# Epipolar Geometry and Stereo

CS 4391 Introduction Computer Vision

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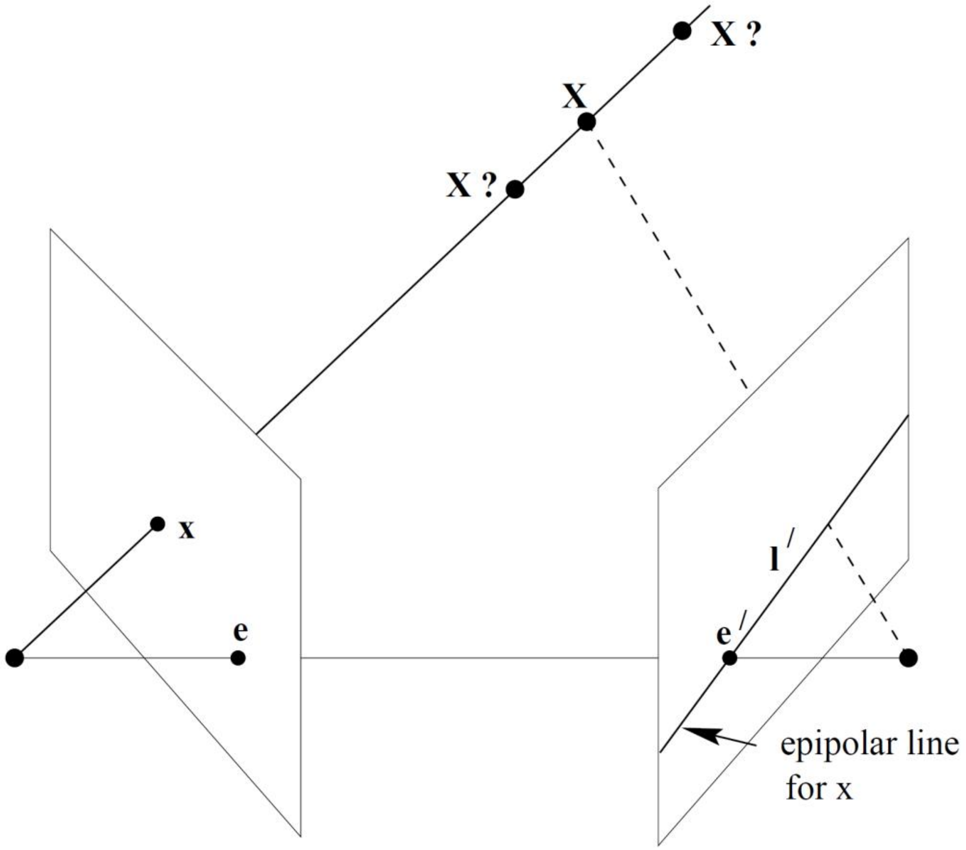
# Recall Fundamental Matrix




- Epipolar line $\mathbf{l}' = F\mathbf{x}$

$$\mathbf{l} = F^T\mathbf{x}'$$

- Fundamental matrix

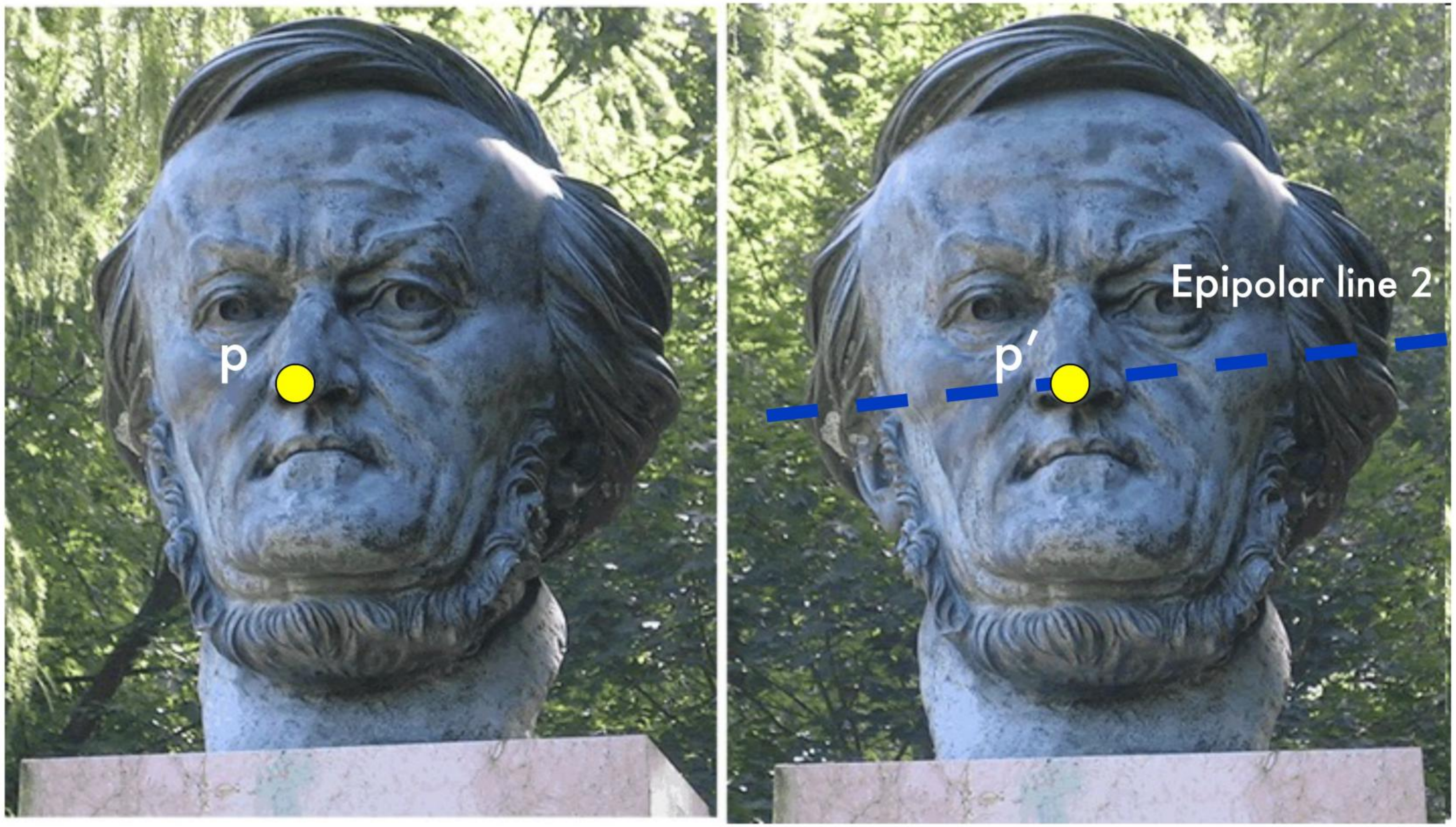
$$F = [\mathbf{e}']_\times P'P^+$$

3x3

Epipole $\mathbf{e}' = (P'C)$

$$P^+ = P^T(PP^T)^{-1}$$

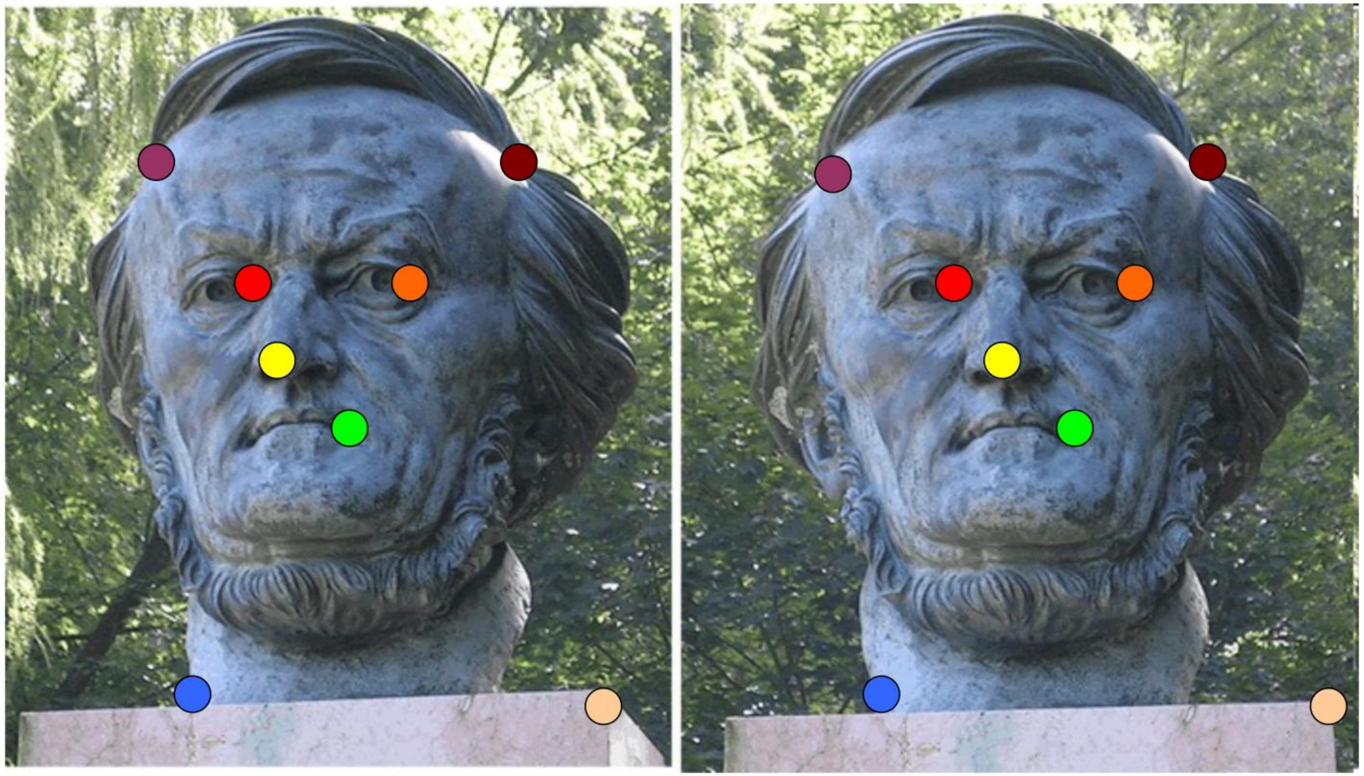
# Why the Fundamental Matrix is Useful?




$$\mathbf{l}' = F\mathbf{p}$$

# Estimating the Fundamental Matrix

- The 8-point algorithm



$$\mathbf{l}' = F\mathbf{x}$$

$$\mathbf{x}'^{\mathsf{T}}\mathbf{F}\mathbf{x} = 0$$

# Estimating the Fundamental Matrix

$$\mathbf{x}'^{\mathsf{T}}\mathbf{F}\mathbf{x} = 0 \qquad \mathbf{x} = (x, y, 1)^{\mathsf{T}} \qquad \mathbf{x}' = (x', y', 1)^{\mathsf{T}}$$

$$x'xf_{11} + x'yf_{12} + x'f_{13} + y'xf_{21} + y'yf_{22} + y'f_{23} + xf_{31} + yf_{32} + f_{33} = 0$$

$$(x'x, x'y, x', y'x, y'y, y', x, y, 1)\,\mathbf{f} = 0$$

n correspondences

$$\mathtt{A}\mathbf{f} = \begin{bmatrix} x'_1x_1 & x'_1y_1 & x'_1 & y'_1x_1 & y'_1y_1 & y'_1 & x_1 & y_1 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \\ x'_nx_n & x'_ny_n & x'_n & y'_nx_n & y'_ny_n & y'_n & x_n & y_n & 1 \end{bmatrix}\mathbf{f} = \mathbf{0}$$

# Linear System

$$\mathbf{A}\mathbf{f} = 0$$

$$n \times 9 \quad 9 \times 1$$

- Find non-zero solutions
- If f is a solution, k×f is also a solution for $k \in \mathcal{R}$
- If the rank of A is 8, unique solution (up to scale)
- Otherwise, we can seek a solution $||\mathbf{f}|| = 1$

$$\min ||\mathbf{A}\mathbf{f}||$$

Subject to $||\mathbf{f}|| = 1$

Solution: $A = UDV^T$ SVD decomposition of A

$$n \times n \quad n \times 9 \quad 9 \times 9$$

f is the last column of V

A5.3 in HZ

# Estimating the Fundamental Matrix

- The singularity constraint $\det \mathbf{F} = 0$

$$\min \|F - F'\|$$

Subject to $\det F' = 0$

$$F = UDV^T$$

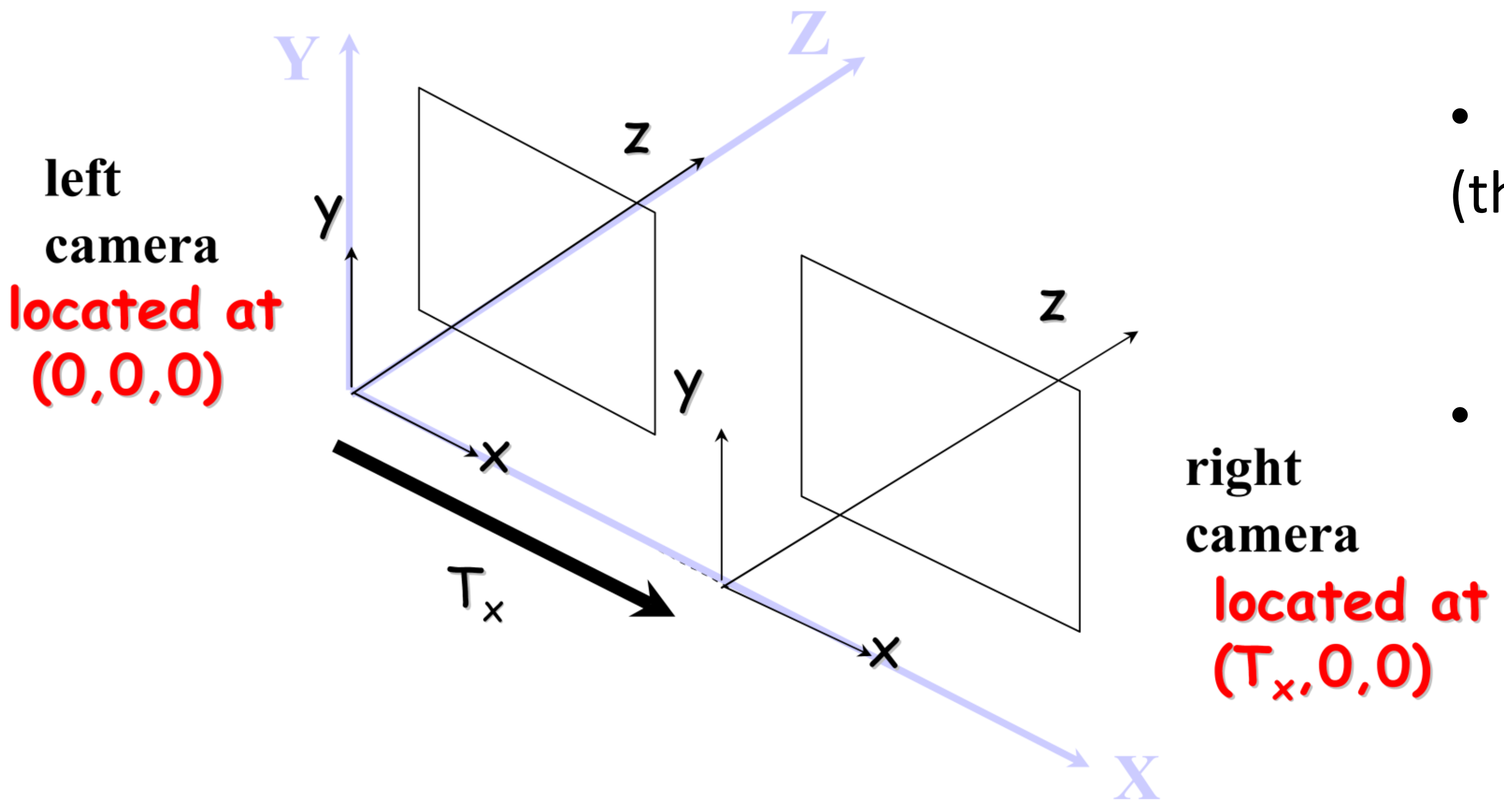
$$\mathtt{D} = \mathrm{diag}(r, s, t)$$

$$r \geq s \geq t.$$

Solution:

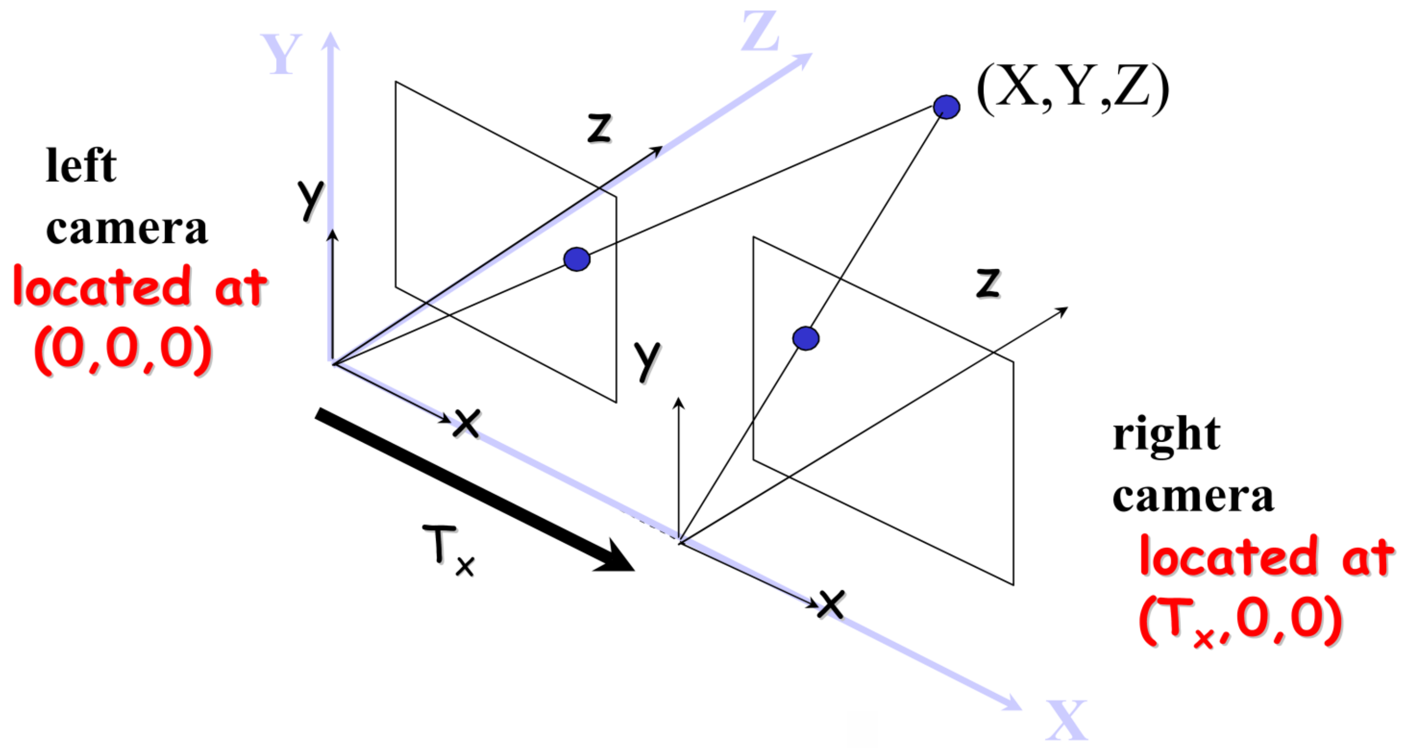
$$\mathtt{F}' = \mathtt{U}\mathrm{diag}(r, s, 0)\mathtt{V}^\mathsf{T}$$

# Special Case: A Stereo System




- The right camera is shifted by $T_x$ (the stereo baseline)
- The camera intrinsics are the same

# Special Case: A Stereo System




- Left camera

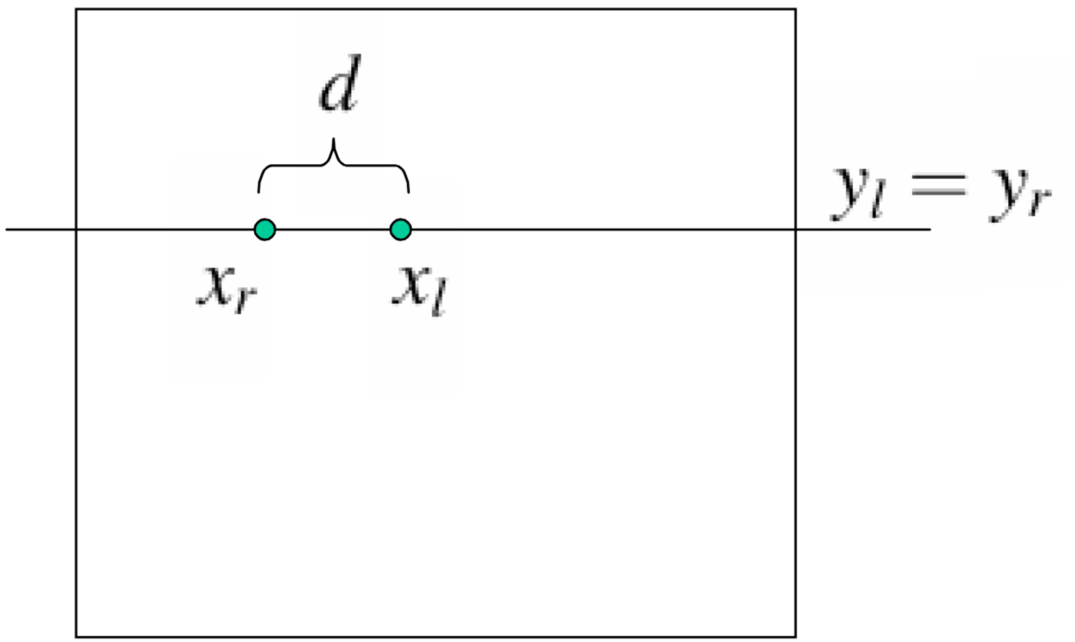
$$x_l = f\frac{X}{Z} + p_x \qquad y_l = f\frac{Y}{Z} + p_y$$

- Right camera

$$x_r = f\frac{X - T_x}{Z} + p_x$$

$$y_r = f\frac{Y}{Z} + p_y$$

# Stereo Disparity




- Disparity

$$
\begin{aligned}
d &= x_l - x_r \\
&= (f\frac{X}{Z} + p_x) - (f\frac{X - T_x}{Z} + p_x) \\
&= f\frac{T_x}{Z}
\end{aligned}
$$

- Depth

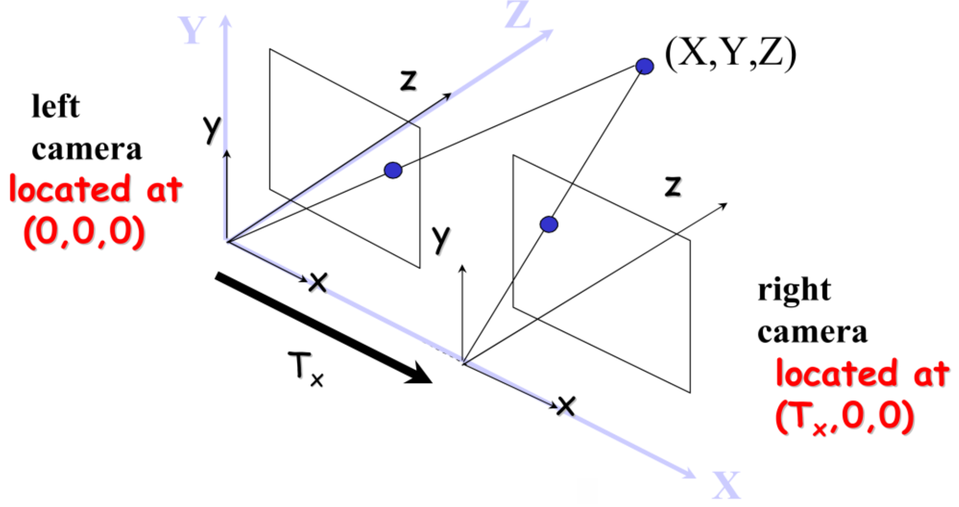
$$Z = f\frac{T_x}{d}$$

Baseline ($T_x$)

Disparity ($d$)

Recall motion parallax: near objects move faster (large disparity)

# Special Case: A Stereo System




$$\mathtt{P} = \mathtt{K}[\mathtt{I} \mid \mathbf{0}] \qquad \mathtt{P}' = \mathtt{K}[\mathtt{I} \mid \mathbf{t}]$$

$$\mathtt{F} = [\mathbf{e}']_\times \mathtt{K}'\mathtt{R}\mathtt{K}^{-1} = \mathtt{K}'^{-\mathsf{T}}[\mathbf{t}]_\times \mathtt{R}\mathtt{K}^{-1} = \mathtt{K}'^{-\mathsf{T}}\mathtt{R}[\mathtt{R}^\mathsf{T}\mathbf{t}]_\times \mathtt{K}^{-1} = \mathtt{K}'^{-\mathsf{T}}\mathtt{R}\mathtt{K}^\mathsf{T}[\mathbf{e}]_\times$$

$$\mathtt{F} = [\mathbf{e}']_\times \mathtt{K}\mathtt{K}^{-1} = [\mathbf{e}']_\times$$

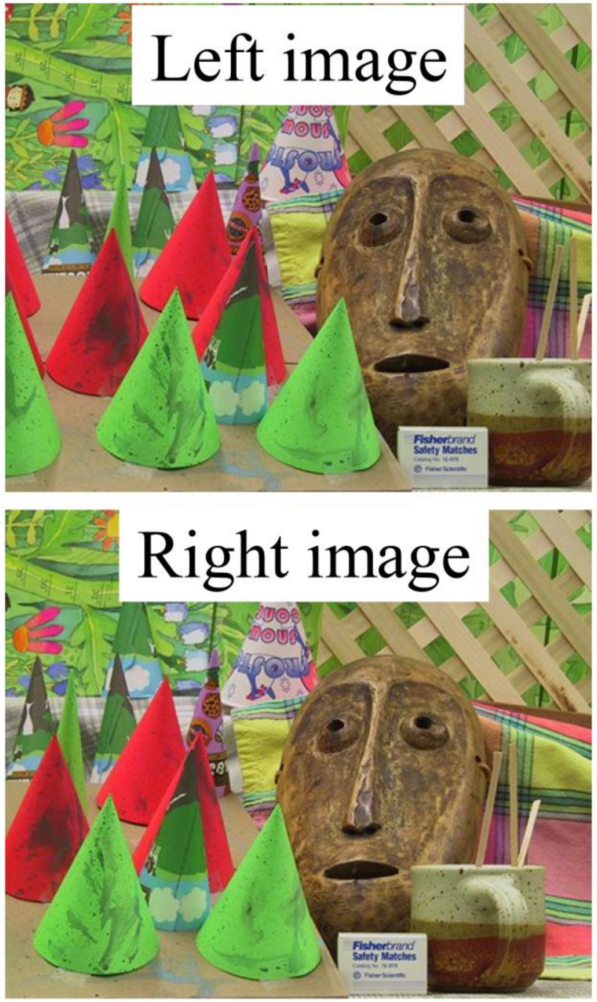
$$\mathbf{e}' = (P'C) \qquad \mathbf{C} = \begin{pmatrix} \mathbf{0} \\ 1 \end{pmatrix}$$

$$K = \begin{bmatrix} f_x & 0 & p_x \\ 0 & f_y & p_y \\ 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{e}' = \begin{bmatrix} f_x T_x \\ 0 \\ 0 \end{bmatrix} \qquad F = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -f_x T_x \\ 0 & f_x T_x & 0 \end{bmatrix}$$

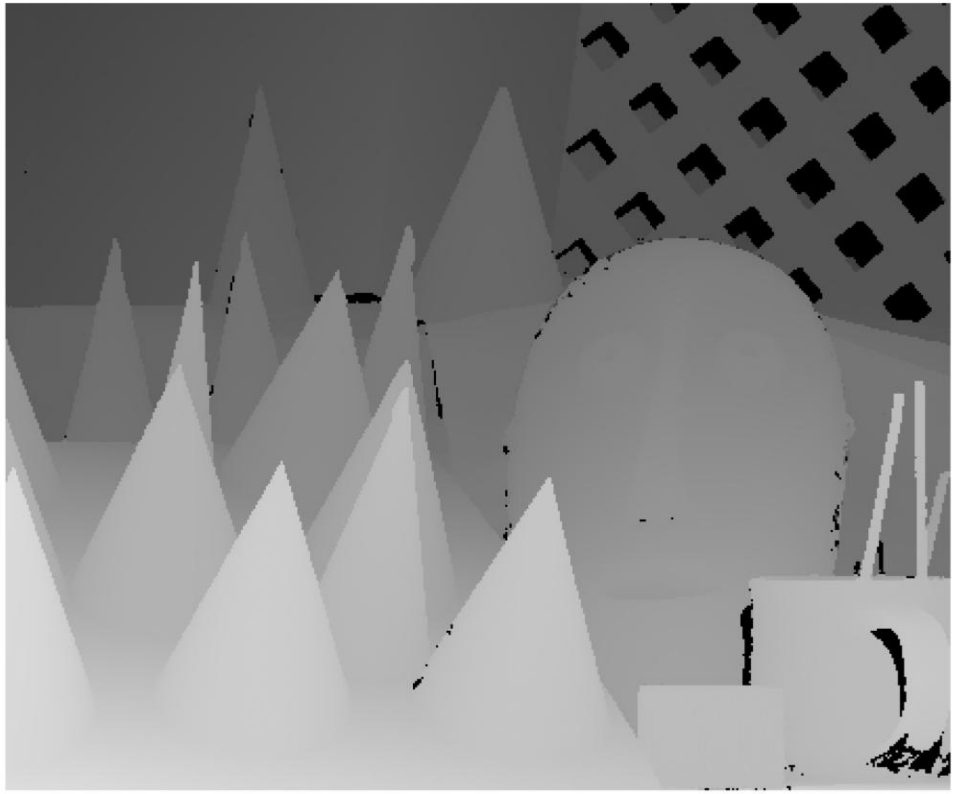
$$\mathbf{x}'^T F \mathbf{x} = 0$$

$$y = y'$$

# Stereo Example




Disparity values (0-64)

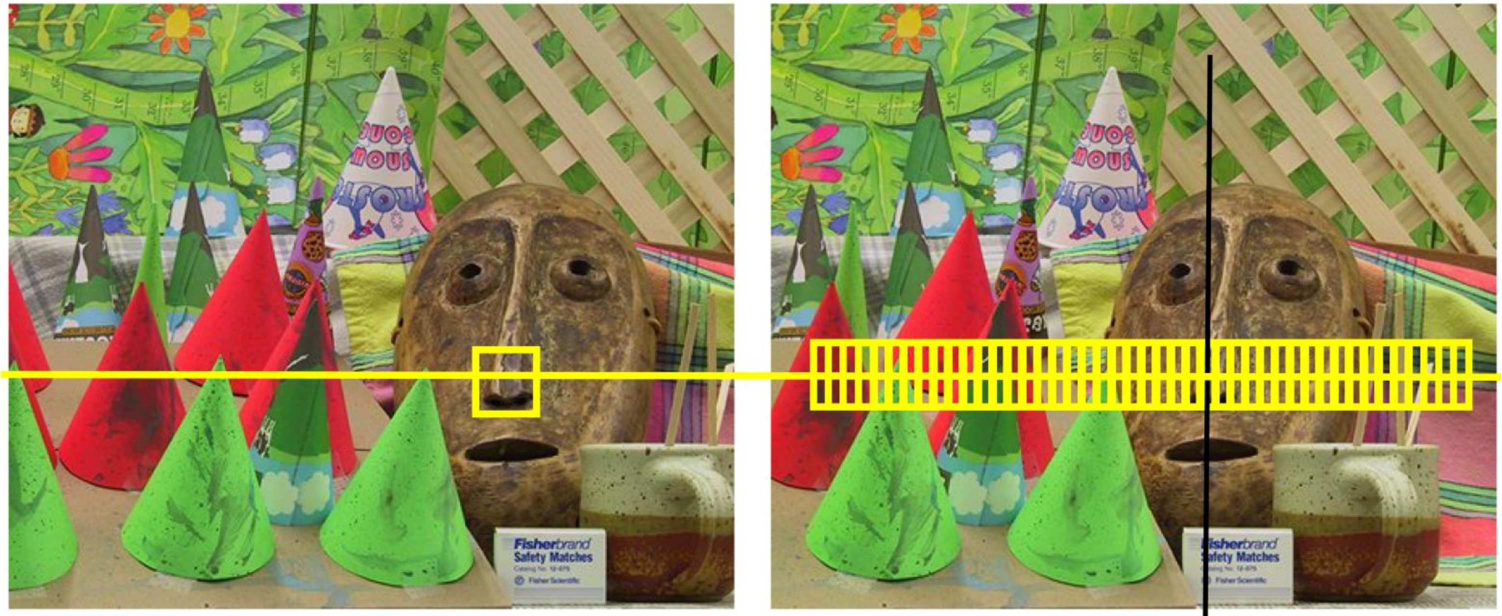


$$d = f\frac{T_x}{Z}$$

Note how disparity is larger (brighter) for closer surfaces.

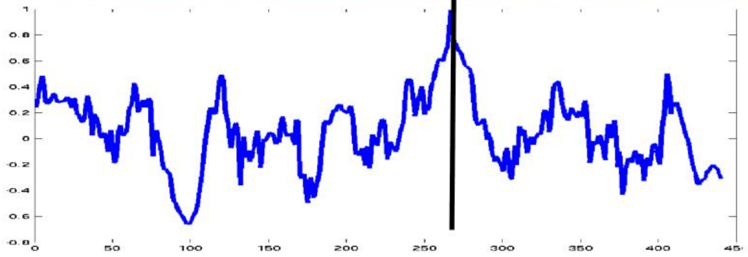
# Computing Disparity

Left Image　　　　　　　　　　Right Image



For a patch in left image

Compare with patches along same row in right image



Match Score Values

- Eipipolar lines are horizontal lines in stereo
- For general cases, we can find correspondences on eipipolar lines
- Depth from disparity

$$Z = f \frac{T_x}{d}$$

# Further Reading

- Stanford CS231A: Computer Vision, From 3D Reconstruction to Recognition, Lecture 5 https://web.stanford.edu/class/cs231a/syllabus.html

- Multiview Geometry in Computer Vision, Richard Hartley and Andrew Zisserman, Chapter 9, Epipolar Geometry and Fundamental Matrix

- https://nvlabs.github.io/FoundationStereo/