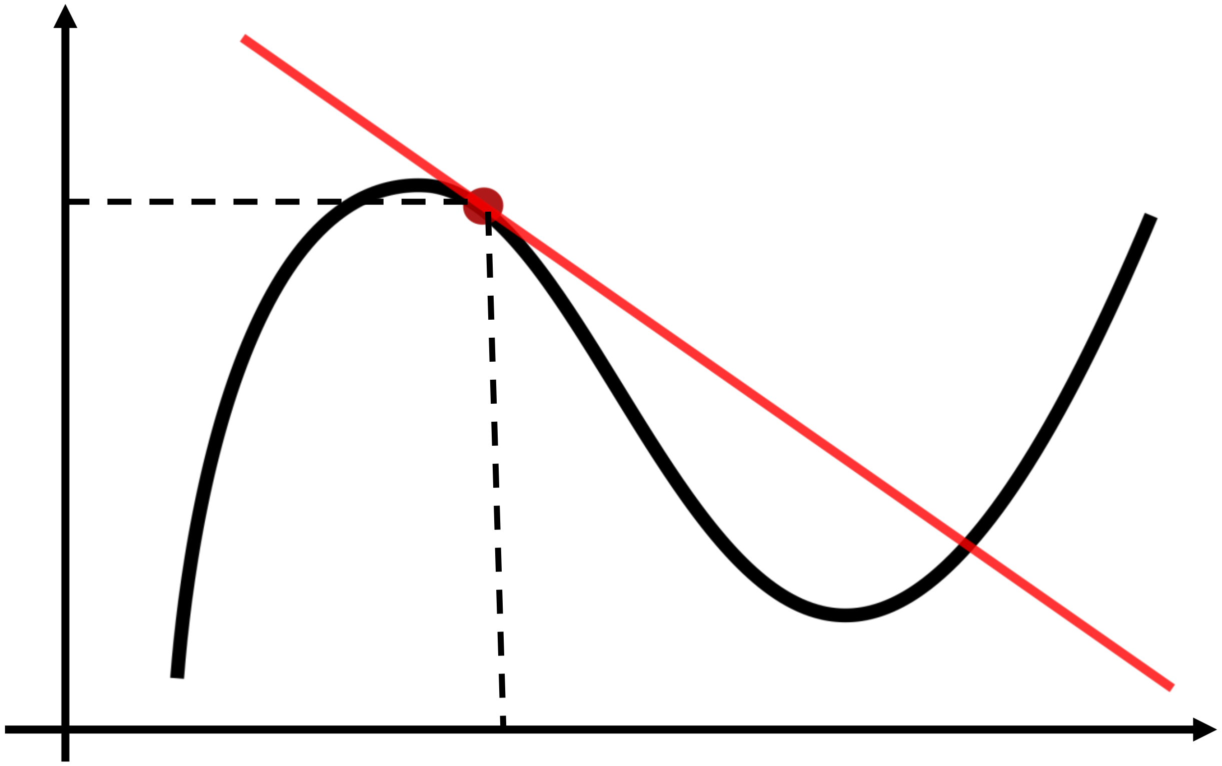
Function $I(\mathbf{x})$ $f(\mathbf{x})$

$I(x, y)$ $f(x, y)$

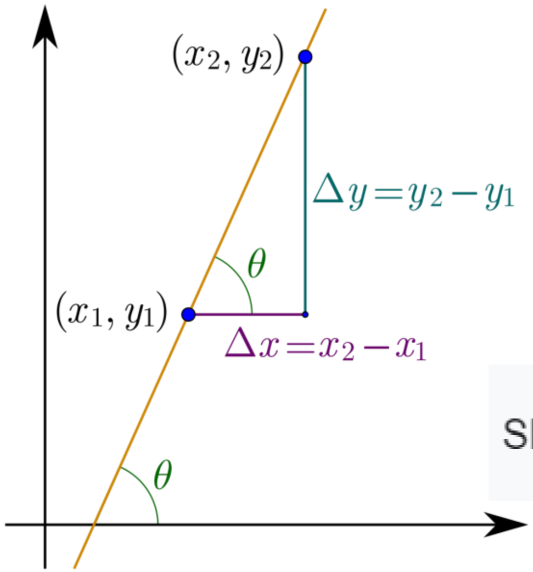
Derivative

- Derivative of a 1D function

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$



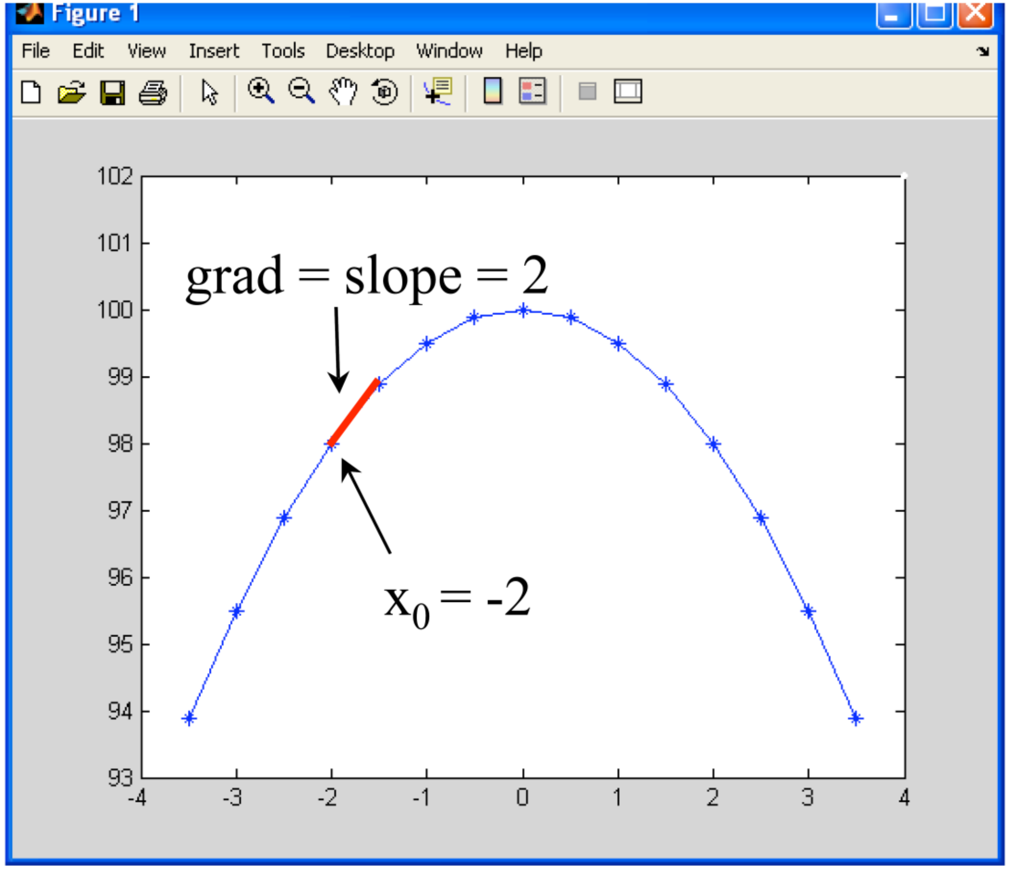
The slope of the tangent line is equal to the derivative of the function at the marked point.



Slope: $m = \dfrac{\Delta y}{\Delta x} = \tan(\theta)$

Derivative

$f(x) = 100 - 0.5 * x^2$ $df(x)/dx = - x$



Partial Derivative

- Multivariable function

$$\frac{\partial}{\partial x_i} f(\mathbf{a}) = \lim_{h \to 0} \frac{f(a_1, \ldots, a_{i-1}, a_i + h, a_{i+1}, \ldots, a_n) - f(a_1, \ldots, a_i, \ldots, a_n)}{h}$$
$$= \lim_{h \to 0} \frac{f(\mathbf{a} + h\mathbf{e_i}) - f(\mathbf{a})}{h} .$$

- Example $f(x, y) = x^2 + xy + y^2$

- Total differential $dy = \frac{\partial y}{\partial x_1} dx_1 + \cdots + \frac{\partial y}{\partial x_n} dx_n$

Gradient

- For a function $f\colon \mathbb{R}^n \to \mathbb{R}$, gradient at a point $p = (x_1, \ldots, x_n)$

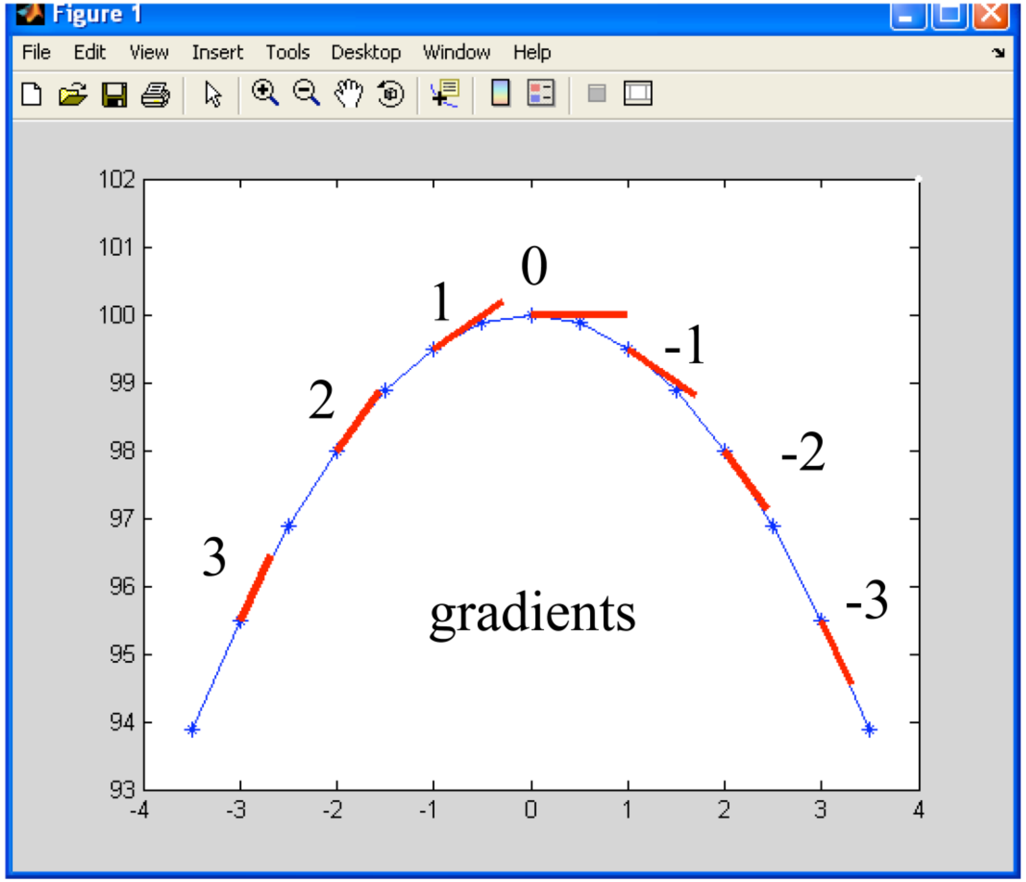
$$\nabla f(p) = \begin{bmatrix} \frac{\partial f}{\partial x_1}(p) \\ \vdots \\ \frac{\partial f}{\partial x_n}(p) \end{bmatrix}$$

- Gradient vs. total differential

$$df_p = \begin{bmatrix} \frac{\partial f}{\partial x_1}(p) & \cdots & \frac{\partial f}{\partial x_n}(p) \end{bmatrix}$$

1D Gradient

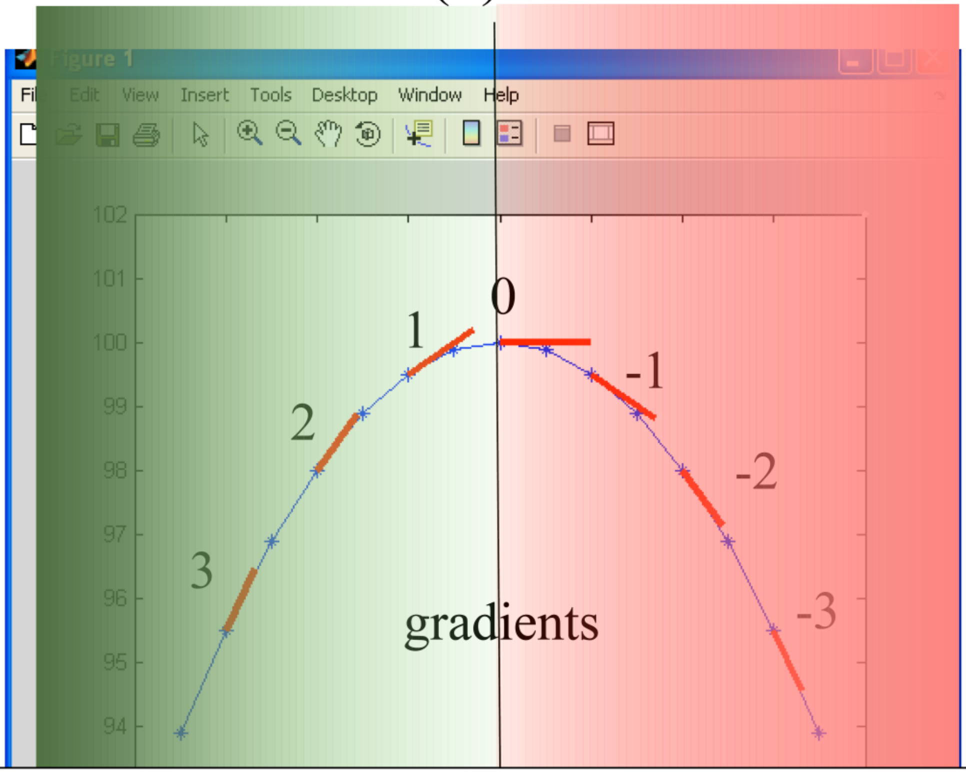
$$f(x) = 100 - 0.5 * x^2 \qquad df(x)/dx = - x$$



1D Gradient

f(x) = 100 - 0.5 * x^2   df(x)/dx = - x

Gradients on this side of peak are positive



Gradients on this side of peak are negative

Note: Sign of gradient at point tells you what direction to go to travel "uphill"

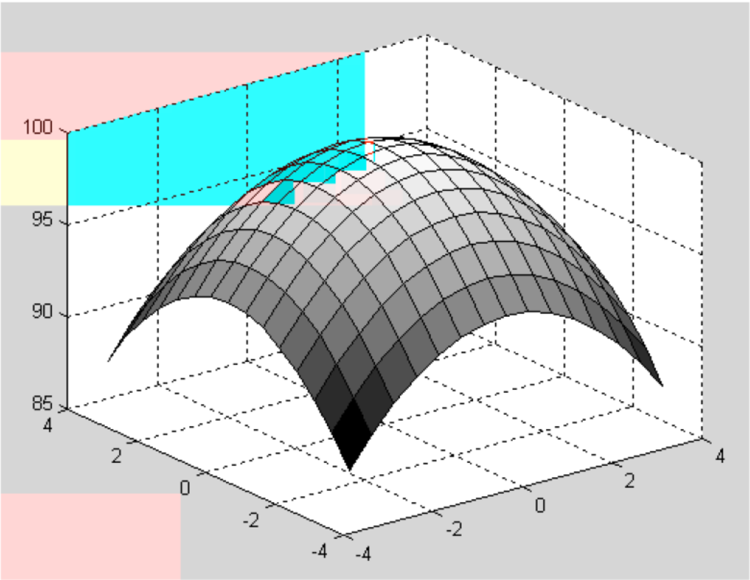
2D Gradient

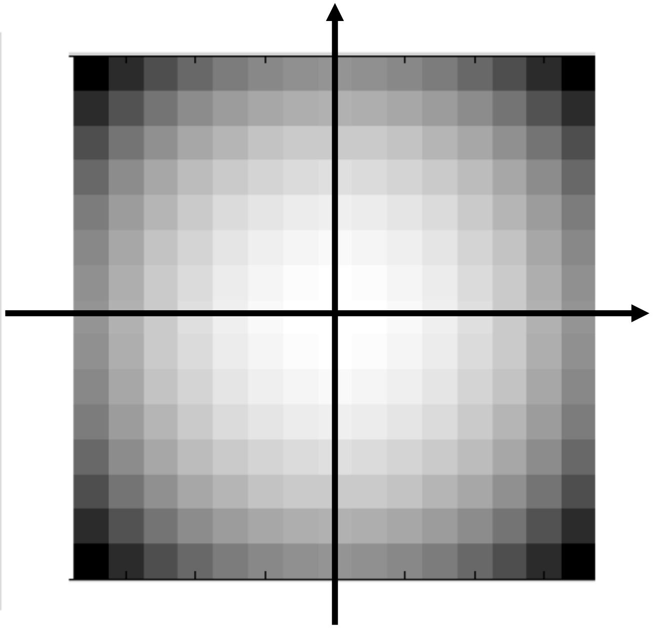
$$f(x,y) = 100 - 0.5 * x^2 - 0.5 * y^2$$

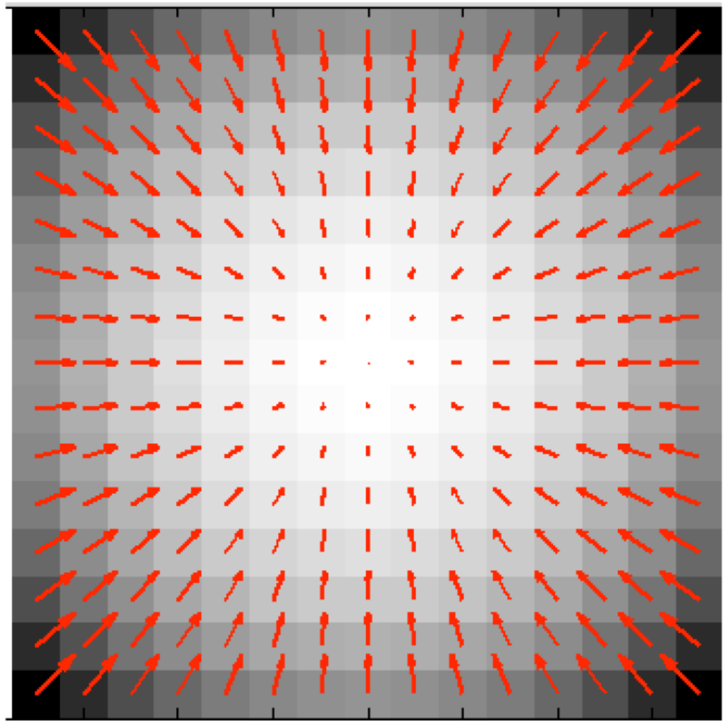
$$df(x,y)/dx = -x \qquad df(x,y)/dy = -y$$

$$\text{Gradient} = [df(x,y)/dx\ ,\ df(x,y)/dy] = [-x\ ,\ -y]$$

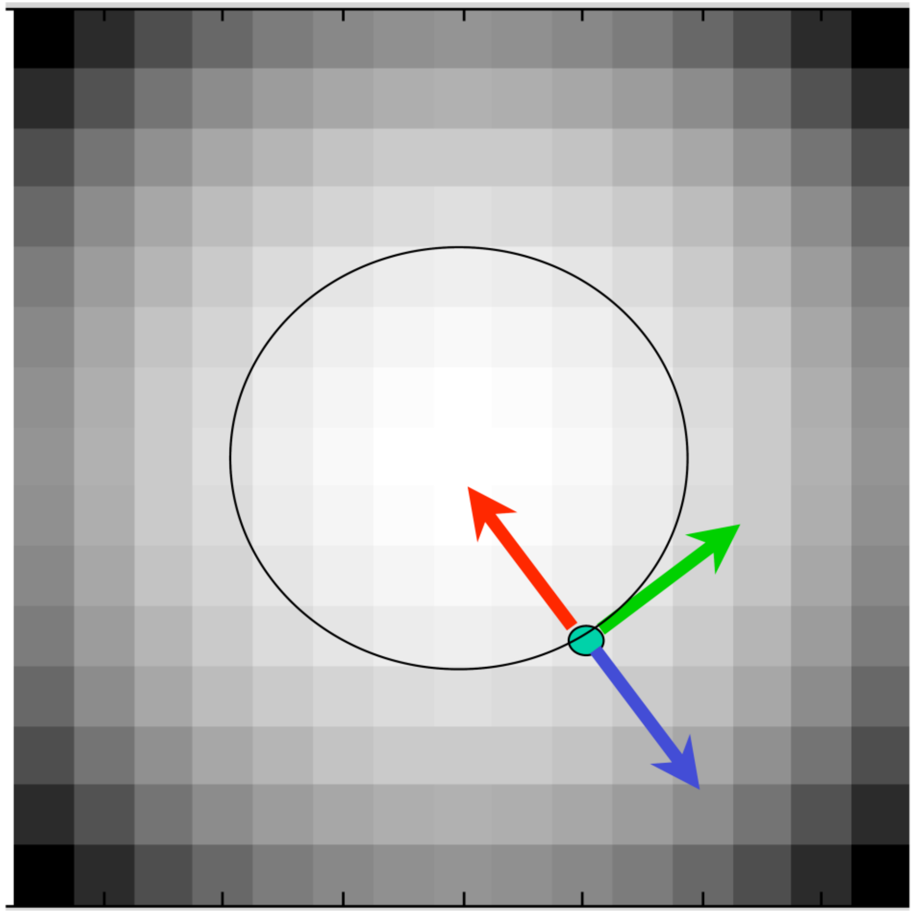
The gradient indicates the direction of steepest ascent.







2D Gradient



Let $g=[g_x,g_y]$ be the gradient vector at point/pixel (x_0,y_0)

Vector g points uphill

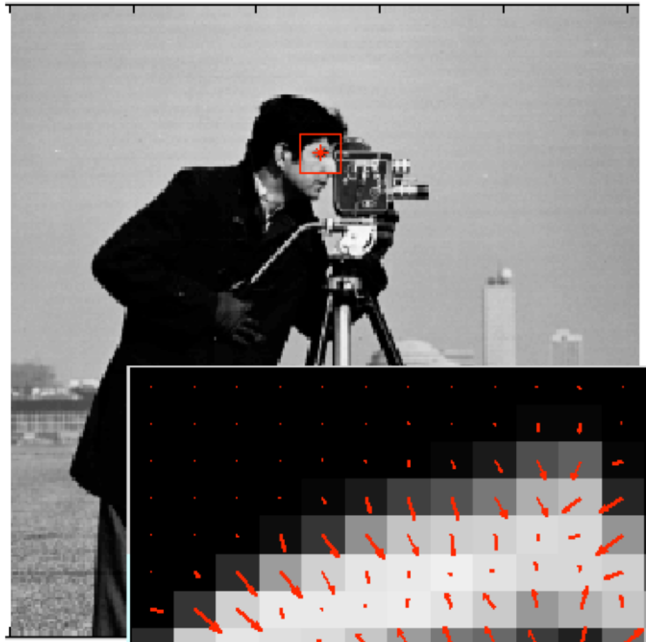
(direction of steepest ascent)

Vector - g points downhill

(direction of steepest descent)

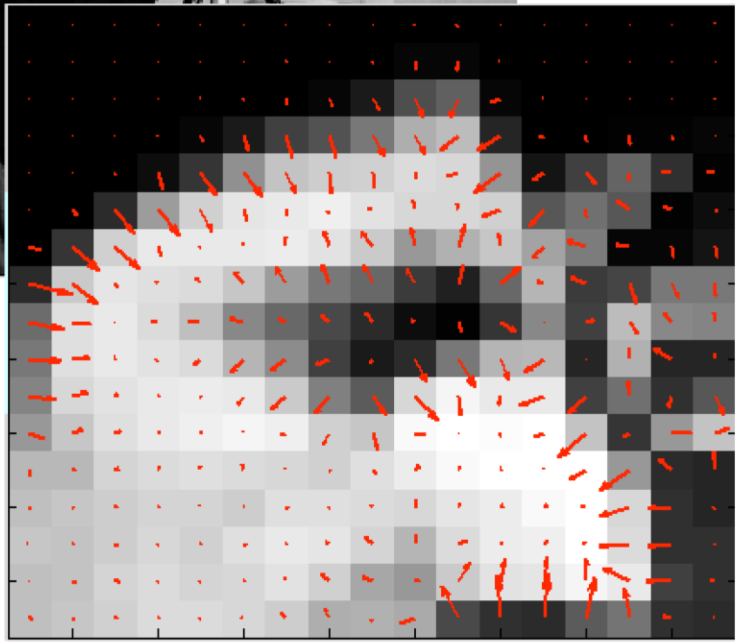
Vector $[g_y, -g_x]$ is perpendicular, and denotes direction of constant elevation. i.e. normal to contour line passing through point (x_0,y_0)

Image Gradient



The same is true of 2D image gradients.

The underlying function is numerical (tabulated) rather than algebraic. So need numerical derivatives.



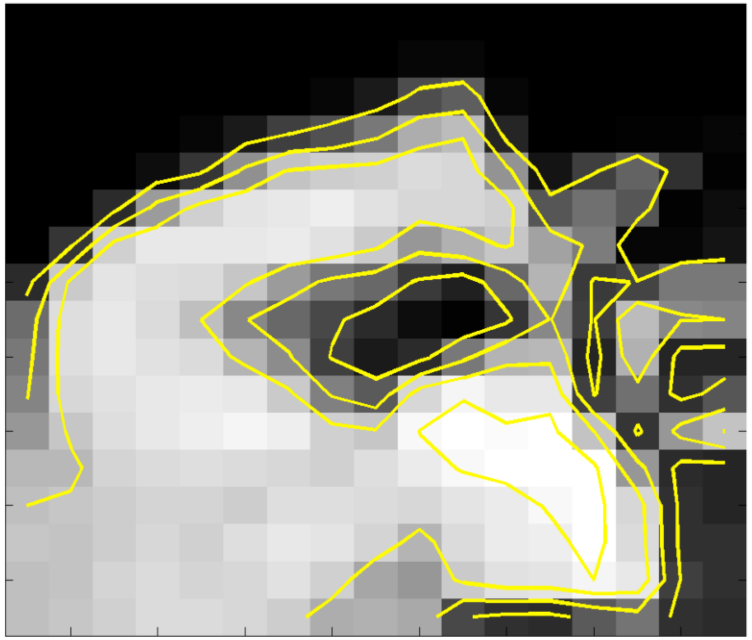


Image Gradient

- Derivative of a function

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

- Central difference is more accurate

$$f'(x) = \lim_{h \to 0} \frac{f(x+0.5h) - f(x-0.5h)}{h}$$

Image Gradient

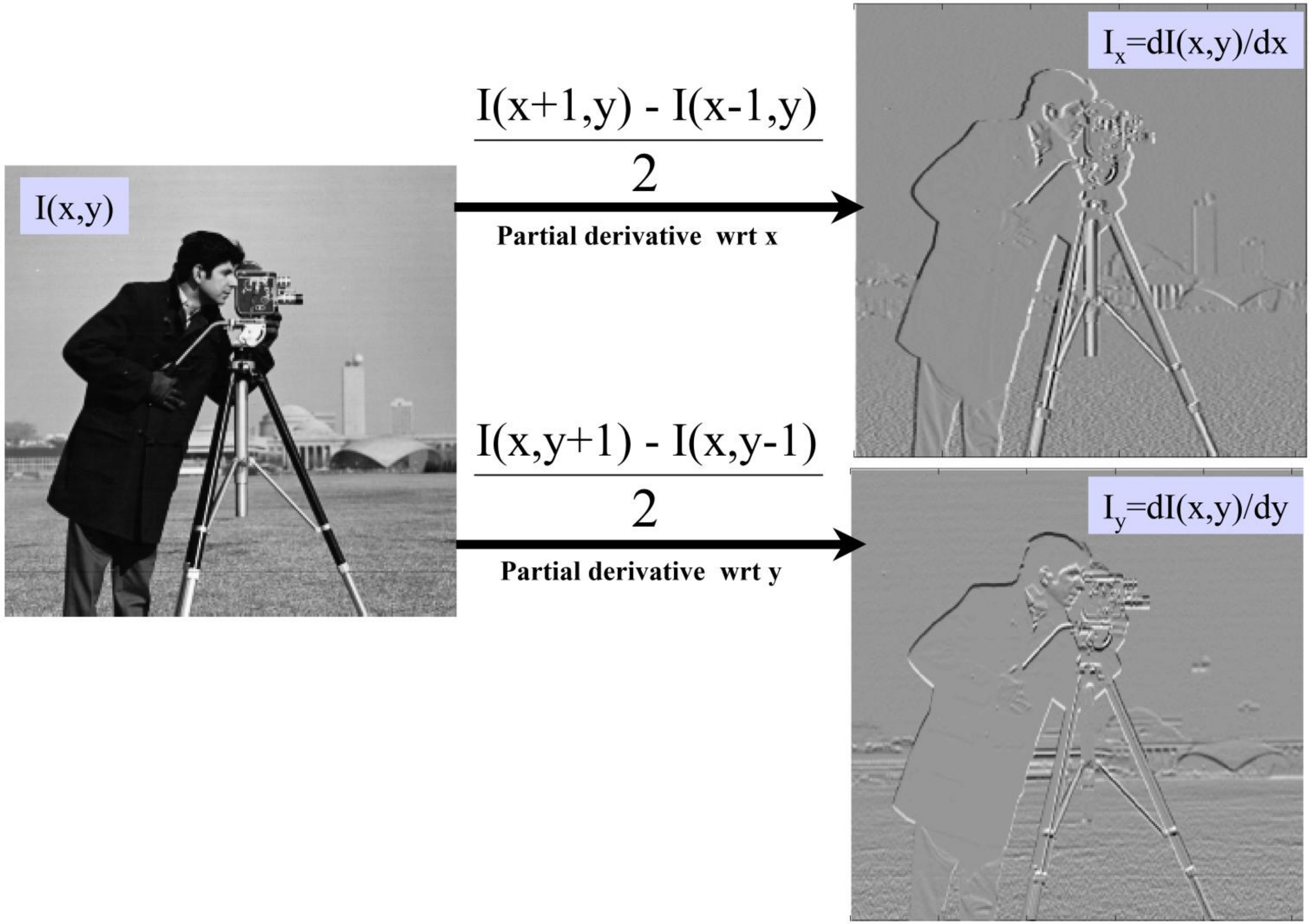
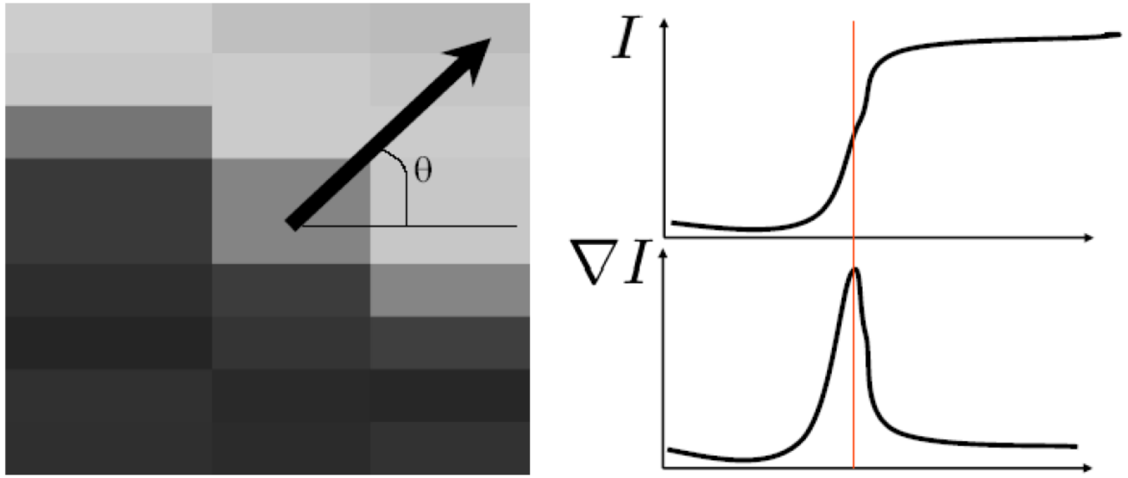


Image Gradient

Gradient Vector: $\nabla I = [\frac{\partial I}{\partial x}, \frac{\partial I}{\partial y}]^T$



$$|\nabla I| = \sqrt{\left(\frac{\partial I}{\partial x}\right)^2 + \left(\frac{\partial I}{\partial y}\right)^2}$$

Magnitude:

$$\theta = atan2\left(\frac{\partial I}{\partial y}, \frac{\partial I}{\partial x}\right)$$

Orientation

Further Reading

- Chapter 3.1, Richard Szeliski
- Slope https://en.wikipedia.org/wiki/Slope
- Gradient https://en.wikipedia.org/wiki/Gradient
- Matplotlib 3D surface: https://matplotlib.org/stable/gallery/mplot3d/surface3d.html