Camera Projection

CS 4391 Introduction Computer Vision

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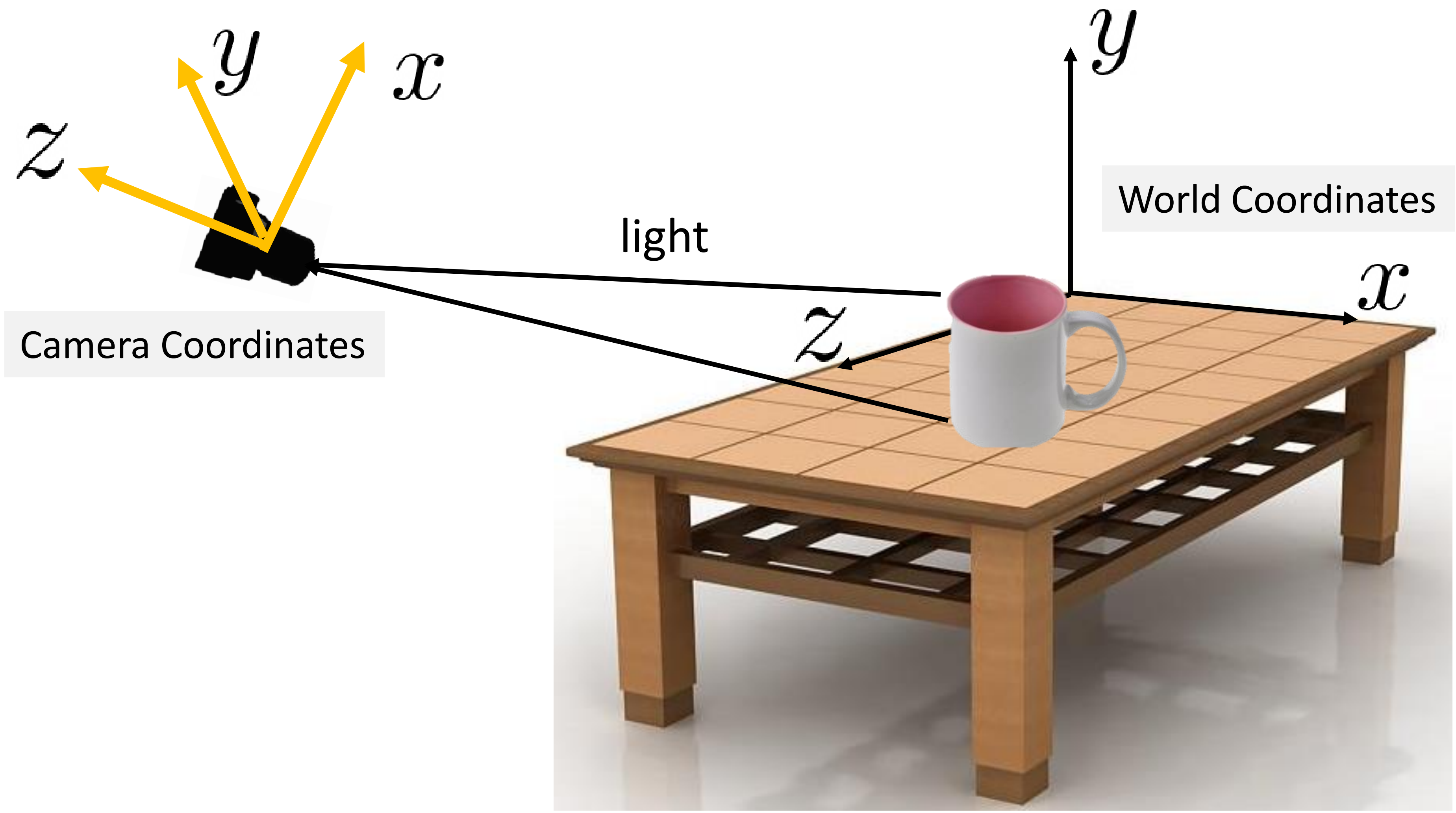
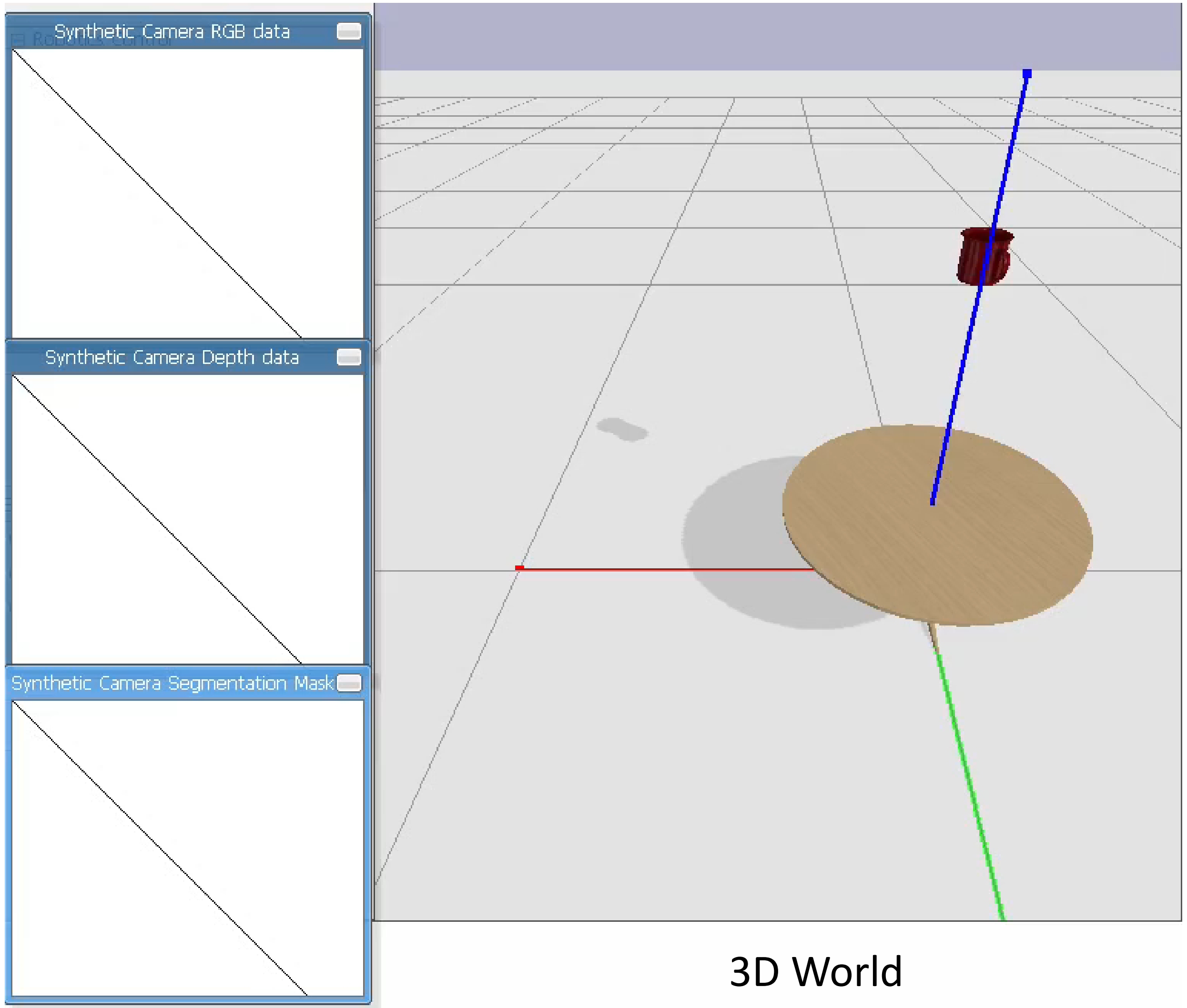
Some slides of this lecture are courtesy Silvio Savarese

A Camera in the 3D World



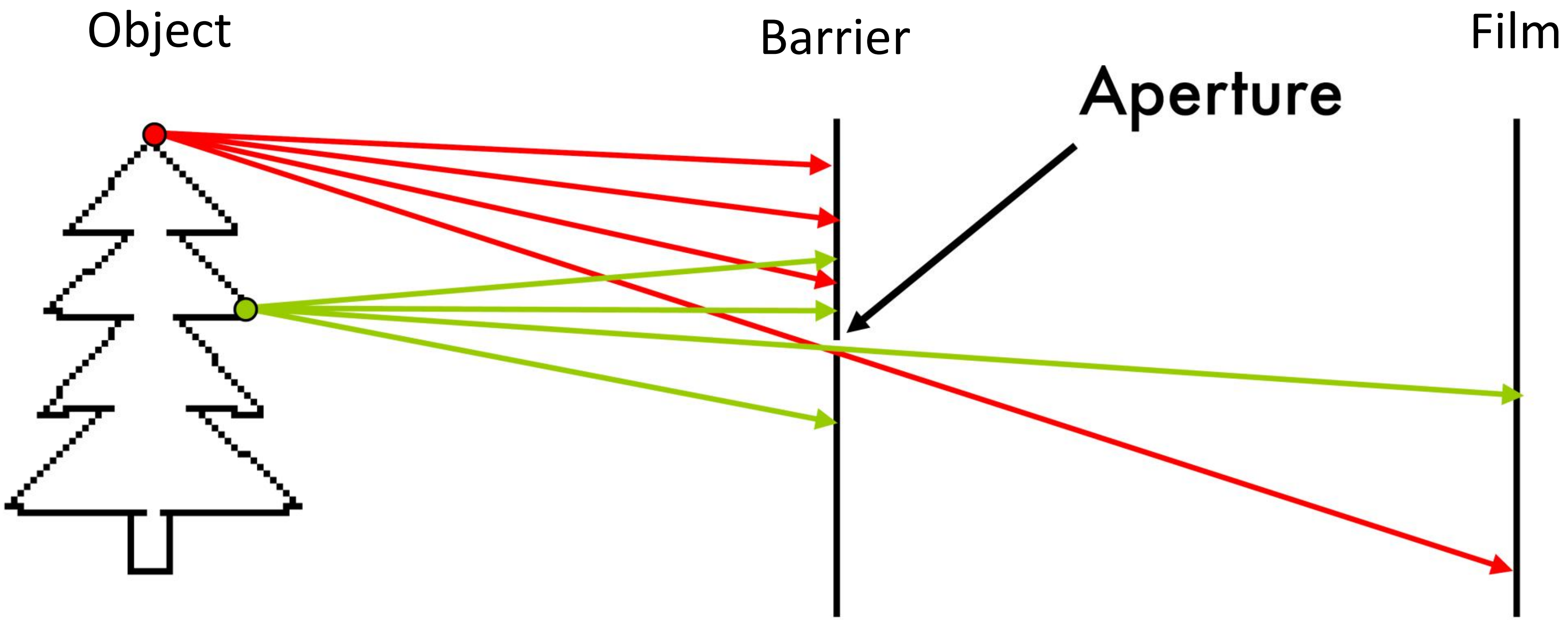
PyBullet with a Camera

Camera View

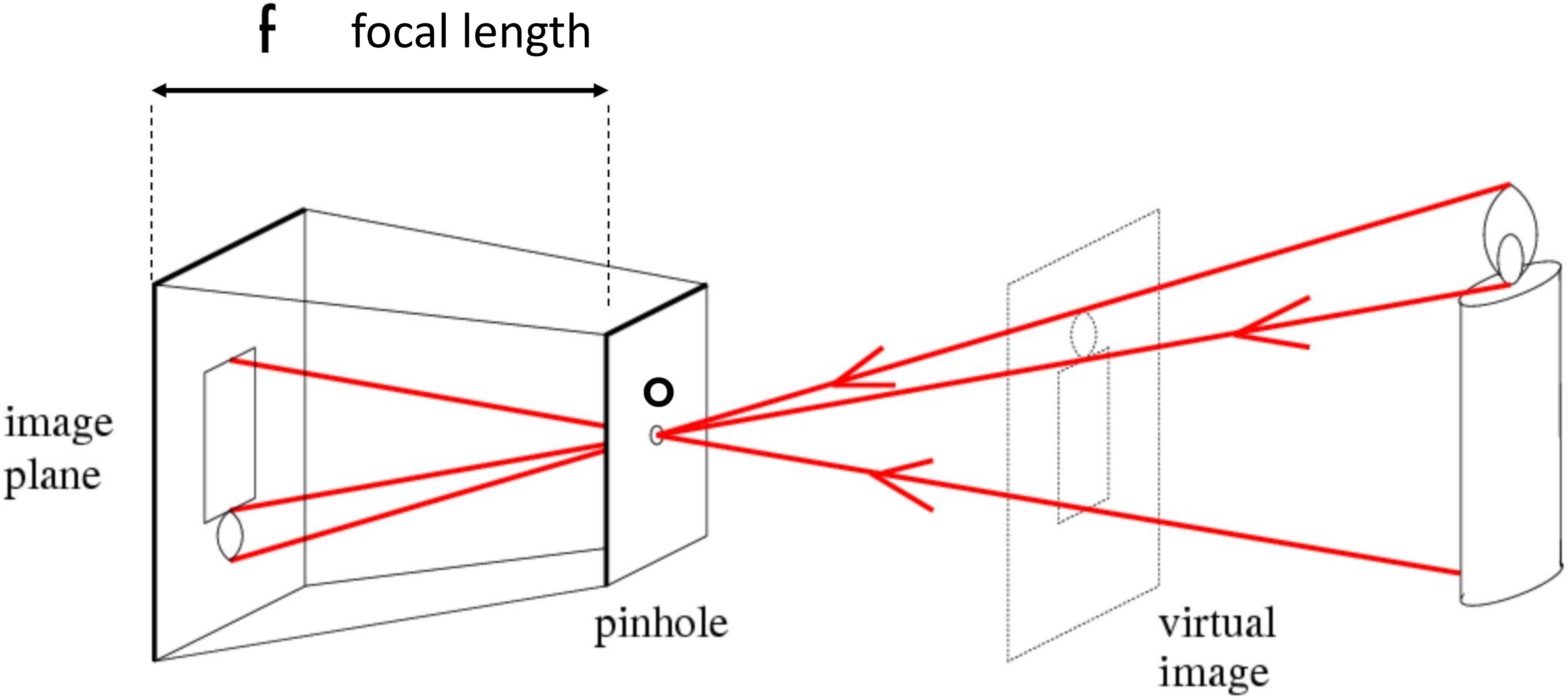


3D World

Pinhole Camera



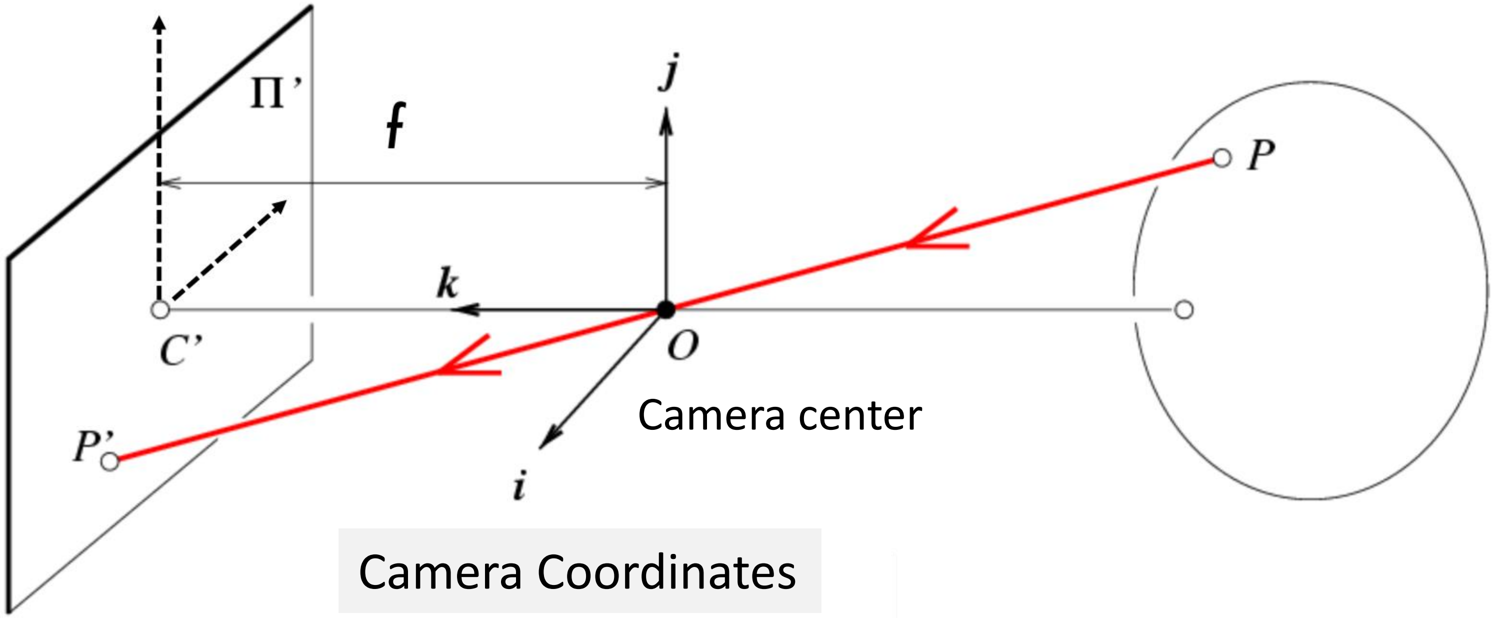
Pinhole Camera

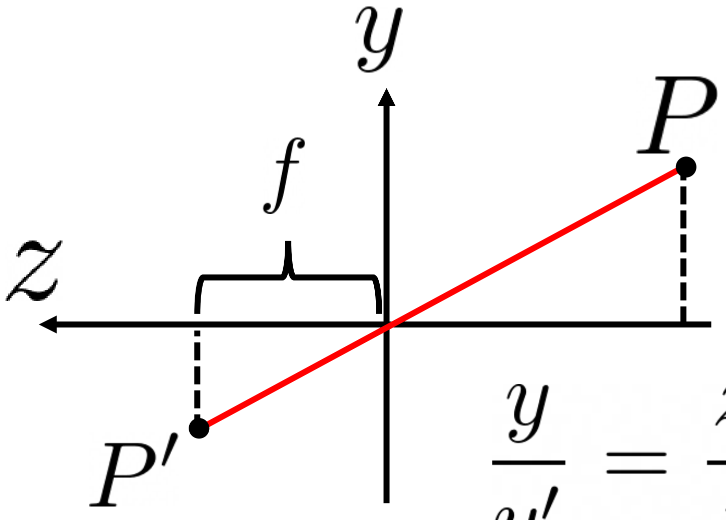


Rotate the image plane by 180°

Cannot be implemented in practice
Useful for theoretic analysis

Central Projection in Camera Coordinates





$$\frac{y}{y'} = \frac{z}{f}$$

Camera coordinates $\mathrm{P} = \begin{bmatrix} \mathrm{x} \\ \mathrm{y} \\ \mathrm{z} \end{bmatrix} \rightarrow \mathrm{P}' = \begin{bmatrix} \mathrm{x}' \\ \mathrm{y}' \end{bmatrix}$

$$z' = f$$

$$\begin{cases} \mathrm{x}' = \mathrm{f}\,\dfrac{\mathrm{x}}{\mathrm{z}} \\ \mathrm{y}' = \mathrm{f}\,\dfrac{\mathrm{y}}{\mathrm{z}} \end{cases}$$

Central Projection with Homogeneous Coordinates

$$\mathrm{P} = \begin{bmatrix} \mathrm{x} \\ \mathrm{y} \\ \mathrm{z} \end{bmatrix} \rightarrow \mathrm{P}' = \begin{bmatrix} \mathrm{x}' \\ \mathrm{y}' \end{bmatrix}$$

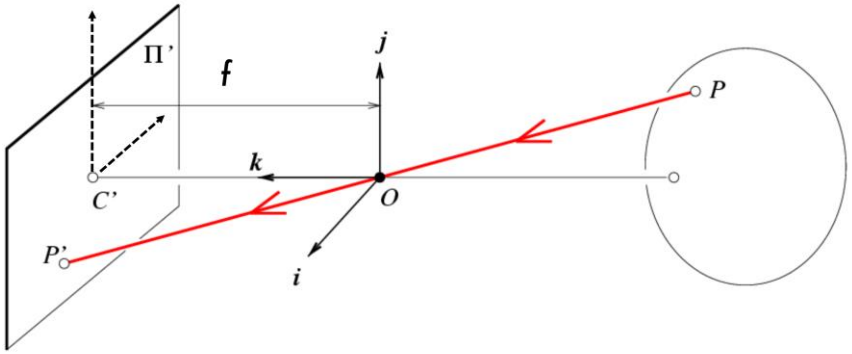
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \longrightarrow \begin{bmatrix} f\frac{x}{z} \\ f\frac{y}{z} \end{bmatrix}$$

Central projection

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \longrightarrow \begin{bmatrix} fx \\ fy \\ z \end{bmatrix} = \begin{bmatrix} f & & & 0 \\ & f & & 0 \\ & & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

3x4 matrix

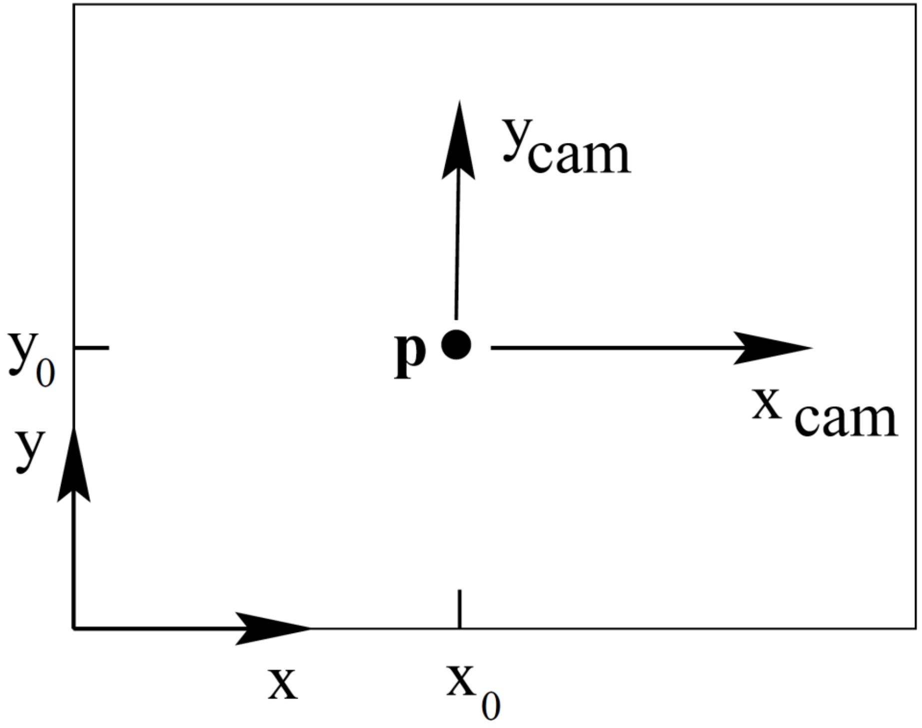
Principal Point Offset



Principal point $\mathbf{p} = (p_x, p_y)$

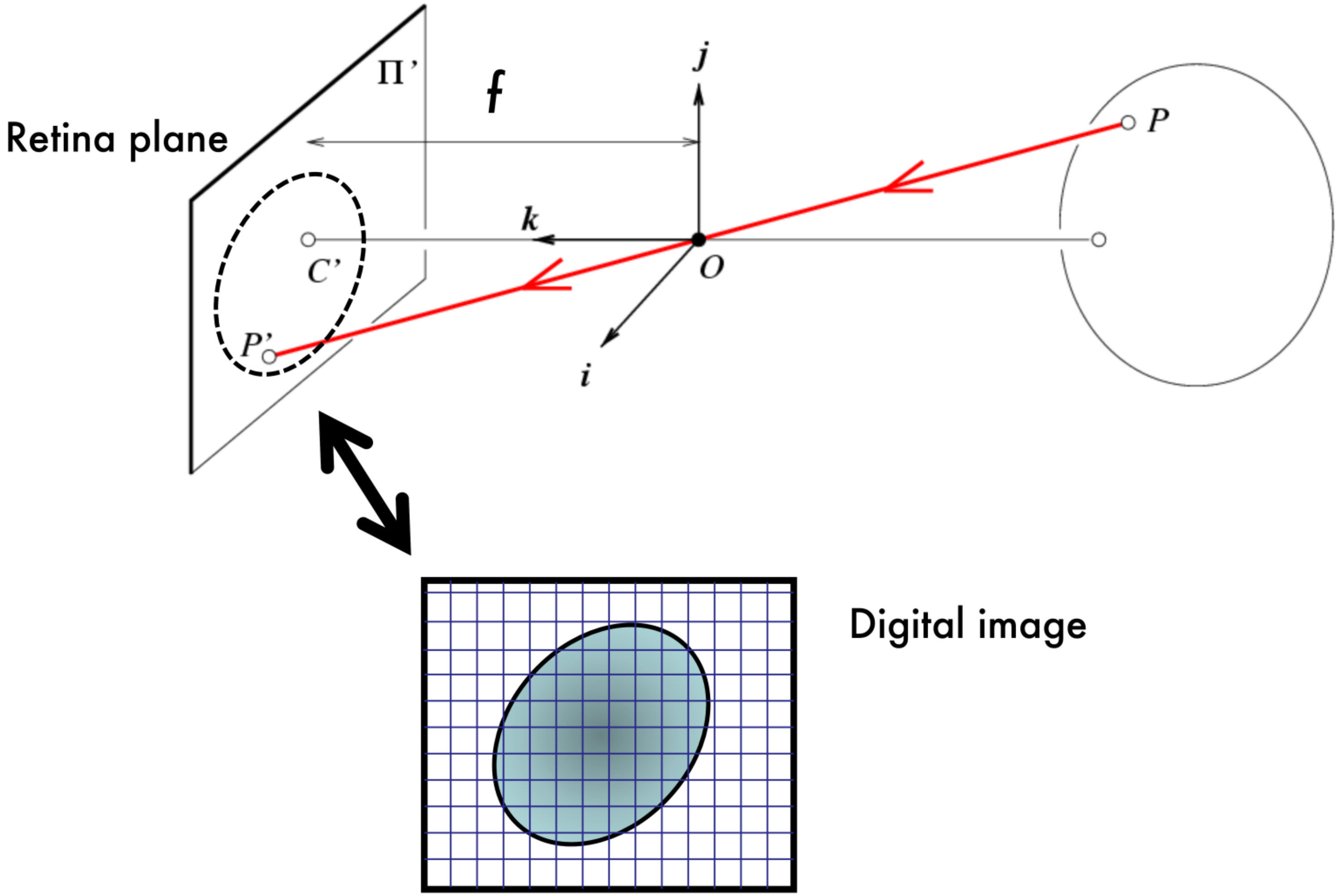
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \longrightarrow \begin{bmatrix} f\frac{x}{z} + p_x \\ f\frac{y}{z} + p_y \end{bmatrix}$$

$$\begin{bmatrix} f & & p_x & 0 \\ & f & p_y & 0 \\ & & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$



Principle point: projection of the camera center

From Metric to Pixels



Pixels, bottom-left coordinate systems

From Metric to Pixels

- Metric space, i.e., meters

$$\begin{bmatrix} f & & p_x & 0 \\ & f & p_y & 0 \\ & & 1 & 0 \end{bmatrix}$$

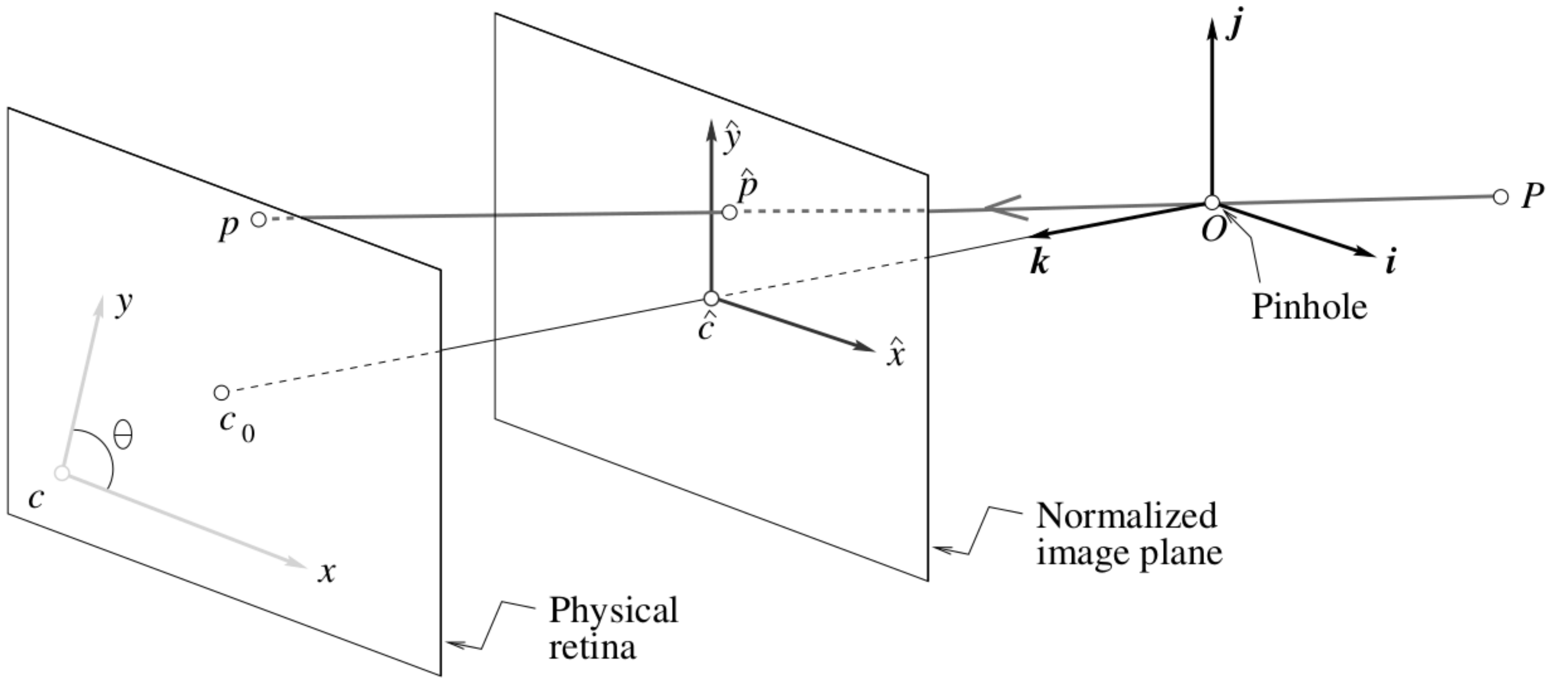
- Pixel space

$$\begin{bmatrix} \alpha_x & & x_0 & 0 \\ & \alpha_y & y_0 & 0 \\ & & 1 & 0 \end{bmatrix}$$

$$\alpha_x = f m_x$$
$$\alpha_y = f m_y$$
$$x_0 = p_x m_x$$
$$y_0 = p_y m_y$$

m_x, m_y Number of pixel per unit distance

Axis Skew



The skew parameter will be zero for most normal cameras.

$$\begin{bmatrix} \alpha_x & & x_0 & 0 \\ & \alpha_y & y_0 & 0 \\ & & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \longrightarrow \begin{bmatrix} \alpha_x \frac{x}{z} + x_0 \\ \alpha_y \frac{y}{z} + y_0 \end{bmatrix}$$

$$\begin{bmatrix} \alpha_x & -\alpha_x \cot(\theta) & x_0 & 0 \\ & \frac{\alpha_y}{\sin(\theta)} & y_0 & 0 \\ & & 1 & 0 \end{bmatrix}$$

https://blog.immenselyhappy.com/post/camera-axis-skew/

Camera Intrinsics

$$\begin{bmatrix} \alpha_x & -\alpha_x \cot(\theta) & x_0 & 0 \\ & \frac{\alpha_y}{\sin(\theta)} & y_0 & 0 \\ & & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Camera intrinsics

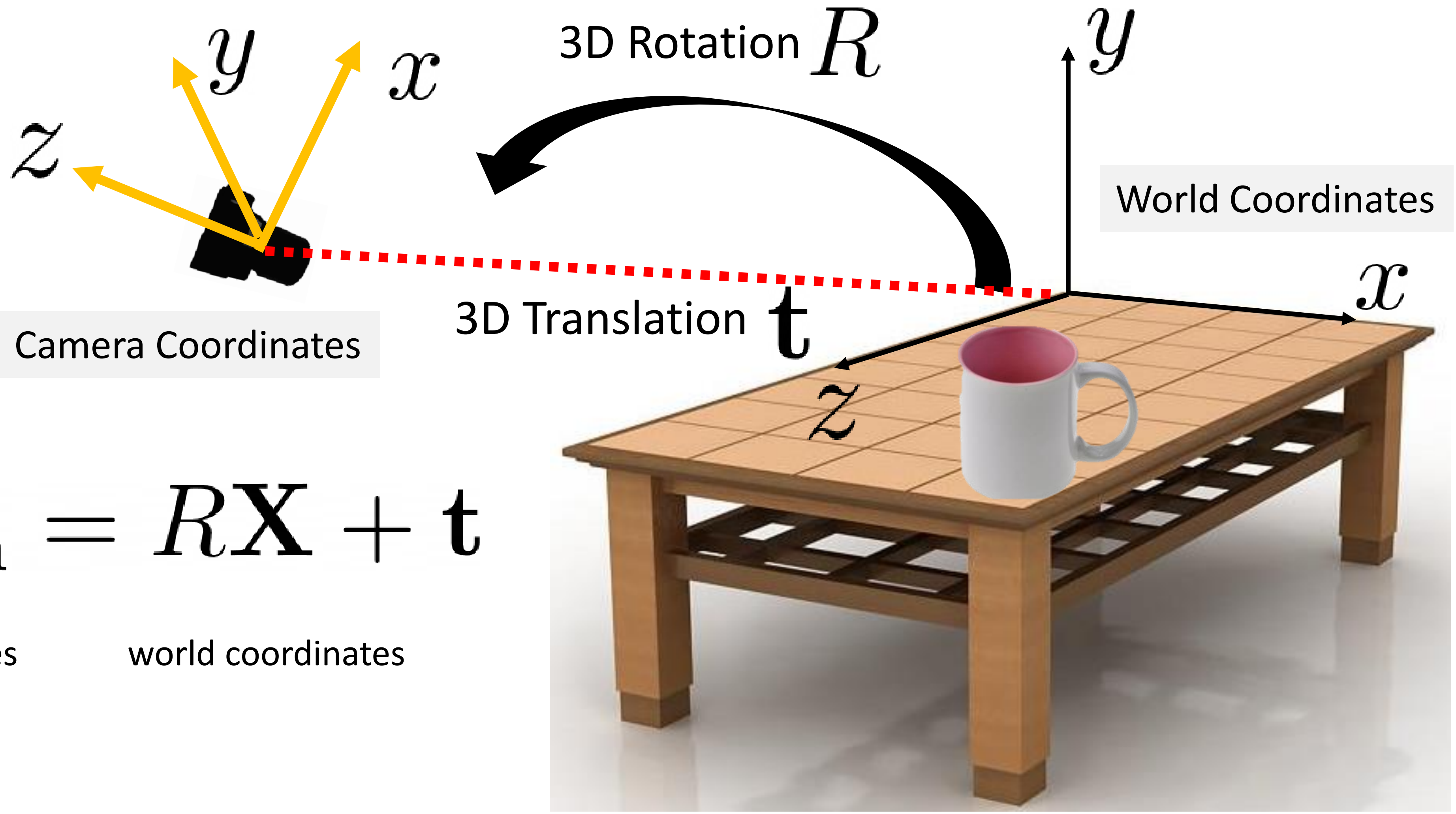
$$K = \begin{bmatrix} \alpha_x & s & x_0 \\ & \alpha_y & y_0 \\ & & 1 \end{bmatrix}$$

$$\mathbf{x} = K[I|\mathbf{0}]\mathbf{X}_{\mathrm{cam}}$$

3x1 3x3 3x4 4x1

Homogeneous coordinates

Camera Extrinsics: Camera Rotation and Translation



$$\mathbf{X}_{\text{cam}} = R\mathbf{X} + \mathbf{t}$$

camera coordinates world coordinates

Camera Projection Matrix $P = K[R|\mathbf{t}]$

- Homogeneous coordinates

$$\mathbf{x} = K[I|\mathbf{0}]\mathbf{X}_{\mathrm{cam}}$$

$$= K[R|\mathbf{t}]\mathbf{X}$$

$$K = \begin{bmatrix} \alpha_x & s & x_0 \\ & \alpha_y & y_0 \\ & & 1 \end{bmatrix}$$

3x1 3x3 3x4 4x1

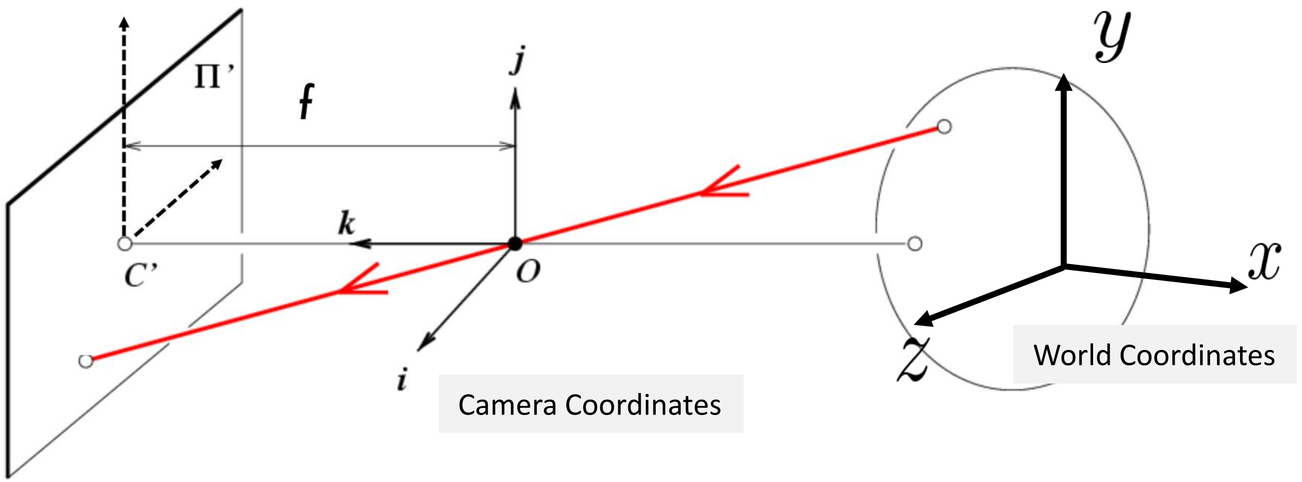
Image coordinates

World coordinates

Camera intrinsics

Camera extrinsics: rotation and translation

Back-projection to a Ray in the World Coordinate



$$P = K[R|\mathbf{t}]$$

$$\mathbf{x} = P\mathbf{X}$$

- A pixel on the image backprojects to a ray in 3D

- The camera center O is on the ray
- $P^+\mathbf{x}$ is on the ray

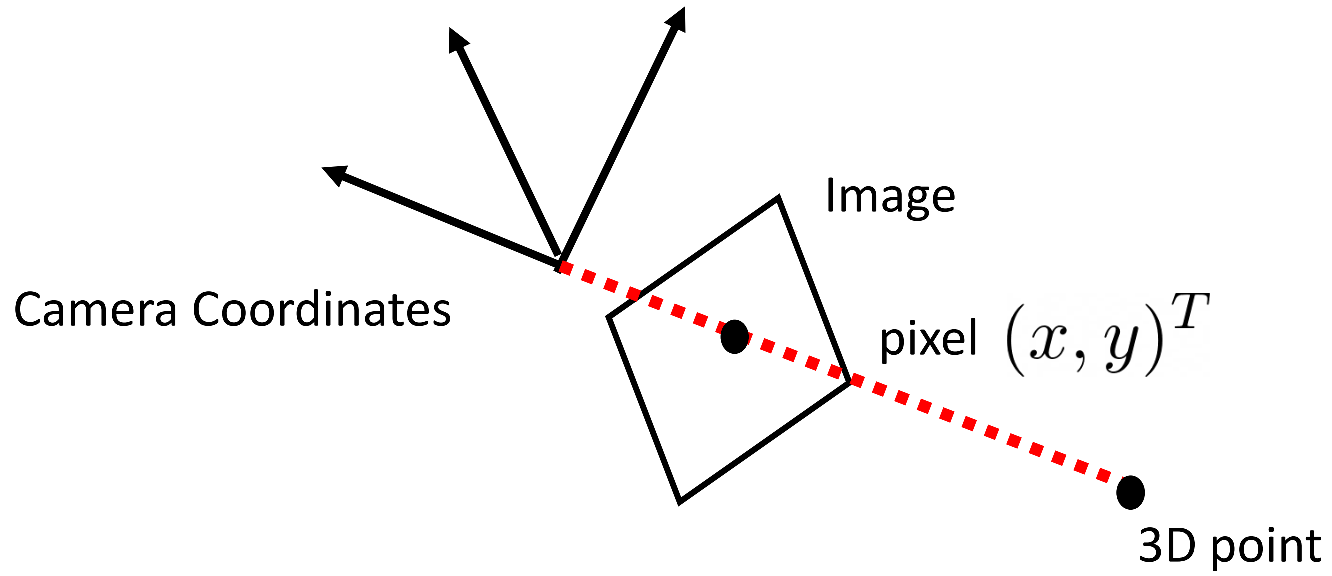
$$P^+ = P^T(PP^T)^{-1}$$

Pseudo-inverse

The ray can be written as

$$\mathbf{X}(\lambda) = (1-\lambda)P^+\mathbf{x} + \lambda O$$

Back-projection to a 3D Point in Camera Coordinates



$$P = K[I|\mathbf{0}]$$

$$\mathbf{x} = K[I|\mathbf{0}]\mathbf{X}_{\text{cam}}$$

$$\begin{bmatrix} f_x & 0 & p_x \\ 0 & f_y & p_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

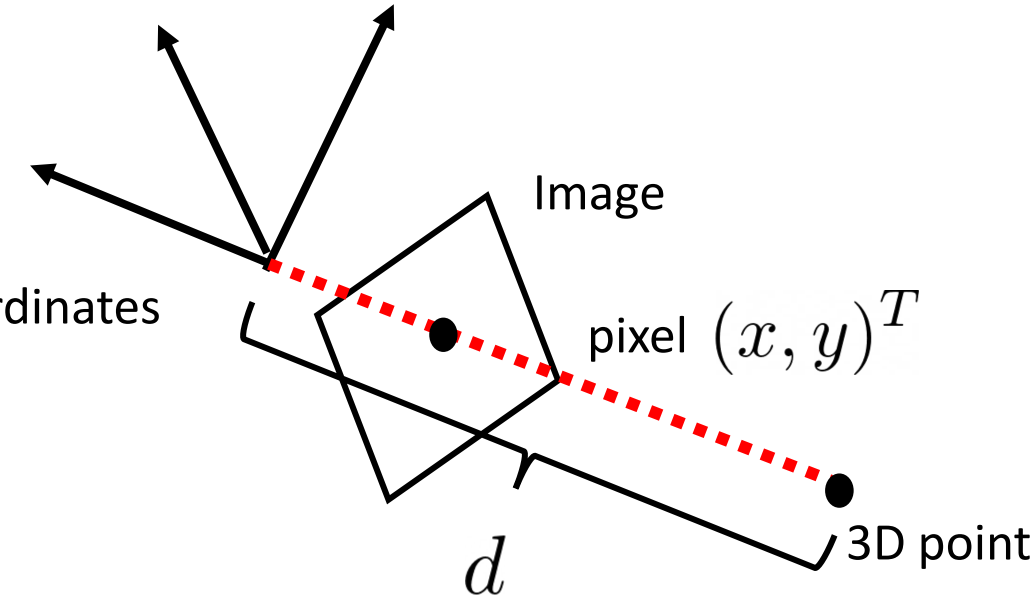
$$x = f\frac{X}{Z} + p_x \qquad \frac{X}{Z} = \frac{x - p_x}{f}$$

$$y = f\frac{Y}{Z} + p_y \qquad \frac{Y}{Z} = \frac{y - p_y}{f}$$

3D Point $\begin{bmatrix} \frac{x-p_x}{f} Z \\ \frac{y-p_y}{f} Z \\ Z \end{bmatrix}$

We need to know the depth of the pixel

Back-projection to a 3D Point in Camera Coordinates



3D camera coordinates $\begin{bmatrix} d\frac{x-p_x}{f_x} \\ d\frac{y-p_y}{f_y} \\ d \end{bmatrix}$

Equivalently

$$\mathbf{x} = K[I|\mathbf{0}]\mathbf{X}_{\text{cam}}$$

$$K^{-1}\mathbf{x}$$

3D point with depth d : $dK^{-1}\mathbf{x}$

$$K = \begin{bmatrix} f_x & 0 & p_x \\ 0 & f_y & p_y \\ 0 & 0 & 1 \end{bmatrix}$$

3D Point $\begin{bmatrix} \frac{x-p_x}{f} Z \\ \frac{y-p_y}{f} Z \\ Z \end{bmatrix}$

The Pinhole Camera Model

- Camera projection matrix: intrinsics and extrinsics

$$P = K[R|\mathbf{t}]$$

3x3 — Camera intrinsics

3x4 — Camera extrinsics: rotation and translation

Further Reading

- Section 2.1, Computer Vision, Richard Szeliski
- Multiview Geometry in Computer Vision, Richard Hartley and Andrew Zisserman, Chapter 6, Camera Models
- Stanford CS231A: Computer Vision, From 3D Reconstruction to Recognition, Lecture 2 https://web.stanford.edu/class/cs231a/syllabus.html
- Image formation by lenses https://courses.lumenlearning.com/physics/chapter/25-6-image-formation-by-lenses/
- Distortion (Wikipedia) https://en.wikipedia.org/wiki/Distortion_(optics)