Epipolar Geometry and Stereo

CS 4391 Introduction Computer Vision

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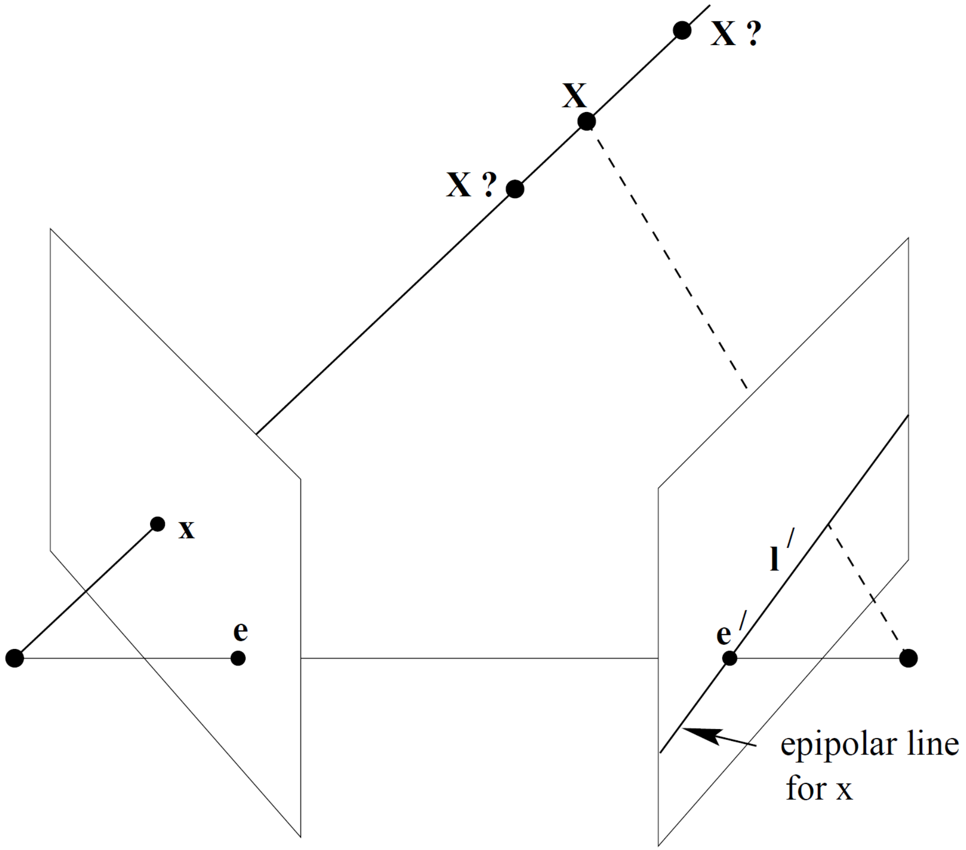
Recall Fundamental Matrix



- Epipolar line $\mathbf{l}' = F\mathbf{x}$

$$\mathbf{l} = F^T\mathbf{x}'$$

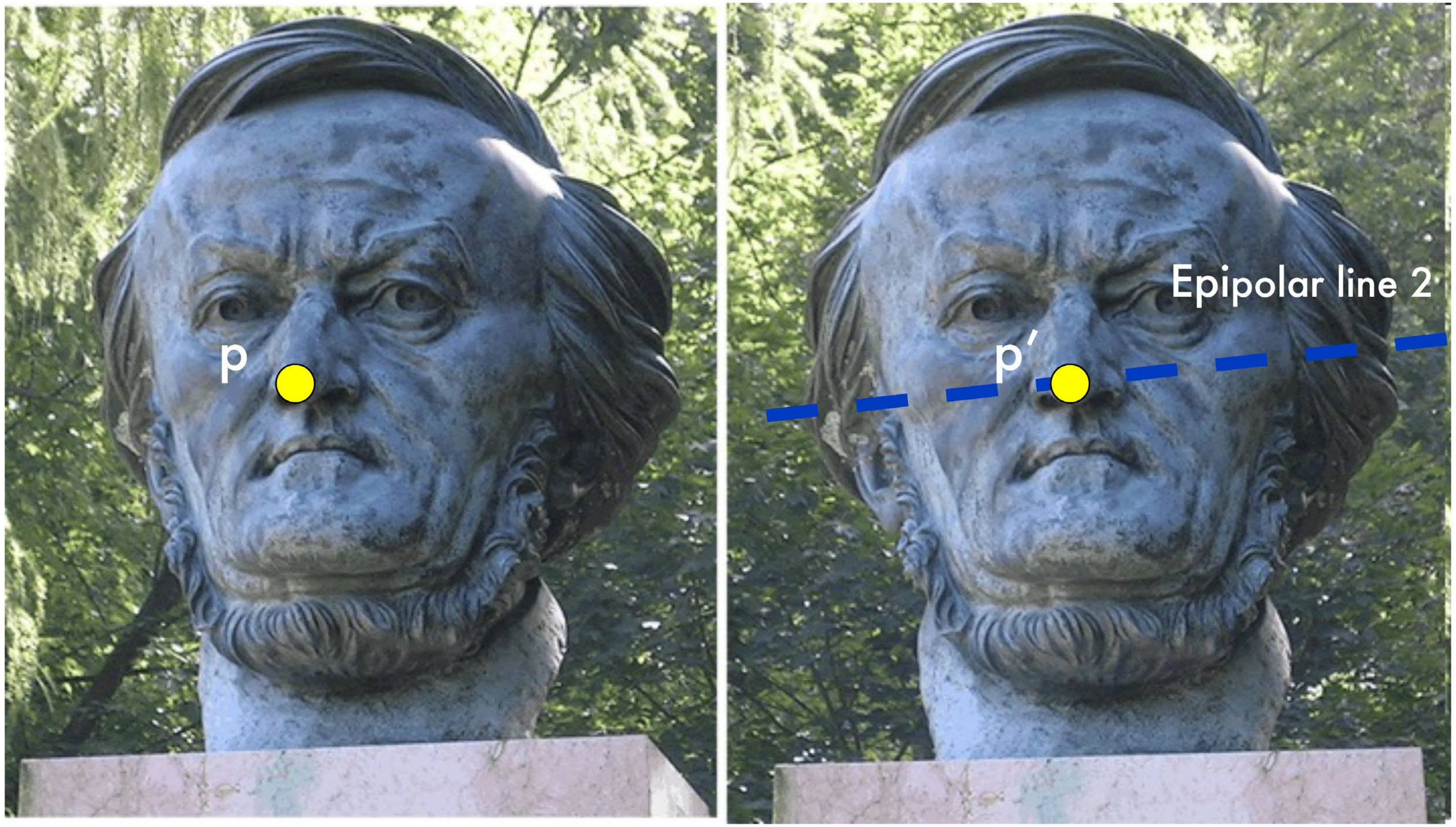
- Fundamental matrix

$$\underset{3x3}{F} = [\mathbf{e}']_\times P'P^+$$

Epipole $\mathbf{e}' = (P'C)$

$$P^+ = P^T(PP^T)^{-1}$$

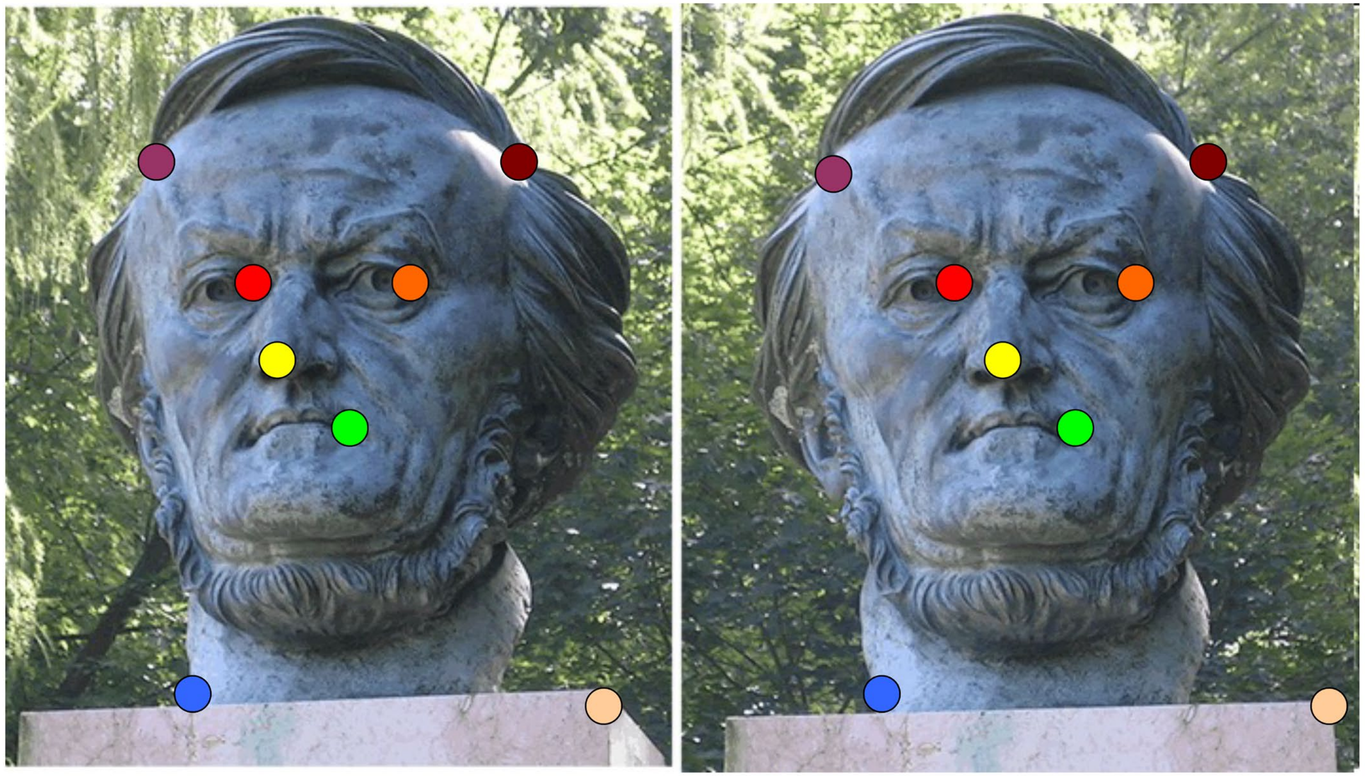
Why the Fundamental Matrix is Useful?



$$\mathbf{l}' = F\mathbf{p}$$

Estimating the Fundamental Matrix

- The 8-point algorithm



$$\mathbf{l}' = F\mathbf{x}$$

$$\mathbf{x}'^{\top}\mathbf{F}\mathbf{x} = 0$$

Estimating the Fundamental Matrix

$$\mathbf{x}'^{\mathsf{T}}\mathbf{F}\mathbf{x} = 0 \qquad \mathbf{x} = (x, y, 1)^{\mathsf{T}} \qquad \mathbf{x}' = (x', y', 1)^{\mathsf{T}}$$

$$x'xf_{11} + x'yf_{12} + x'f_{13} + y'xf_{21} + y'yf_{22} + y'f_{23} + xf_{31} + yf_{32} + f_{33} = 0$$

$$(x'x, x'y, x', y'x, y'y, y', x, y, 1)\,\mathbf{f} = 0$$

n correspondences

$$\mathtt{A}\mathbf{f} = \begin{bmatrix} x'_1x_1 & x'_1y_1 & x'_1 & y'_1x_1 & y'_1y_1 & y'_1 & x_1 & y_1 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \\ x'_nx_n & x'_ny_n & x'_n & y'_nx_n & y'_ny_n & y'_n & x_n & y_n & 1 \end{bmatrix}\mathbf{f} = \mathbf{0}$$

Linear System

$$\underset{n \times 9}{A}\underset{9 \times 1}{\mathbf{f}} = 0$$

- Find non-zero solutions
- If f is a solution, k×f is also a solution for $k \in \mathcal{R}$
- If the rank of A is 8, unique solution (up to scale)
- Otherwise, we can seek a solution $||\mathbf{f}|| = 1$

$$\min ||A\mathbf{f}||$$

Subject to $||\mathbf{f}|| = 1$

Solution: $A = \underset{n \times n}{U}\,\underset{n \times 9}{D}\,\underset{9 \times 9}{V^T}$ SVD decomposition of A

f is the last column of V

A5.3 in HZ

Estimating the Fundamental Matrix

- The singularity constraint $\det \mathbf{F} = 0$

$$\min \|F - F'\|$$

Subject to $\det F' = 0$

$$F = UDV^T$$

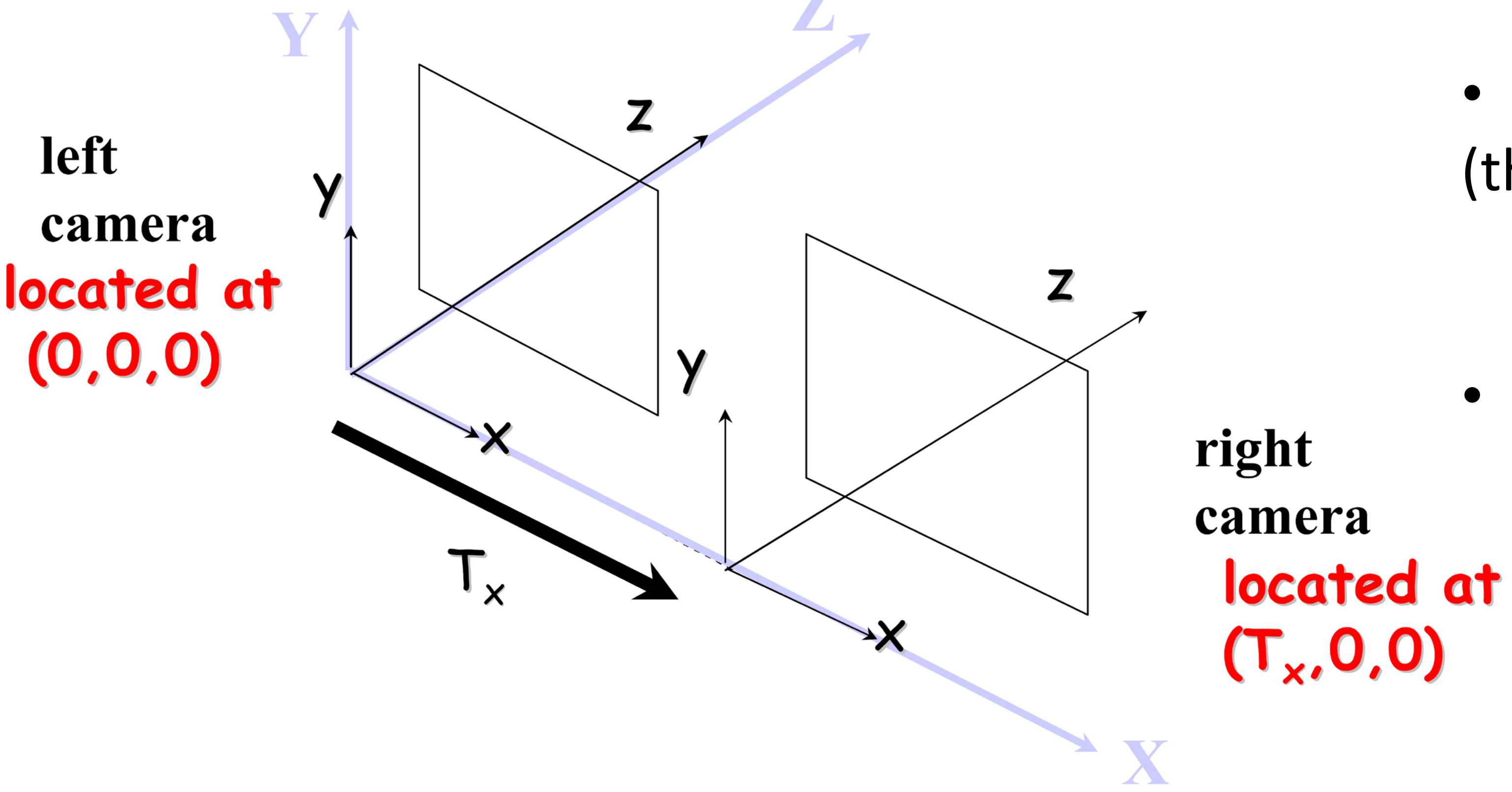
$$\mathtt{D} = \mathrm{diag}(r, s, t)$$

$$r \geq s \geq t.$$

Solution:

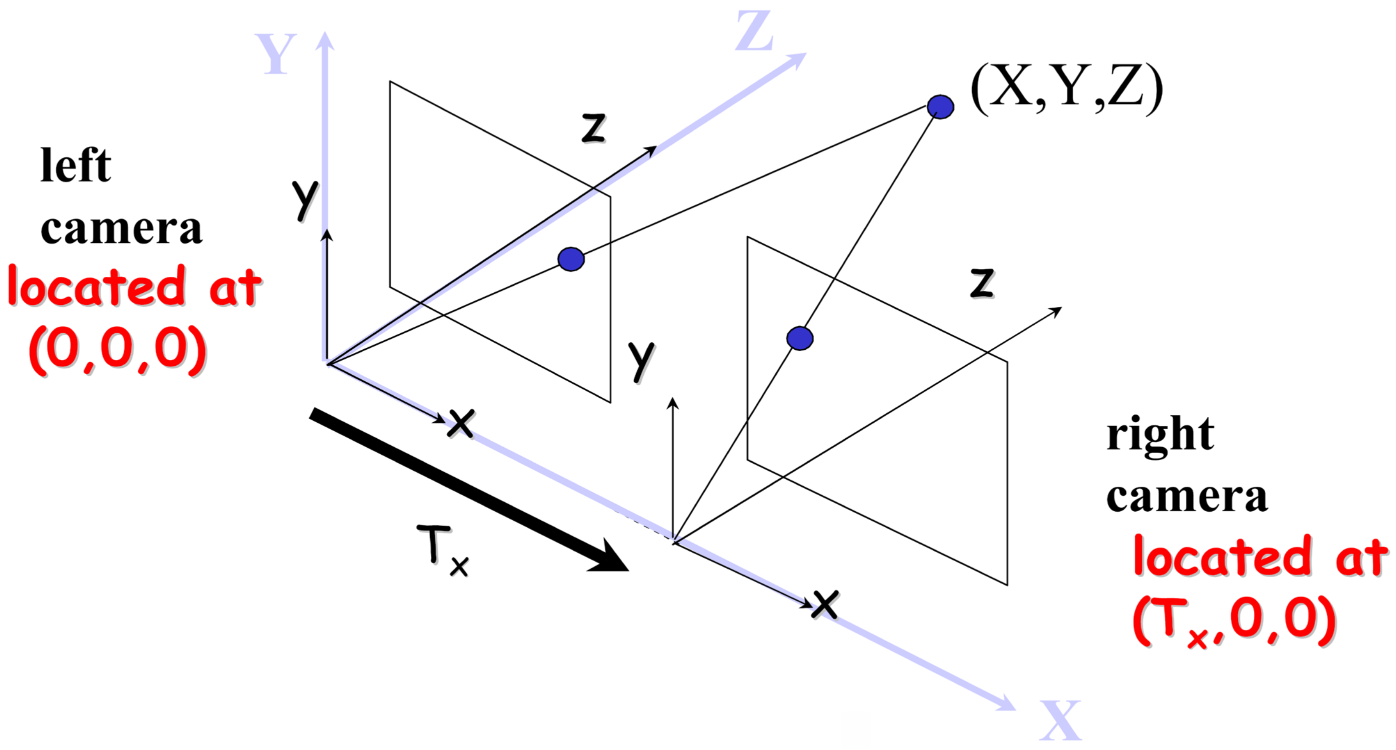
$$\mathtt{F}' = \mathtt{U}\mathrm{diag}(r, s, 0)\mathtt{V}^{\mathsf{T}}$$

Special Case: A Stereo System



- The right camera is shifted by T_x (the stereo baseline)
- The camera intrinsics are the same

Special Case: A Stereo System



left camera located at (0,0,0)

right camera located at (T_x,0,0)

- Left camera

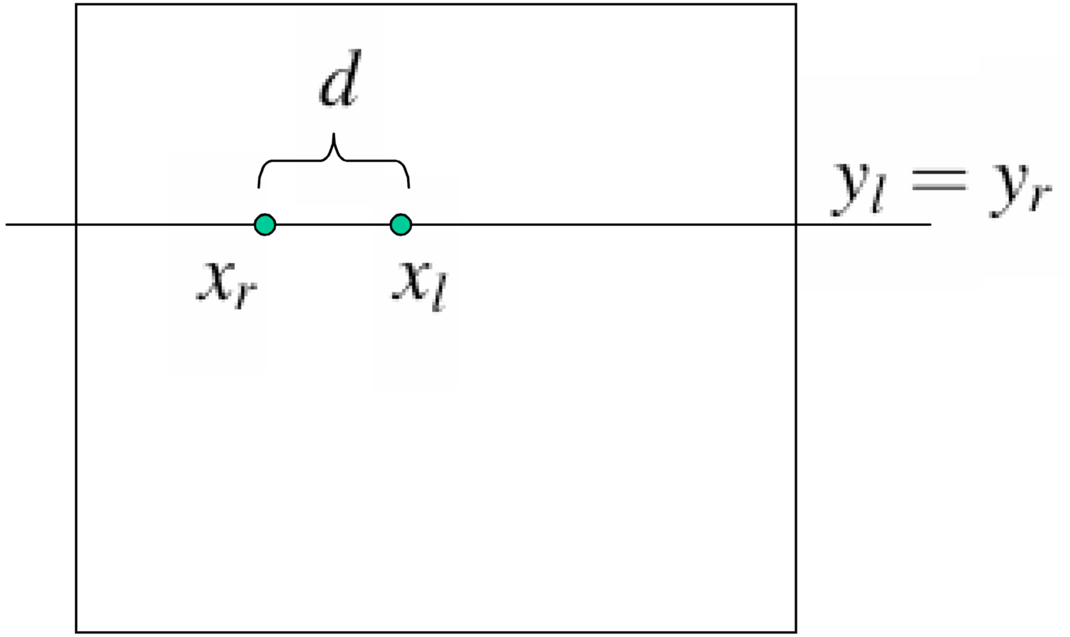
$$x_l = f\frac{X}{Z} + p_x \qquad y_l = f\frac{Y}{Z} + p_y$$

- Right camera

$$x_r = f\frac{X - T_x}{Z} + p_x$$

$$y_r = f\frac{Y}{Z} + p_y$$

Stereo Disparity



- Disparity

$$\begin{aligned} d &= x_l - x_r \\ &= \left(f\frac{X}{Z} + p_x\right) - \left(f\frac{X - T_x}{Z} + p_x\right) \\ &= f\frac{T_x}{Z} \end{aligned}$$

- Depth

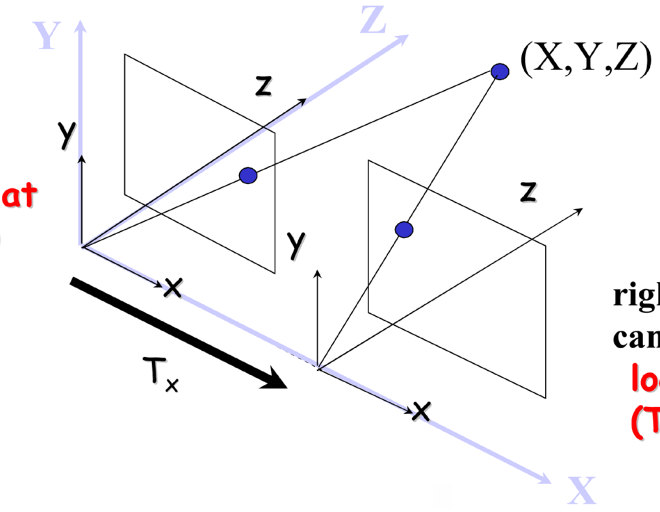
$$Z = f\frac{T_x}{d}$$

Baseline

Disparity

Recall motion parallax: near objects move faster (large disparity)

Special Case: A Stereo System



left camera located at (0,0,0)

right camera located at (T_x,0,0)

$$\mathrm{P} = \mathrm{K}[\mathrm{I} \mid \mathbf{0}] \qquad \mathrm{P}' = \mathrm{K}[\mathrm{I} \mid \mathbf{t}]$$

$$\mathrm{F} = [\mathbf{e}']_\times \mathrm{K}'\mathrm{R}\mathrm{K}^{-1} = \mathrm{K}'^{-\mathsf{T}}[\mathbf{t}]_\times \mathrm{R}\mathrm{K}^{-1} = \mathrm{K}'^{-\mathsf{T}}\mathrm{R}[\mathrm{R}^\mathsf{T}\mathbf{t}]_\times \mathrm{K}^{-1} = \mathrm{K}'^{-\mathsf{T}}\mathrm{R}\mathrm{K}^\mathsf{T}[\mathbf{e}]_\times$$

$$\mathbf{F} = [\mathbf{e}']_\times \mathbf{K}\mathbf{K}^{-1} = [\mathbf{e}']_\times$$

$$\mathbf{e}' = (P'C) \qquad \mathbf{C} = \begin{pmatrix} \mathbf{0} \\ 1 \end{pmatrix}$$

$$K = \begin{bmatrix} f_x & 0 & p_x \\ 0 & f_y & p_y \\ 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{e}' = \begin{bmatrix} f_x T_x \\ 0 \\ 0 \end{bmatrix} \qquad F = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -f_x T_x \\ 0 & f_x T_x & 0 \end{bmatrix}$$

$$\mathbf{x}'^T F \mathbf{x} = 0$$

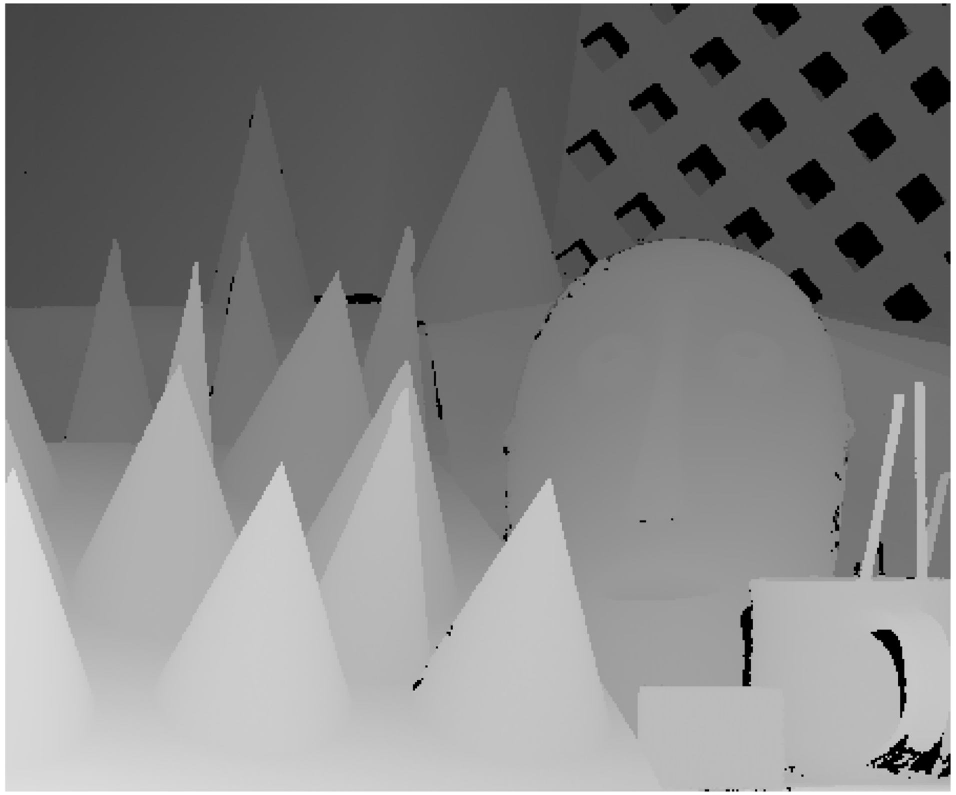
$$y = y'$$

Stereo Example


Left image


Right image

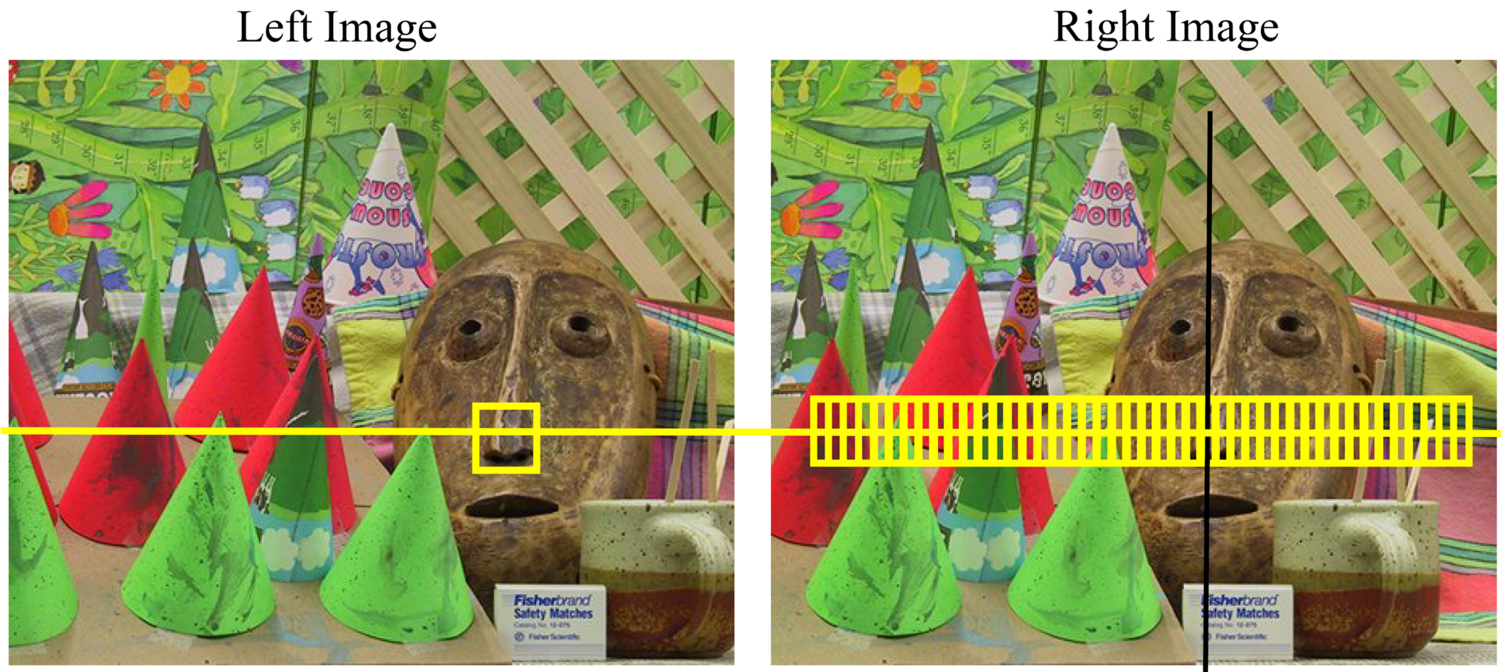
Disparity values (0-64)



$$d = f\frac{T_x}{Z}$$

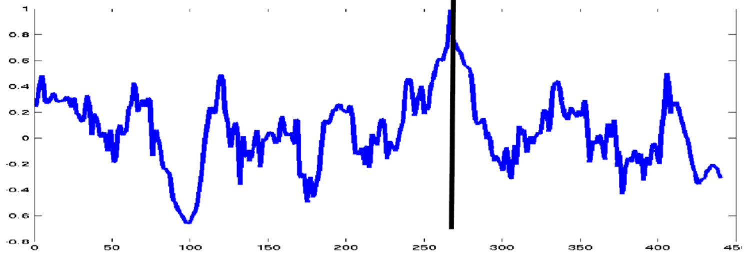
Note how disparity is larger (brighter) for closer surfaces.

Computing Disparity



For a patch in left image

Compare with patches along same row in right image



Match Score Values

- Eipipolar lines are horizontal lines in stereo
- For general cases, we can find correspondences on eipipolar lines
- Depth from disparity

$$Z = f\frac{T_x}{d}$$

Further Reading

- Stanford CS231A: Computer Vision, From 3D Reconstruction to Recognition, Lecture 5 https://web.stanford.edu/class/cs231a/syllabus.html

- Multiview Geometry in Computer Vision, Richard Hartley and Andrew Zisserman, Chapter 9, Epipolar Geometry and Fundamental Matrix