Supplementary Material for the Paper "Semantic Context Modeling with Maximal Margin Conditional Random Fields for Automatic Image Annotation"

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1. Proof of Proposition 1

Proposition 1. If $m \le \sum_{i\in\mathcal{S},\bar{y}_i^t=-y_i^t} m_i + \sum_{i\in\mathcal{S},j\in\mathcal{N}_i,\bar{y}_i^t\bar{y}_j^t=-y_i^t y_j^t} m_{ij}$, then $L'_{\text{hinge}}(\mathbf{x}^t,\mathbf{y}^t,\mathbf{w},\mathbf{b})$ is an upper bound of $L_{\text{hinge}}(\mathbf{x}^t,\mathbf{y}^t,\mathbf{w},\mathbf{b})$.

Proof. Let

$$\begin{aligned} D &= L'_{\text{hinge}}(\mathbf{x}^t,\mathbf{y}^t,\mathbf{w},\mathbf{b}) - L_{\text{hinge}}(\mathbf{x}^t,\mathbf{y}^t,\mathbf{w},\mathbf{b}) \qquad (1)\\ &= \sum_{i\in\mathcal{S}} \max\Big(0, m_i - 2y_i^t(\mathbf{w}_i^T\phi_i(\mathbf{x}^t)+b_i)\Big) + \sum_{i\in\mathcal{S}}\sum_{j\in\mathcal{N}_i} \max\Big(0, m_{ij} - 2y_i^t y_j^t \mathbf{w}_{ij}^T\phi_j(\mathbf{x}^t)\Big)\\ &\quad - \max\Big(0, m + \sum_{i\in\mathcal{S}}(\bar{y}_i^t - y_i^t)(\mathbf{w}_i^T\phi_i(\mathbf{x}^t)+b_i) + \sum_{i\in\mathcal{S}}\sum_{j\in\mathcal{N}_i}(\bar{y}_i^t\bar{y}_j^t - y_i^t y_j^t)\mathbf{w}_{ij}^T\phi_j(\mathbf{x}^t)\Big). \qquad (2)\end{aligned}$$

Case 1: If

$$m + \sum_{i\in\mathcal{S}}(\bar{y}_i^t - y_i^t)(\mathbf{w}_i^T\phi_i(\mathbf{x}^t)+b_i) + \sum_{i\in\mathcal{S}}\sum_{j\in\mathcal{N}_i}(\bar{y}_i^t\bar{y}_j^t - y_i^t y_j^t)\mathbf{w}_{ij}^T\phi_j(\mathbf{x}^t) \le 0, \qquad (3)$$

then

$$D = L'_{\text{hinge}}(\mathbf{x}^t,\mathbf{y}^t,\mathbf{w},\mathbf{b}) \ge 0. \qquad (4)$$

Case 2: If

$$m + \sum_{i\in\mathcal{S}}(\bar{y}_i^t - y_i^t)(\mathbf{w}_i^T\phi_i(\mathbf{x}^t)+b_i) + \sum_{i\in\mathcal{S}}\sum_{j\in\mathcal{N}_i}(\bar{y}_i^t\bar{y}_j^t - y_i^t y_j^t)\mathbf{w}_{ij}^T\phi_j(\mathbf{x}^t) > 0, \qquad (5)$$

then

$$\begin{aligned} D = -m &+ \sum_{i\in\mathcal{S}}\Big[\max\Big(0, m_i - 2y_i^t(\mathbf{w}_i^T\phi_i(\mathbf{x}^t)+b_i)\Big) - (\bar{y}_i^t - y_i^t)(\mathbf{w}_i^T\phi_i(\mathbf{x}^t)+b_i)\Big]\\ &+ \sum_{i\in\mathcal{S}}\sum_{j\in\mathcal{N}_i}\Big[\max\Big(0, m_{ij} - 2y_i^t y_j^t\mathbf{w}_{ij}^T\phi_j(\mathbf{x}^t)\Big) - (\bar{y}_i^t\bar{y}_j^t - y_i^t y_j^t)\mathbf{w}_{ij}^T\phi_j(\mathbf{x}^t)\Big]. \qquad (6)\end{aligned}$$

Let

$$\begin{aligned} D_i &= \max\Big(0, m_i - 2y_i^t(\mathbf{w}_i^T\phi_i(\mathbf{x}^t)+b_i)\Big) - (\bar{y}_i^t - y_i^t)(\mathbf{w}_i^T\phi_i(\mathbf{x}^t)+b_i), \qquad (7)\\ D_{ij} &= \max\Big(0, m_{ij} - 2y_i^t y_j^t\mathbf{w}_{ij}^T\phi_j(\mathbf{x}^t)\Big) - (\bar{y}_i^t\bar{y}_j^t - y_i^t y_j^t)\mathbf{w}_{ij}^T\phi_j(\mathbf{x}^t), \qquad (8)\end{aligned}$$

then

$$D = -m + \sum_{i\in\mathcal{S}} D_i + \sum_{i\in\mathcal{S}}\sum_{j\in\mathcal{N}_i} D_{ij}. \qquad (9)$$

Consider D_i in two cases:
Case 2.1: If

$$m_i - 2y_i^t(\mathbf{w}_i^T\phi_i(\mathbf{x}^t) + b_i) \leq 0, \tag{10}$$

then

$$\begin{aligned} D_i &= -(\bar{y}_i^t - y_i^t)(\mathbf{w}_i^T\phi_i(\mathbf{x}^t) + b_i) \\ &= \begin{cases} 0 & \text{if } \bar{y}_i^t = y_i^t \\ 2y_i^t(\mathbf{w}_i^T\phi_i(\mathbf{x}^t) + b_i) \geq m_i & \text{if } \bar{y}_i^t = -y_i^t \end{cases} . \end{aligned} \tag{11}$$

Case 2.2: If

$$m_i - 2y_i^t(\mathbf{w}_i^T\phi_i(\mathbf{x}^t) + b_i) > 0, \tag{12}$$

then

$$\begin{aligned} D_i &= m_i - (\bar{y}_i^t + y_i^t)(\mathbf{w}_i^T\phi_i(\mathbf{x}^t) + b_i) \\ &= \begin{cases} m_i - 2y_i^t(\mathbf{w}_i^T\phi_i(\mathbf{x}^t) + b_i) > 0 & \text{if } \bar{y}_i^t = y_i^t \\ m_i & \text{if } \bar{y}_i^t = -y_i^t \end{cases} . \end{aligned} \tag{13}$$

By combining case 2.1 and 2.2, we can get

$$D_i \begin{cases} \geq 0 & \text{if } \bar{y}_i^t = y_i^t \\ \geq m_i & \text{if } \bar{y}_i^t = -y_i^t \end{cases} . \tag{14}$$

Similarly, we can get

$$D_{ij} \begin{cases} \geq 0 & \text{if } \bar{y}_i^t\bar{y}_j^t = y_i^t y_j^t \\ \geq m_{ij} & \text{if } \bar{y}_i^t\bar{y}_j^t = -y_i^t y_j^t \end{cases} . \tag{15}$$

Now, we have

$$\begin{aligned} D &= -m + \sum_{i\in\mathcal{S},\bar{y}_i^t=y_i^t} D_i + \sum_{i\in\mathcal{S},\bar{y}_i^t=-y_i^t} D_i + \sum_{i\in\mathcal{S},j\in\mathcal{N}_i,\bar{y}_i^t\bar{y}_j^t=y_i^t y_j^t} D_{ij} + \sum_{i\in\mathcal{S},j\in\mathcal{N}_i,\bar{y}_i^t\bar{y}_j^t=-y_i^t y_j^t} D_{ij} \\ &\geq -m + \sum_{i\in\mathcal{S},\bar{y}_i^t=-y_i^t} m_i + \sum_{i\in\mathcal{S},j\in\mathcal{N}_i,\bar{y}_i^t\bar{y}_j^t=-y_i^t y_j^t} m_{ij} \\ &\geq 0. \end{aligned} \tag{16}$$

By combining Case 1 and 2, we have $D \geq 0$, which proves the proposition. □