Keypoint Features II

CS 6384 Computer Vision
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Some slides of this lecture are courtesy Kris Kitani
Feature Detection and Matching

Geometry-aware Feature Matching for Structure from Motion Applications. Shah et al, WACV’15

Applications: stereo matching, image stitching, 3D reconstruction, camera pose estimation, object recognition
Feature Detectors

• How to find image locations that can be reliably matched with images?
Feature Detectors

(a) Corner

(b) Edge

(c) Textureless region
Harris Corner Detector

\[ f(\Delta x, \Delta y) \approx \sum_{x,y} w(x, y) (I_x(x, y) \Delta x + I_y(x, y) \Delta y)^2 \]

\[ f(\Delta x, \Delta y) \approx (\Delta x \quad \Delta y) M \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix} \]

\[ M = \sum_{x,y} w(x, y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} = \begin{bmatrix} \sum_{x,y} w(x, y) I_x^2 & \sum_{x,y} w(x, y) I_x I_y \\ \sum_{x,y} w(x, y) I_x I_y & \sum_{x,y} w(x, y) I_y^2 \end{bmatrix} \]
Invariance

• Can the same feature point be detected after some transformation?
  • Translation invariance
    Are Harris corners translation invariance?
  • 2D rotation invariance
    Are Harris corners rotation invariance?
  • Scale invariance
    Are Harris corners scale invariance?

No
Scale Invariance

• Solution 1: detection features in all scales, matching features in corresponding scale (for small scale change)

Multi-scale oriented patches (MOPS) extracted at five pyramid levels (Brown, Szeliski, and Winder 2005)
Scale Invariance

• Solution 2: detect features that are stable in both location and scale

Intuition: Find local maxima in both position and scale

What filter can we use for scale selection?
Recall Derivative Filter

Central difference

\[ f'(x) = \lim_{{h \to 0}} \frac{f(x + 0.5h) - f(x - 0.5h)}{h} \]

| -1 | 0 | 1 |

X derivative

Find edge
Image Gradient

Gradient in $x$ only
\[ \nabla f = \left[ \frac{\partial f}{\partial x}, 0 \right] \]

Gradient in $y$ only
\[ \nabla f = \left[ 0, \frac{\partial f}{\partial y} \right] \]

Gradient in both $x$ and $y$
\[ \nabla f = \left[ \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right] \]

Gradient direction
\[ \theta = \tan^{-1} \left( \frac{\frac{\partial f}{\partial y}}{\frac{\partial f}{\partial x}} \right) \]

Gradient magnitude
\[ ||\nabla f|| = \sqrt{\left( \frac{\partial f}{\partial x} \right)^2 + \left( \frac{\partial f}{\partial y} \right)^2} \]
Signal Noises

• Derivative filters are sensitive to noises

Intensity plot

Derivative plot

How to deal with noises?
Gaussian Filter

• Smoothing

1D
\[ g(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{x^2}{2\sigma^2}} \]

2D
\[ g(x, y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}} \]

Image \( f \)

Gaussian Filter \( h \)

Convolution \( h \ast f \)

Derivative \( \frac{\partial}{\partial x} (h \ast f) \)

Peak = edge location
Derivative of Gaussian Filter

- Convolution is associative

\[
\frac{\partial}{\partial x} (h \ast f) = \left( \frac{\partial}{\partial x} h \right) \ast f
\]

Smoothing and derivative

\[
\frac{\partial}{\partial x} h
\]

\[
\left( \frac{\partial}{\partial x} h \right) \ast f
\]
Derivative of Gaussian Filter

• Convolution is associative
  \[
  \frac{\partial}{\partial x}(h \ast f) = \left( \frac{\partial}{\partial x} h \right) \ast f
  \]

  \[
  g_x(x,y) = \frac{\partial g(x,y)}{\partial x} = -x \frac{x^2+y^2}{2\pi\sigma^4 e^{\frac{x^2+y^2}{2\sigma^2}}}
  \]

  \[
  g_y(x,y) = \frac{\partial g(x,y)}{\partial y} = -y \frac{x^2+y^2}{2\pi\sigma^4 e^{\frac{x^2+y^2}{2\sigma^2}}}
  \]

Gaussian

\[
g(x,y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}
\]
Laplace Filter

first-order finite difference

\[ f'(x) = \lim_{h \to 0} \frac{f(x + 0.5h) - f(x - 0.5h)}{h} \]

Derivative filter

\[-1 \quad 0 \quad 1\]

second-order finite difference

\[ f''(x) \approx \frac{\delta^2 h[f](x)}{h^2} = \frac{f(x+h) - 2f(x) + f(x-h)}{h^2} \]

Laplace filter

\[ 1 \quad -2 \quad 1 \]
Laplace Filter

- 2D

$$\nabla^2 I = \frac{\partial^2 I}{\partial x^2} + \frac{\partial^2 I}{\partial y^2}$$

1D Laplace filter

2D Laplace filter
Laplacian of Gaussian Filter

\[ \nabla^2 I = \frac{\partial^2 I}{\partial x^2} + \frac{\partial^2 I}{\partial y^2} \]

\[ \nabla^2 I \circ g = \nabla^2 g \circ I \]

\[ \nabla^2 g = \frac{x^2 + y^2 - 2\sigma^2}{\sigma^4} g(x, y) \]

Smoothing and second derivative
Laplacian of Gaussian Filter

\[ f \]

\[ \frac{\partial}{\partial x} h \]

Derivative of Gaussian

\[ \left( \frac{\partial}{\partial x} h \right) \ast f \]

\[ \frac{\partial^2}{\partial x^2} h \]

Laplacian of Gaussian

\[ \left( \frac{\partial^2}{\partial x^2} h \right) \ast f \]

Zero crossings
Laplacian of Gaussian for Scale Selection

Laplacian filter

Highest response when the signal has the same **characteristic scale** as the filter
Laplacian of Gaussian for Scale Selection

characteristic scale
Search over different scales $\sigma$
Laplacian of Gaussian for Scale Selection

Multi-scale 2D Blob detection
Laplacian of Gaussian for Scale Selection

cross-scale maximum

local maximum

local maximum

local maximum
Scale Invariance Feature Transform (SIFT)

- Keypoint detection
- Compute descriptors
- Matching descriptors

David Lowe, Distinctive Image Features from Scale-Invariant Keypoints. IJCV, 2004
SIFT: Scale-space Extrema Detection

• Difference of Gaussian (DoG)

\[ G(x, y, \sigma) = \frac{1}{2\pi\sigma^2} e^{-(x^2+y^2)/2\sigma^2} \]

\[ L(x, y, \sigma) = G(x, y, \sigma) \ast I(x, y) \]

\[ D(x, y, \sigma) = (G(x, y, k\sigma) - G(x, y, \sigma)) \ast I(x, y) = L(x, y, k\sigma) - L(x, y, \sigma). \]

Approximate of Laplacian of Gaussian (efficient to compute)
SIFT: Scale-space Extrema Detection

• Gaussian pyramid

• Gaussian filters

\[ L(x, y, \sigma) = G(x, y, \sigma) * I(x, y) \]

\[ G_{\sigma_1} * G_{\sigma_2} = G_{\sigma} \quad \sigma^2 = \sigma_1^2 + \sigma_2^2 \]

• Sub-sampling by a factor of 2
  • Multiple the Gaussian kernel deviation by 2
SIFT: Scale-space Extrema Detection

Maxima and minima of DoG images

\[ L(x, y, \sigma) = G(x, y, \sigma) \ast I(x, y) \]
\[ G(x, y, \sigma) = \frac{1}{2\pi \sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}} \]
\[ D(x, y, \sigma) = (G(x, y, k\sigma) - G(x, y, \sigma)) \ast I(x, y) \]
\[ = L(x, y, k\sigma) - L(x, y, \sigma). \]
SIFT Descriptor

• Image gradient magnitude and orientation

\[ L(x, y, \sigma) = G(x, y, \sigma) \ast I(x, y) \]

\[ m(x, y) = \sqrt{(L(x + 1, y) - L(x - 1, y))^2 + (L(x, y + 1) - L(x, y - 1))^2} \]

\[ \theta(x, y) = \tan^{-1}((L(x, y + 1) - L(x, y - 1))/(L(x + 1, y) - L(x - 1, y))) \]
SIFT Descriptor

- Divide the 16x16 window into a 4x4 grid of cells (2x2 case shown below)
- Compute an orientation histogram for each cell
- 16 cells * 8 orientations = 128 dimensional descriptor

Using the scale of the keypoint to select the level of Gaussian blur for the image
SIFT: Rotation Invariance

• Rotate all orientations by the dominant orientation
SIFT: Rotation Invariance

• Rotate all orientations by the dominant orientation
SIFT Properties

• Can handle change in viewpoint (up to about 60 degree out of plane rotation)

• Can handle significant change in illumination

• Relatively fast < 1s for moderate image sizes

• Lots of code available
  • E.g., https://www.vlfeat.org/overview/sift.html
SIFT Matching Example

https://www.vlfeat.org/overview/sift.html
SIFT Matching Example
Further Reading

• Section 7.1, Computer Vision, Richard Szeliski

• David Lowe, Distinctive Image Features from Scale-Invariant Keypoints. IJCV, 2004

• ORB: An efficient alternative to SIFT or SURF. Rublee et al., ICCV, 2011